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BIOMETRIKA.

RESULTS OF CROSSING GREY (HOUSE) MICE WITH ALBINOS.

BY E. H. J. SCHUSTER, M.A., New College, Oxford.

THE experiments here recorded were commenced in the department of comparative anatomy in Oxford in the autumn of 1902, but various personal circumstances have delayed the publication of any account of them for more than a year after their completion.

They were made at the suggestion and with the advice and help of Professor Weldon, and I have therefore numerous reasons for thanking him.

Source of Material.

The original stock of white mice was obtained from various dealers, among whom Jeunet, of Paris, figures most largely. Some also were given to me by Mr Darbishire.

The grey mice were caught in numerous houses in Oxford, and were brought to me by their captors in answer to an advertisement in *The Oxford Times*. In order to avoid, as much as possible, any possibility of using grey mice with "fancy" blood in their veins, those caught in the precincts of the Museum, where Mr Darbishire's experiments had been going on for some months, were in all cases rejected.

Previous Experiments of a Similar Nature.

Breeding experiments similar to mine have been made by Cuénot ('02 and '03) and Allen ('04), and the fact that my results do not seem to be in exact accordance with theirs must serve for an apology for the appearance of an otherwise superfluous publication. That such differences should have arisen at first seemed surprising; but an ingenious theory of Cuénot's explains the possibility of their occurrence, and I should like, before going on to the consideration of results, to give a short account of this theory.

Cuénot ('04) suggests that all mice, including albinos, have some definite colour in them, but that it needs a special faculty to make this latent colour apparent on their coats, and that the difference between all coloured mice and all albino mice lies in the possession or non-possession of this faculty.

Coat-colour, then, is a compound character which must be split up primarily into two component parts: (1) nature of the colour; (2) power, or lack of power of producing colour. He represents the constitution of the gametes of a mouse with regard to these two things by two letters, the first indicating their condition with regard to (2). When they are colour-producing the letter *C* (chromogène) is used, when non-colour-producing *A* (albino). The second letter denotes the nature of the colour. Thus *G* grey, *B* black, and *Y* yellow.

For example:

CG represents an ordinary house mouse.

AG represents an albino mouse whose gametes would produce the grey colour if possessed of the colour-producing faculty.

CB = a pure black mouse.

AB a mouse similar to *AG*, only with Black for the latent colour.

CY a pure yellow mouse.

AY albino with yellow latent.

The six symbols described above denote six pure races of mice. When any two of these are mated together hybrids will of course result whose gamete formula is a combination of the gamete formula of the two parents. The colour of the hybrids is determined by the relative dominance: (1) of colour productiveness and albinism; (2) of the nature of the two colours latent in the two parental gametes. As in all cases colour dominates albinism, it only remains to be considered which of the three colours employed is dominant over which of the others. Here we find *G* dominant over *B*, *Y* dominant over *B*, *Y* dominant over *G*. Thus a grey *CG* crossed with a white *AY* will produce yellow mice whose formula is *CG-AY*. A black *CB* crossed with either a grey *CG* or a white *AG* will produce grey mice, though in the first case the formula will be *CB-CG*, and in the second it will be *CB-AG*.

All this is not a modification nor an extension of Mendel's laws, but merely an application of them to a special case; Cuénot's theory as to the two factors which determine coat colour renders such an application possible. The further behaviour of the hybrids when crossed among themselves or with pure mice can therefore be predicted by any one conversant with these laws and possessed of a knowledge of the gametic formula of the hybrids used.

A third element is introduced by the fact that the colour of a mouse's coat is not always uniform, but may be marked with white. This marking may occur with mice of any colour; it is transmitted independently of the colour and may be latent in albinos. A uniform coat dominates a piebald coat.

Thus if one mates a piebald yellow mouse with an albino with uniform greyness

latent in it one will obtain a uniform yellow hybrid, for yellow dominates grey, uniformity piebaldness, and colour-productiveness albinism. If such yellow hybrids are paired together and a sufficient number of young obtained they should be in the following proportion: 16 albinos, 9 piebald yellow, 27 uniform yellow, 3 piebald grey, and 9 uniform grey: see Appendix II, p. 12. Eight of these 64 mice would be pure, all of different constitutions, and there would be two of each of 12 different kinds of hybrids, four of 6 different kinds, and eight of one kind. Thus a reasonable degree of complication is obtained when only two colours are employed; when more than two colours are introduced the complication is largely increased. I have brought forward this example to explain my reasons for not examining in detail the occurrence of each of the various types of mice produced in my experiments. For in addition to the inherent difficulties of such an examination, I am in total ignorance as to colours latent in the original stock of white mice.

Types of mice produced.

Bateson ('03) gives a list of a number of types of Fancy mice with a statement of the colour of the actual pigments contained in each, and almost all the mice produced in these experiments can be referred to one or other of the groups in this list; I cannot, however, guarantee that I have referred them rightly, as no microscopic examination of them has been made, so that the colour as it appears to the naked eye is the only basis of the classification.

After the name of each colour is given the abbreviation employed as a symbol for it. In giving instances of Cuénot's theory, for the sake of uniformity I have used my own abbreviations and not those employed in his original paper.

(1) *Grey* (*G*) (Cinnamon or Agouti) figure 4.

(2) *Golden agouti* (*GA*).

(3) *Yellow* (*Y*). In this are included both yellow and cream, as these shade insensibly into one another and form indeed a highly variable group of colours.

Figures 5, 7, 8 represent three conditions of yellowness, but the lightest yellow of all obtained was almost completely white, with but the faintest suggestion of pigment. It had, however, perfectly black eyes like all the coloured mice produced.

(4) *Sooty Yellow* (*SY*).

(5) *Chocolate* (*Ch.*) figure 6.

(6) *Chinchilla* (*Cc.*) figure 2. I cannot find this anywhere on Bateson's list. In the best specimens it is a beautiful silvery grey in colour but it shades insensibly into the ordinary grey coat, figure 3 being one of the intermediate forms. Allen ('04) records having obtained some light grey mice in crossing house mice with albinos, these may possibly have been of the same nature as my Chinchillas. His apparently died in childhood so that he had no opportunity of breeding from them.

Intermediate forms between pairs of these colours did occur and are represented by the symbols of the two colours joined by a hyphen. Thus figure 1 represents

a mouse intermediate in colour between a black and a grey, and would therefore be denoted by *B-G*.

Each of these colours occurred both uniform and marked with white. The white marking, as Cuénot observes, is subject to continuous variation. The parts first affected are the tail and the paws; then a small white mark may be found on the belly and on the top of the head. The white mark on the belly may grow forwards and backwards, so as to cover the whole ventral surface; laterally it sends out arms of whiteness immediately in front of the hind legs, these grow dorsalwards and may meet and form a complete white belt. The mark in the head spreads forwards till it forms a good white blaze on the forehead, or further still till the front of the muzzle is completely white, and eventually it becomes continuous with the ventral white mark.

The letter *P* in conjunction with the symbol for a colour means that the mouse is piebald and of this colour; no attempt is made to indicate the degree to which the white marking has spread, though of course a record of this has been kept.

Results.

106 white mice were mated with grey mice and of these crosses 74 proved fertile.

70 families contained only grey mice. Of the remaining four 2 contained mice of varying shades of yellow in addition to grey, and the original chinchillas appeared in the other two. Thus the first generation of hybrids consisted of 342 mice, of which 329 were grey, seven yellow, and six chinchilla.

The appearance of yellow in this generation is easily understood, if we assume, in the absence of any definite information on the subject, that the albino parents in each case were hybrids containing yellow and some other colour latent. Such mice would according to Cuénot's hypothesis produce some yellow and some grey when mated with pure grey mice, as yellow is dominant over grey and grey over any other colour which they might contain.

Having no sort of evidence either as to the nature of the chinchilla coat, or as to its behaviour when pure and mated with other colours, it would be futile to indulge in speculation on the possible reasons for its occurrence here.

The yellow mice when mated together produced yellow and grey and white. Thirteen young from four families were obtained, and of these five were grey, six yellow and two white. These numbers are of course too small for the proportions to have any significance, but the theoretical expectation for sixteen mice would be four albino, nine yellow and three grey.

From the original seven yellow mice all the yellows and sooty yellows occurring in the experiments were produced.

The original pairs are labelled *A* and *B* in the Appendix, the *A*s having the ♂ white and the *B*s the ♀.

The offspring of the *A* and *B* crosses were mated with white mice and with one another.

In the former case the crosses produced are labelled *C* and *D*, *C* having the ♂ a product of an *A* or *B* cross and the ♀ an albino, and *D* being the reciprocal of this.

The *C*s produced 282 young of which 130 were albino and 152 coloured, the *D*s produced 255 young of which 131 were albino and 124 coloured. Thus together we have 537 young of which 261 are albinos and 276 coloured. Considering merely the characters, colour productiveness and albinism, the Mendelian expectation would be 268·5 of each, so that the actual result obtained is in fairly close agreement with it.

If we go on to an enumeration of the actual colours obtained, we find among the *C*s 25 black, 102 grey, 2 intermediate between grey and chinchilla, 2 golden agouti, 16 grey marked with white, and 7 black marked with white.

Among the *D*s we have 26 black, 77 grey, 1 golden agouti, 1 yellow, 1 sooty yellow, 1 chocolate, 12 grey marked with white and 7 black marked with white.

Speculation as to the proportion of the various colours seems to be useless, but if one assumes that some of the albinos employed both in the *A* and *B* and in the *C* and *D* crosses had any other characters but uniformity and greyness latent in them, then one would expect that these characters would become apparent in the coats of some of the mice produced.

This remark applies with equal force to the consideration of the results of the various other crosses described below, but it will not be repeated again.

The offspring of the *A* and *B* crosses were paired together and these pairs are labelled *E*. The *E*s produced 119 albinos, 27 blacks, 17 intermediate between black and grey, 201 greys, 9 intermediate between chinchilla and grey, 9 chinchilla, 8 golden agouti, 9 yellow, 3 sooty yellow, 1 chocolate, 19 grey marked with white, 3 black marked with white, 1 chocolate marked with white, and 1 chinchilla marked with white. That is to say 119 albinos and 308 coloured. 119 is at any rate a possible approximation to the Mendelian quarter. The 308 coloured mice should be composed of $\frac{1}{3}$ pure colour producers and $\frac{2}{3}$ hybrids between albinos and colour producers.

The *F* and *G* crosses were made to test the truth of this. The *F*s have coloured offspring of the *E*s for ♀s and albinos for ♂s. The *G*s are reciprocal. Out of 87 of these crosses 32 produced no albino young and 55 produced some albino young. 32 is a fair approximation to $\frac{1}{3}$ of 87.

The colours of the young contained in these 32 families is as follows: 18 black, 1 intermediate between black and grey, 95 grey, 2 intermediate between chinchilla and grey, 1 yellow, 8 sooty yellow, 1 golden agouti, 6 chocolate, 20 grey marked with white, 13 black marked with white.

The 55 families produced 141 albinos and 137 coloured mice. If these are to be regarded as hybrids equal numbers of each should be expected; and a fair degree of equality is obtained. If we add these numbers to the results of the *C* and *D* crosses with which they are to be associated, we get out of 815 mice 402 albino and 413 coloured.

The actual colours obtained in the 55 *F* and *G* families are as follows:—23 black, 74 grey, 2 golden agouti, 1 chinchilla, 1 yellow, 6 sooty yellow, 2 chocolate, 1 yellow marked with white, 24 grey marked with white and 3 black marked with white.

Cross *L* was made by breeding together those coloured mice which had been used in the *F*s and *G*s and which had produced only coloured young. Their offspring were all coloured, consisting of 57 grey, 3 grey marked with white, 3 black and 5 sooty yellow.

In cross *M* the remaining coloured mice which had been used for the *F*s and *G*s were mated together and produced 30 albinos and 71 coloured mice, the colours being as follows:—52 grey, 5 grey marked with white, 7 black, 1 golden agouti, 6 sooty yellow.

The white offspring of the *E*s were also mated together, and in all cases bred true.

In conclusion, it is hardly necessary to point out that with regard to characters, colour-productiveness and albinism the mice under consideration here behave in complete accordance with Mendel's laws, both with regard to dominance and to segregation. If one considers the nature of the colour and the question as to whether it is distributed uniformly or is marked with white there is nothing in my results incompatible with similar behaviour with regard to these characters also.

Certain minor difficulties do indeed suggest themselves, for instance the occurrence of intermediate forms and the tendency of yellow mice when crossed either with white or grey hybrids to produce sooty yellow young; this colour figuring somewhat conspicuously in some of the latter generations.

It is only to be regretted that the true nature of the albinos was not known and it is to be hoped that in the future the experiments may be repeated without this very serious drawback.

LIST OF LITERATURE REFERRED TO.

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'03. ——. Ibid. 1903.

'04. ——. Ibid. 1904.

'03. W. Bateson. "Colour Heredity in Fancy Mice and Rats." P.Z.S. 1903, Vol. II.

'04. Glover M. Allen. "The Heredity of Coat-Colour in Mice." Proc. Amer. Acad. Art. and Sci. Vol. XL.

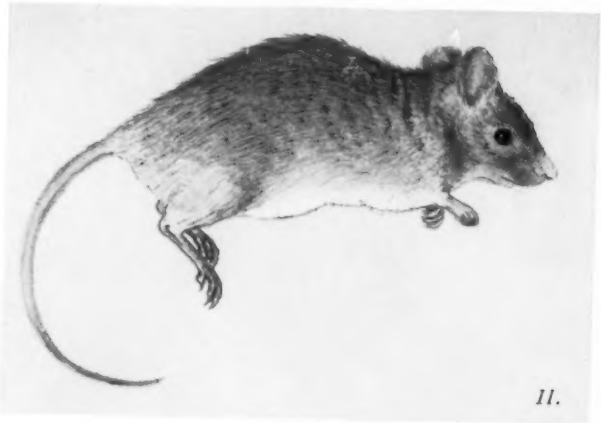
DESCRIPTION OF FIGURES.

These are taken from water-colour drawings ad nat. by Miss Adeline Ritchie.

(1) Intermediate between grey and black.
(2) Chinchilla.
(3) Intermediate between chinchilla and grey.
(4) Grey.
(5), (7), (8) Three shades of yellow.
(6) Chocolate.

PLATE I.

I.



II.

III.

IV.

III.

IV.

V.

VI.

VII.

VIII.

COAT-COLOURS IN MICE.

APPENDIX I.

A List of all the Mice Produced.

The following abbreviations are used in describing the mice:—

A, Albino. *Cc.* Chinchilla. *SY*, Sooty Yellow.
G, Grey. *Ch.* Chocolate.
Y, Yellow (including Cream). *GA*, Golden Agouti.

Intermediate colours are represented by the symbols of the colours between which they lie placed together.

P joined to the symbol for a colour means that the mouse was marked with white.

Crosses *A* and *B*. *A*, Father Albino, Mother Grey. *B*, Mother Albino, Father Grey. 70 *A* and *B* pairs produced only *G* mice.

Family. *A* 32, 3*Y* and 2*G*.
B 17, 4*Cc*.
B 26, 4*Y*, 2*G*.
B 43, 2*Cc*., 2*G*.

Cross C. Products of A and B Crosses mated to Albino Does.

	Results		Results
C 1	2*A*, 1*B*, 1*G*	*C 41*	3*A*, 1*G*
C 2	3*A*, 2*B*	*C 42*	2*A*, 2*G*
C 3	1*A*, 1*GA*, 1*BP*	*C 43*	2*A*
C 4	3*A*, 2*B*, 2*GP*	*C 45*	2*A*, 3*GP*
C 5	2*A*, 2*G*	*C 46*	3*A*, 1*BP*
C 6	4*A*, 1*G*	*C 48*	3*A*, 1*B*, 1*G*
C 7	3*A*, 3*B*	*C 49*	2*A*, 1*G*
C 8	1*G*, 3*G*	*C 47*	1*A*, 1*G*
C 9	3*A*, 2*B*, 1*G*	*C 50*	2*A*, 2*G*, 1*GP*
C 10	6*G*	*C 51*	3*A*, 1*B*
C 13	2*A*, 2*B*, 1*G*, 1*BP*	*C 52*	2*A*, 2*B*
C 12	1*A*, 5*G*	*C 53*	3*G*
C 14	1*A*, 5*G*	*C 54*	4*A*, 4*G*
C 15	2*A*, 1*B*, 1*G*, 1*GP*	*C 57*	2*A*, 1*B*
C 16	2*A*, 1*G*	*C 59*	3*A*, 3*B*
C 19	5*G*	*C 60*	1*A*
C 20	5*G*	*C 63*	4*A*, 2*G*, 2*GP*
C 21	3*A*, 4*G*	*C 62*	3*A*, 2*G*, 2*GP*, 1*BP*
C 22	2*A*, 2*G*, 2*Ch. G*	*C 61*	3*A*, 4*G*, 2*GP*
C 23	5*A*	*C 64*	1*A*, 2*G*, 1*GP*
C 25	2*A*, 2*B*, 2*G*	*C 65*	2*A*, 2*B*
C 26	1*A*, 5*G*	*C 66*	2*A*, 2*G*
C 27	3*A*, 2*G*	*C 67*	1*A*, 3*G*
C 28	1*A*, 2*BP*	*C 68*	4*A*, 1*G*
C 29	4*A*, 1*G*	*C 69*	3*A*
C 30	3*A*, 1*B*, 3*G*, 1*GP*	*C 71*	2*A*, 2*G*
C 32	3*A*	*C 72*	1*G*
C 37	2*A*, 1*B*, 3*G*	*C 73*	4*A*, 1*B*, 1*G*
C 40	3*A*, 3*G*, 1*GP*	*C 74*	1*A*, 1*G*, 1*GA*
C 39	3*A*, 1*G*		
Total of *Cs*	130*A*, 25*B*, 102*G*, 2*Ch. G*, 2*GA*, 16*GP*, 7*BP*		

Cross D. Products of A and B Crosses mated with Albino Bucks.

	Results		Results
D 2	3*G*, 2*G*	*D 33*	2*A*, 2*G*, 1*GA*
D 3	1*A*, 1*GP*	*D 35*	2*A*, 1*B*
D 4	1*A*, 1*G*, 1*GP*, 1*BP*	*D 37*	2*A*, 2*G*
D 6	2*A*, 1*G*	*D 38*	2*A*, 1*B*, 2*G*
D 7	4*A*, 1*B*	*D 40*	3*A*, 1*G*
D 8	1*A*, 4*G*	*D 41*	4*A*, 2*G*
D 9	2*A*, 4*G*, 1*GP*, 1*BP*	*D 42*	3*A*, 1*B*, 2*G*
D 12	2*A*, 1*B*	*D 44*	2*A*, 2*G*, 1*GP*
D 13	2*A*, 3*G*, 1*GP*	*D 45*	2*A*, 1*GP*
D 14	2*A*, 1*B*, 2*G*, 1*BP*	*D 46*	3*G*
D 15	2*A*, 2*G*, 1*BP*	*D 47*	2*A*, 4*B*, 1*G*
D 16	1*A*, 2*B*, 2*GP*	*D 48*	2*A*
D 17	3*A*, 2*B*	*D 52*	3*A*, 2*BP*
D 18	2*A*, 1*B*, 2*G*	*D 50*	2*A*, 3*G*
D 19	2*A*, 2*G*	*D 53*	2*A*, 1*G*, 1*GP*
D 20	5*A*, 1*G*	*D 54*	1*A*, 1*G*, 1*GP*
D 21	3*A*, 1*G*	*D 55*	5*A*, 1*G*
D 22	2*A*, 3*G*	*D 57*	4*A*, 1*B*, 1*BP*
D 23	3*A*	*D 58*	2*A*, 1*B*, 1*G*
D 24	2*A*, 3*G*	*D 51*	2*A*, 1*B*
D 26	1*A*, 3*G*	*D 59*	4*A*
D 27	3*A*, 1*B*, 1*G*	*D 60*	3*A*, 1*Y-G*
D 28	3*A*, 1*B*, 1*G*	*D 61*	2*A*, 1*G*, 1*Y*, 1*GP*
D 29	1*B*, 3*G*	*D 62*	2*A*, 1*G*
D 30	3*A*, 3*G*	*D 64*	3*A*, 1*G*
D 32	4*A*, 3*B*	*D 65*	1*A*, 1*B*
D 66	3*A*, 1*Ch.*	*D 67*	1*A*, 1*B*, 1*GP*, 1*G*
		D 68	4*A*
Total of *Ds*	131*A*, 26*B*, 7*BP*, 77*G*, 12*GP*, 1*GA*, 1*Y*, 1*SY*, 1*Ch.*		
Total of *Cs* and *Ds*	261*A*, 51*B*, 14*BP*, 179*G*, 28*GP*, 2*Cc-G*, 3*GA*, 1*Y*, 1*SY*, 1*Ch.*		

Cross E. Products of A and B Crosses mated with One another.

	Results		Results
E 2	1*A*, 1*B*, 2*G*	*E 61*	1*A*, 2*G*, 1*GA*
E 3	2*A*, 2*G*	*E 62*	1*B*, 2*G*, 2*GA*, 1*GP*
E 5	2*B*, 2*G*	*E 63*	3*A*, 1*B*, 2*G*
E 8	2*G*	*E 64*	1*A*, 2*B*, 2*G*
E 9	2*A*, 1*G*	*E 66*	1*B*, 3*G*
E 10	1*A*, 4*G*	*E 67*	1*A*, 3*G*
E 12	1*A*, 2*B*, 2*G*	*E 68*	2*A*, 2*G*
E 11	4*G*	*E 69*	1*A*, 3*G*
E 14	1*A*, 2*B*, 1*G*	*E 70*	1*A*, 4*G*
E 15	4*G*	*E 71*	1*A*, 2*B*, 4*G*
E 16	4*G*	*E 72*	1*A*, 1*B*, 2*G*
E 17	1*A*, 4*G*	*E 73*	1*A*, 1*CcG*
E 18	3*A*, 3*G*	*E 74*	2*G*, 1*GP*, 2*CcP*
E 19	1*A*, 1*B*, 1*G*	*E 75*	1*A*, 1*CcG*, 1*Cc*
E 20	1*A*, 3*G*, 2*GP*	*E 76*	1*A*, 3*G*
E 21	2*A*, 2*G*, 1*GP*	*E 77*	2*A*, 3*G*
E 22	3*A*	*E 78*	1*A*, 4*G*
E 23	3*G*	*E 79*	3*A*, 2*G*
E 24	2*A*, 2*G*, 1*GP*	*E 80*	1*A*, 3*G*, 1*GP*
E 25	3*A*, 2*G*	*E 82*	1*A*, 3*G*
E 26	1*A*, 2*G*	*E 83*	2*A*, 3*G*
E 27	1*A*, 1*G*	*E 84*	1*A*, 3*B*, 1*GP*
E 28	2*A*, 2*G*	*E 85*	2*A*, 1*G*, 1*Ch.*, 1*GP*
E 29	1*B-G*, 3*G*	*E 86*	2*A*, 2*B-G*, 1*G*
E 30	1*A*, 3*GA*, 1*Ch. P*	*E 87*	2*A*, 3*G*
E 31	1*A*, 3*G*, 1*GA*	*E 88*	1*A*, 2*G*
E 33	3*A*, 1*G*, 2*Y*	*E 89*	2*A*, 3*G*
E 34	1*A*, 4*G*	*E 90*	2*A*, 3*G*
E 35	4*A*, 1*G*	*E 94*	3*B-G*
E 40	1*B*, 3*G*	*E 95*	1*A*, 1*B-G*, 1*GP*
E 41	3*G*	*E 96*	1*A*, 1*GP*
E 42	2*G*, 1*SG*	*E 97*	1*A*, 1*B*, 1*G*
E 44	2*A*, 2*G*	*E 98*	2*A*, 3*G*
E 45	1*A*, 3*G*	*E 99*	1*A*, 3*G*
E 46	2*A*, 2*G*	*E 100*	1*A*, 1*B*, 2*G*
E 47	1*A*, 1*G*, 1*SG*	*E 101*	1*A*, 1*B*, 3*G*
E 48	2*A*, 1*G*, 1*SG*	*E 102*	1*A*, 2*G*
E 49	1*A*, 2*G*	*E 103*	1*A*, 2*G*, 1*GP*
E 50	2*G*, 1*GA*, 1*Y*	*E 104*	3*BG*
E 51	4*G*	*E 108*	5*G*
E 52	1*A*, 1*B*, 1*GP*	*E 105*	5*BG*, 1*G*
E 54	2*A*, 2*GP*	*E 109*	1*BG*, 1*G*, 1*GP*, 2*BP*
*E 54**	1*A*, 1*GP*	*E 110*	2*G*
E 55	3*A*, 1*B*, 1*G*, 1*GP*	*E 111*	1*A*, 2*Y*
E 56	1*A*, 2*G*	*E 112*	1*A*, 3*Cc*
E 57	3*A*, 1*G*	*E 113*	2*G*
E 58	2*A*, 2*GP*	*E 113**	1*A*, 2*CcG*, 1*Cc*
E 59	3*A*, 1*G*	*E 114*	1*A*, 1*G*, 2*Cc*
E 60	5*G*	*E 114**	1*A*, 2*CcG*, 2*Cc*
E 36	3*G*	*E 115*	2*G*, 2*Y*
E 37	2*A*	*E 116*	1*A*, 1*CcG*, 1*Y*
E 38	1*A*, 1*B*, 2*G*	*E 117*	1*G*, 1*Y*
E 39	1*B*, 2*G*		
Total of *Es*	119*A*, 27*B*, 17*B-G*, 201*G*, 9*CcG*, 9*Cc*, 8*GA*, 9*Y*, 3*SY*, 1*Ch.*, 19*GP*, 3*BP*, 1*Ch.P*, 1 *Cc.P*=119 Albino, 308 coloured.		

Cross F. Being coloured ♂ Products of E Crosses mated with Albino ♀s. Those which produced Albinos among their young are taken first.

	Results		Results
F 1	3*A*, 2*G*	*F 23*	3*A*, 1*GP*
F 2	4*A*, 2*G*	*F 24*	1*A*, 4*G*
F 3	2*A*, 2*G*	*F 31*	3*A*, 2*G*
F 4	3*A*	*F 32*	1*A*, 1*B*, 1*GA*
F 5	2*A*, 2*GP*	*F 33*	3*A*
F 6	2*A*, 2*BP*	*F 34*	1*A*, 2*G*
F 10	3*A*, 3*G*	*F 35*	2*A*, 3*G*
F 13	2*A*, 1*GP*, 1*BP*	*F 36*	1*A*, 1*G*
F 14	2*A*	*F 39*	1*A*, 1*Cc*, 2*GP*
*F 14**	4*A*, 1*B*	*F 43*	1*A*, 1*B*, 1*G*, 1*GP*
F 16	4*A*, 1*G*	*F 44*	2*A*, 1*G*
F 22	3*A*		
Total of *F*s containing albinos. 53*A*, 3*B*, 24*G*, 1*Cc*, 1*GA*, 7*GP*, 3*BP*.			

Cross G. Being coloured ♀ Products of E Crosses mated with Albino ♂s. Those which produced Albinos among their Young.

	Results		Results
G 1	4*A*, 1*G*	*G 31*	2*A*, 1*B*, 1*G*
G 2	4*A*, 1*G*, 1*GA*	*G 32*	1*A*, 4*G*
G 3	3*A*, 2*G*	*G 35*	2*A*, 8*GP*
G 6	3*A*, 1*G*, 1*Ch.*	*G 36*	2*A*, 2*B*, 3*G*
G 8	4*A*, 4*B*	*G 37*	4*A*, 2*G*, 1*GP*
G 9	5*A*, 2*G*	*G 38*	2*A*, 2*B*, 2*G*
G 11	4*A*, 1*B*	*G 39*	3*A*, 2*Cc*, 3*GP*
G 12	2*A*, 3*B*	*G 41*	5*A*, 2*B*, 1*G*
G 13	3*A*, 3*G*	*G 42*	5*A*
G 15	1*A*, 4*G*	*G 45*	2*A*, 1*G*
G 17	1*A*, 1*B*, 2*G*	*G 46*	2*A*, 1*B*
G 18	2*A*, 4*G*	*G 50*	4*A*, 1*G*, 1*Ch.*
G 19	2*A*, 5*G*	*G 51*	4*A*, 2*G*, 1*GP*
G 22	1*A*, 2*G*, 5*SY*	*G 53*	1*A*, 1*B*, 3*G*, 1*GP*
G 25	4*A*, 1*Y*, 1*GP*	*G 52*	3*A*, 1*G*, 1*GP*
G 28	2*A*, 1*G*, 1*SY*, 1*GP*, 1*YP*	*G 47*	1*A*, 2*B*, 1*G*
Total	88*A*, 20*B*, 50*G*, 1*GA*, 1*Y*, 6*SY*, 2*Ch.*, 17*GP*, 1*YP*		

Fs and Gs which have produced no Albino Young.

	Results		Results
F 9	7*G*	*G 4*	2*G*
F 7	4*G*	*G 5*	5*B*, 3*G*
F 11	2*G*, 1*GA*	*G 7*	10*G*
F 15	1*B*, 3*G*	*G 10*	2*B*, 3*G*
F 19	2*G*, 2*GP*	*G 14*	3*B*, 1*G*, 3*GP*
F 20	2*G*	*G 20*	8*G*, 1*GP*
F 26	1*BP*	*G 23*	1*B*, 1*G*, 3*BP*, 2*GP*
F 27	3*G*, 2*GP*	*G 24*	6*BP*
F 29	2*B*, 2*G*	*G 26*	5*G*
F 30	2*B*, 2*G*	*G 27*	3*G*, 4*GP*
F 37	1*G*, 1*Cc.G*, 1*Ch.*	*G 29*	2*G*, 4*SY*
F 38	4*G*, 1*BP*	*G 30*	3*G*, 1*GP*
F 41	2*G*, 1*Ch.*, 1*BP*	*G 33*	3*G*, 1*GP*, 1*BP*
F 42	1*G*	*G 34*	5*G*, 2*GP*
F 45	2*G*, 3*Ch.*, 1*GP*	*G 48*	3*G*, 1*Y*, 4*SY*
F 46	1*B-G*, 1*Ch.*, 1*GP*	*G 49*	2*B*, 4*G*
Total	5*B*, 1*B-G*, 37*G*, 2*Cc-G*, 1*GA*, 6*Ch.*, 6*GP*, 3*BP*	Total	13*B*, 56*G*, 1*Y*, 8*SY*, 14*GP*, 10*BP*

Cross K. Pairs of Albino Young from Cross E.

These gave all Albino Young.

Cross L. Pairs made of the Coloured Offspring of E pairs which have given all Coloured Young when Crossed with Whites.

	Results		Results
L 1	4*G*	*L 9*	4*G*
L 2	9*G*	*L 10*	1*B*, 4*G*
L 3	6*G*	*L 11*	3*G*
L 4	5*G*	*L 13*	1*B*, 4*G*
L 5	5*G*, 1*GP*	*L 14*	3*G*, 2*GP*
L 7	2*G*, 5*SY*	*L 15*	3*G*
L 8	1*B*, 5*G*		
Total	57*G*, 3*GP*, 3*B*, 5*SY*.		

Cross M. Pairs made of Coloured Offspring of E pairs which have given some White Young when Crossed with White.

	Results		Results
M 1	3*A*, 3*SY*	*M 11*	1*A*, 1*G*
M 2	1*GA*, 2*G*, 1*GP*	*M 12*	2*A*, 2*G*
M 3	3*A*, 1*GP*, 2*G*	*M 13*	5*G*
M 4	1*A*, 6*G*	*M 14*	1*A*, 4*B*, 2*G*
M 5	3*A*, 3*G*	*M 15*	1*A*, 1*B*, 3*G*
M 6	2*A*, 2*G*, 1*B*	*M 16*	5*G*
M 7	2*A*, 3*G*	*M 17*	1*A*, 2*G*
M 8	2*A*, 2*GP*, 1*G*	*M 18*	3*G*
M 9	1*A*, 5*G*, 1*GP*	*M 21*	1*A*, 3*G*, 1*SY*
M 10	3*A*, 2*SY*, 1*G*	*M 20**	3*A*, 1*B*, 1*G*
Total	30*A*, 52*G*, 5*GP*, 7*B*, 1*GA*, 6*SY*.		

APPENDIX II. (See p. 3.)

The eight pure mice would have the following gametic formulae *CGU, CGP, CYU, CYP, AGU, AGP, AYU, AYP.* The last four being white, and the first four coloured respectively, uniform grey, piebald grey, uniform yellow and piebald yellow. Of the 56 hybrids, there are 24 which are hybrids with regard to only one pair of characters;—two of each of the following 12 different kinds: *CGU-CGP, CGU-AGU,* uniform grey in colour; *CGU-CYU, CYU-CYP, CYU-AYU,* uniform yellow; *CGP-AGP,* piebald grey; *CGP-CYP, CYP-AYP,* piebald yellow; *AGU-AGP, AGU-AYU, AGP-AYP, AYU-AYP,* albino. There are 24 which are hybrids with regard to two pairs of characters, 4 of each of the following six different kinds, *CGU-CYP, AYU-CGU, AYP-CYU,* uniform yellow; *AGP-CGP,* uniform grey; *AYP-CGP,* piebald yellow; *AGU-AYP,* albino. Also 8 hybrids with regard to all three pairs of characters; of these there can be only one kind, which is uniform yellow in colour.

BIOMETRICAL STUDIES ON MAN.

I. VARIATION AND CORRELATION IN BRAIN-WEIGHT*.

By RAYMOND PEARL, Ph.D.

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1. *Introduction.*

THE subject of brain-weight in man has for a long time been given considerable attention by anatomists and anthropologists. The reason for this is obvious. Since the brain is the organ of the mind it appeared to earlier workers that size of brain ought to be an index of intellectual capacity. The substance of the brain is of such physical homogeneity that its weight is a fairly accurate, as well as simple, "size" measure of the organ. Consequently, by earlier workers, brain-weights were collected, studied and discussed to a considerable extent as measures

* Contributions from the Zoological Laboratory of the University of Michigan. No. 86.

or indices of brain power. It became evident after a time, however, that there was not the close and definite relationship between brain-weight and intellectual capacity which had been supposed to exist. Individuals of marked intellectual power were found in not a few instances to have brain-weights below the average, while on the other hand it was not at all difficult to find individuals of very mediocre intellectual attainment who possessed brains of unusually large size and weight. These results still held even after rough corrections were made for bodily size, age, etc. While it is thus evident that brain-weight cannot be taken as a close index of intellectual power, holding for individual instances, yet it is clear that, considered from the phylogenetic standpoint, increase in brain-weight and in psychic capacity have in general gone hand in hand, and the weight of the brain increases quite regularly as we go up the taxonomic scale*.

The present trend of investigation in this subject, is rather from the point of view of anthropology than of psychology. Evidently brain-weight is an important and interesting anthropological character, and in this field we may expect significant results.

The method of investigation which has been almost universally followed in brain-weight work has been to tabulate large masses of statistics of weighings, compute means for various groupings, and draw the conclusions which appeared to follow from the tabulations and averages. In other words the only statistical methods which in most cases have been applied to the data have been those of the sociological statistician. Such methods serve fairly well, of course, when only the "types" are wanted, but they are quite inadequate for some of the work which many neurologists have wished to do in this field. Practically all students of the subject have attempted to determine in one way or another the degree of correlation which exists between brain-weight and other physical characters and also age. Knowledge of these correlations is of course much to be desired. The human brain is justly to be regarded as the highest product of organic evolution. Any contribution to a knowledge of the laws governing its variation and correlation cannot fail to be of the greatest interest. Again, only through a knowledge of the degree of the correlation of brain-weight with other characters of the body is it possible to make scientifically such suitable corrections for bodily differences as will make fair any comparison of the brain-weights of different races, or of different groups of the same race. Now, as everyone knows who has even an elementary knowledge of statistics, it is possible to make the same statistical material lead to quite different conclusions, by grouping it in different ways, when the tabulations and averages are the only sources from which conclusions may be drawn. As a matter of fact this has happened in work on brain-weights. Different investigators, working in different ways, have arrived at

* Cf. Ziehen, Th.: "Das Gehirn, Massverhältnisse," Bardeleben's *Handbuch der Anatomie des Menschen*, Bd. IV. 1—3 Abtheilung, pp. 353—386, 1899, p. 362 et seq.; and more recently Spitzka, E. A.: "Brain-weight of Animals with special reference to the Weight of the Brain in the Macaque Monkey," *Jour. Comp. Neurol.* Vol. XIII. pp. 9—17, 1903.

quite different numerical appreciations of the relation existing between brain-weight and other characters. The very interesting question of variation in brain-weight has been scientifically investigated only by Pearson*, so far as is known to the writer.

The reason which led me to undertake the present piece of work was primarily a desire to apply adequate statistical methods to a biometrical problem of peculiar intrinsic interest, and for which fairly large collections of reliable data were available. It was desired to determine as a part of a general plan of work outlined as exactly as possible from the data at hand, what were the actual conditions of variation and correlation in brain-weight. It was hoped that by such work some light might be thrown on the question of the evolution of man's brain. The work was originally planned to include the analysis of only one large collection of brain-weight data (see below p. 16) but when the constants for this material had been determined certain of the results were seen to be rather peculiar. Not being satisfied that these indicated the true state of affairs, but thinking that they might be caused by some hidden anomalies of the raw material, it was decided to extend the work to other data in order to test the first results.

The specific problems to which attention has been directed in this work are:

1. The amount of variation in the weight of adult male and female brains.
2. The relation of brain-weight to age in the adult.
3. The relation of brain-weight to stature and body-weight in the adult.
4. The relation of brain-weight to skull characters.
5. The nature of the regression of brain-weight on these other characters.
6. The effect of natural selection on brain-weight.
7. The value of brain-weight statistics from the anthropological standpoint.

At this point I wish to acknowledge my indebtedness and express my thanks to the officials of the Carnegie Institution for a grant through which I obtained a large Brunsviga arithmometer and other necessary apparatus for statistical work. This aid enabled me to extend this work far beyond what would have been possible had I been restricted to "long-hand" methods of computation. The amount of arithmetical calculation necessary in any extensive biometrical investigation is so great that it may almost be said that such work cannot profitably be undertaken on any large scale unless one has at command the various mechanical aids which reduce to a minimum the time necessary to carry out the computations. To my wife I am especially indebted for much aid in the computing, and for the diagrams which accompany the paper. To Prof. H. H. Donaldson I am indebted for the loan of some unique and valuable manuscript records of brain-weighings, as well as for numerous helpful suggestions regarding the work as a whole.

* Pearson, K.: "Variation in Man and Woman," *The Chances of Death*, Vol. I. pp. 319—323.

2. *Data.*

The statistical data for this study were obtained from several sources, as has been mentioned above. The work was begun on Marchand's statistics, then extended to include the old, but still valuable collection of brain-weights by Bischoff. With the results from these two series in hand, I was still not satisfied that I had analyzed sufficient material to be certain of the conclusions. So I determined to include the following material in the discussion: (1) the admirable series of brain-weights of Swedes collected by Retzius. (2) Matiegka's Bohemian series. (3) The Boyd-Marshall data observed at the St Marylebone Infirmary. As will appear later the last mentioned series was used only for a special purpose. The general results of this paper are based on the analysis of the Marchand, Bischoff, Retzius and Matiegka series.

Marchand's* data were obtained from a series of brain-weighings made between the years 1885–1900 in the Pathological Institute at Marburg. The total number of brains weighed was 1234. Of this number 389 were from individuals under 15 years of age, and hence were unavailable for adult brain-weight studies. The data recorded from each individual in Marchand's work included, in addition to brain-weight, sex, age, and stature. It was not possible, however, to determine all of these points in every case, so that omissions of one or more records for an individual occur not infrequently in these statistics. The same is of course true of all extensive series of brain-weight data ever collected. After all such incomplete records have been thrown out there remain in Marchand's tables 475 male and 281 female complete records, or a total of 756 of both sexes. Marchand did not include body-weight in his determination, because he considered that: "das Körpergewicht bei dem Leichenmaterial der Krankenhäuser ein so wechselnder Faktor ist, dass bestimmte Beziehungen zum Gehirngewichte sich daraus kaum ableiten lassen würden."

The brains were weighed in the fresh condition, usually immediately after removal from the cranial cavity. The membranes were not removed before weighing. The weighings were made in the majority of cases to the nearest 5 grams, greater accuracy not being possible on account of such unavoidable sources of error as varying amounts of blood in the vessels of the brain, amount of fluid in the ventricles and membranes, etc. These sources of error cause variations in the total weight outside the limits of 5 grams. Marchand's material comes from the "hessische Bevölkerung."

Bischoff's† data comprise the results of the weighing of something over 800 brains of individuals between the ages of 17 and 85. The attempt was to determine for each individual the following characters in addition to brain-weight: sex, age, cause of death, stature and body-weight. Omissions in the records were

* Marchand, F.: "Ueber das Hirngewicht des Menschen," *Abhandl. d. math.-phys. Cl. d. Königl. Sachs. Gesellsch. d. Wiss.* Bd. XXVII. No. IV. pp. 393—482, 1902.

† Bischoff, T. L. W. v.: *Das Hirngewicht des Menschen*, Bonn, 1880, Pp. vi. and 171, plus Tables.

fairly numerous, so it was only possible to extract full data on sex, stature, body-weight and brain-weight for 365 males and 241 females. This material I have referred to in this paper as the "short" Bavarian series. For determining the correlation between brain-weight and age I was able to extract 529 male and 323 female records from Bischoff's tables. This larger group I have called the "long" Bavarian series. The brains were weighed in the fresh condition, soon after removal and with the membranes. Nearly all the individuals had died in hospitals and a considerable number of them were convicts. Practically all were members of the middle and lower classes of society. The causes of death for this and Marchand's series were very varied, including a wide range of diseases and accidents, so that the material is not vitiated by the great preponderance of some one cause of death which might have a specific effect on brain-weight. The great majority of the individuals included in Bischoff's series were Bavarians.

The Swedish data used were collected by Retzius*. The source of the material was in the main the autopsies at the great Sabbatsberg Krankenhaus in Stockholm, although some of the returns were from autopsies at the Maria Krankenhaus. This series of brain-weights is undoubtedly one of the best which has ever been made with reference to accuracy in the individual weighings and uniformity in the conditions and method of weighing. It is only to be regretted that the series is not more extensive. Regarding the racial homogeneity of the material Retzius says (*loc. cit.* p. 55): "ich überzeugte mich aber in den allermeisten Fällen davon, dass sie eine echt schwedishe Herkunft anzeigten." The method of weighing was as follows (*loc. cit.* p. 55): "Die Gehirne wurden ohne Abnahme der weichen Hirnhaute, in der Regel kurz nach der Herausnahme, d. h. nur nach dem Verlust einer dabei stets ausfliessenden geringen Menge Blut und Cerebrospinalflüssigkeit, direct auf die Waageschale gelegt." The series as published by Retzius gives the weights of 450 male brains, and 250 female. In addition to brain-weight, age, stature, and cause of death are recorded. After throwing out individuals incompletely recorded, and those falling outside the age limits 20—80, there was left available for this study 416 male records and 233 female.

Matiegka† has furnished some very valuable data for the student of brain-weight. His material came from the autopsies at two institutions in Prague, the Institut für gerichtliche Medizin, and the Pathologisch-Anatomische Institut. The series from the former source is the larger and more complete, and is the only one which is used in the present discussion. This includes the brain-weights of 372 males and 197 females, together with records of age, stature, skull-length and skull-breadth. The individuals were all adults between the ages of 20 and 80. The great majority of them were Czechs. The method of weighing the brain

* Retzius, A.: "Ueber das Hirngewicht der Schweden," *Biol. Untersuchungen*, N. F. Bd. IX., Cap. IV. pp. 51—68, 1900.

† Matiegka, H.: "Über das Hirngewicht, die Schädelkapacität und die Kopfform, sowie deren Beziehungen zur psychischen Thätigkeit des Menschen," *Sitzber. des kön. böhmischen Gesellsch. d. Wissensch. Mathem.-Naturwiss. Classe*, Jahrg. 1902, No. XX. pp. 1—75.

was that usually followed: the pia and arachnoid were not removed before weighing. The special value of this material of Matiegka's lies in the fact that it is the most extensive series of brain-weights in which skull characters are so recorded as to make it possible to determine their correlation with the weight of the brain. In addition, data are given for the determination of the correlation between brain-weight and stature, and brain-weight and age. Unfortunately Matiegka's material is open to criticism in certain important respects. In the first place detailed tables showing the exact records for each individual are not given, but instead the material is thrown into the form of correlation tables. In making up these tables the units of grouping were not altogether wisely chosen. This results in introducing a certain error into the absolute values of the constants deduced from these tables. The exact cause, nature and amount of this error will be pointed out later in the paper. Again, the skull-length and skull-breadth were measured in a very unreliable manner. Regarding the method of making these measurements Matiegka says (*loc. cit.* p. 46): "Bei dem von mir verarbeiteten Materiale und zwar in beiden Instituten wurde gewöhnlich auch die *Länge* und *Breite* des *Schädeldaches* d. i. an dem behufs der Hirnentnahme vorgenommenen *Horizontalschnitte* gemessen. Das so erlangte Längenmass ist daher bedeutend *kürzer* als die *grösste Schädellänge*, die Breite wohl häufig etwas *kleiner* als die grösste Schädelbreite*." The effect of this procedure on the biometric constants is discussed later in this paper. My chief reason for including this somewhat questionable material was for the sake of comparison, and because reasonably long series of brain weighings are not so plentiful as to allow one who would study the subject a great range of choice in material.

The data on the brain-weight of the English were very kindly placed at my disposal by Prof. H. H. Donaldson. He was able to obtain, some years ago, a copy of the original detailed manuscript tables which the English anatomist Marshall had compiled from Boyd's original data collected at the St Marylebone Infirmary and the Somerset County Lunatic Asylum. These detailed tables were never published, though it was on them that Marshall's well-known paper, "On the Relations between the Weight of the Brain and its Parts and Stature and Mass of the Body in Man †," was based. For the privilege of examining and using this unique and valuable material I am very grateful to Prof. Donaldson.

From the biometrical standpoint the raw material available for a statistical study of the weight of the brain is peculiarly complex. The great bulk of the statistics consists of material gathered at the autopsies performed in large general hospitals or other public institutions of similar character. The result of this is that the only large collections of brain-weight data available are not representative "random" samples of the general population. Instead they represent a group of the population which has been subjected to a rather stringent selective process of a peculiar kind. It would be very difficult, if not impossible, to formulate all

* Italics Matiegka's.

† *Jour. Anat. and Physiol.* Vol. xxvi. pp. 445—500, 1892.

the factors which contribute to the differentiation of the "general hospital population" from the "general population" of the same locality. Of some of them, however, we may be fairly certain. In the first place, there are many classes (social) of the general population which will never be represented in any significant proportion in the general hospital population. Furthermore, the nature of the injury or disease from which an individual is suffering in many cases determines whether the individual shall be in a hospital. On this point, Greenwood*, in an able discussion of the matter, says: "Evidently the population of a general hospital will chiefly consist of: (i) persons acutely ill, (ii) those suffering from surgical injuries or diseases, (iii) sufferers from medical affections requiring special treatment. Chronic maladies of old age, such as bronchitis, indeed, any highly chronic disease, will be under-represented in comparison with the general death-rate. Similarly, the number of cases of valvular heart disease and rarer disorders, such as *Diabetes Mellitus* or *Insular Sclerosis* and other nervous lesions, will be above the general average."

In addition to the selection which occurs in the formation of the general hospital population there is a still farther weeding out when we come to deal with autopsy records, for the reason that not every individual dying in a hospital is subjected to a post-mortem examination. Whether there shall be an autopsy or not in a given case depends on several factors, one of which is the cause of death. The individuals whose last illnesses have given doubtful or rare clinical features are more likely to be subjected to a post-mortem examination, other things being equal, than the individuals in whose case the fatal illness has run a perfectly typical clearly cut course of some common disease. Consequently there is a tendency for the statistics to be unduly weighted with deaths from some of the rarer maladies. There can be no doubt that the statistical constants relating to any character of the body which is in any way affected by the disease causing death will be different, in material coming from hospital and pathological laboratory autopsy returns, from those which would be obtained could we get a perfectly random sample of the general population dying outside a hospital, and subjected to post-mortem examination regardless of the cause of death.

These peculiarities attendant upon hospital autopsy records are of particular importance when the character studied is the weight of the brain, for the reason that this character is definitely affected both by the age of the individual, and, in certain cases, by the nature of the disease causing death. It is evident that there will be a more or less definite relation between the nature of the cause which brings an individual into a general hospital and the age of the individual. This combined relation of age and disease is a factor of importance in analysing brain-weight data. In what manner will be apparent from a moment's consideration. Suppose we take the first of Greenwood's classes of the general

* Greenwood, M.: "A First Study of the Weight, Variability and Correlation of the Human Viscera, with special reference to the Healthy and Diseased Heart." *Biometrika*, III. pp. 63—83, 1904, p. 65.

hospital population mentioned above, namely, "persons acutely ill." It is at once clear that this will include two sub-groups. First in number and importance will be the group of individuals suffering from diseases medically classified as "acute" in the strict sense of the term, for example, typhoid fever. Second will be the group of individuals acutely ill at the termination of a long-standing chronic disease. These individuals in most instances have been cared for at home as long as the disease remained in the chronic or sub-acute form, and only go into the hospital when it becomes acute. A good example of such cases is afforded by Bright's disease. There can be little doubt on *à priori* grounds that the mean age of the individuals will be lower in the first as compared with the second of these groups. This is shown to be the case by the actual statistics in the brain-weight series where "cause of death" is tabulated. In the lower age groups the causes of death returned are preponderantly "acute" in the strict sense. In the higher age groups we get a preponderance of the chronic affections. Now it is altogether likely that the acute illness which leads to death in a comparatively short time has much less effect on the weight of the brain than the chronic "wasting" sickness. So, then, the general result is that in the higher age classes where the brain-weight has decreased as the natural result of senescence there is a further artificial lowering on account of the preponderance of individuals who have been afflicted with "wasting" diseases. On the other hand the returns during early adult life give mean values for the brain-weight which are probably nearer the true normal value for the general population.

In the present paper it was desired to consider only adult brain-weights and consequently only material which fell in age between 20 and 80 years was usually used. It is the general opinion of anatomists (cf. for example, Ziehen, *loc. cit.* p. 359, and Marchand, *loc. cit.* p. 404), that after the age period 15—20 there is very little increase of weight in the brain with advancing age. In other words the brain is considered to attain practically its complete growth in about the first 20 years of life. According to Marchand (*loc. cit.* p. 402 *et seq.*) the maximum brain-weight remains practically constant till the individual is about 50 years old. After that age he believes that senile degeneration begins. Accordingly, in the discussion of the Marchand data, I have considered the period from 15—80 years to constitute the "adult period," so far as brain-weight is concerned. As a matter of fact, the result would have been but little different had the period 20—80 years been used in this case (as it was with all the other data), because there are only 36 males and 17 females falling in the age class 15—19, and these individuals are fairly evenly distributed among the brain-weight classes as will be seen by consulting Tables 17 and 18 of the Appendix. When the work was begun I had, of course, no means of knowing how it would turn out, so it was necessary to decide how to treat the material on the basis simply of a careful general inspection of the statistics and of the opinion of other workers on the subject. As one point which I wished especially to investigate was the change of brain-weight with age, it became a nice problem as to how to handle the

material so as to best get at this. I finally decided after a good deal of consideration to adopt provisionally Marchand's view stated above, viz., that the weight of the brain reaches a maximum between the ages 15 and 20, remains constant till about age 50, then declines through the old age period. Acting on this plan I separated the material in each case into what I have called a "young" and the "total" group. The "young" series included the individuals falling between the ages 20 to 50 (in the Hessian material, 15—50, and in the Bohemian 20—59 were the limits, in the former case from choice, in the latter from necessity). The "total" series included all the individuals between ages 20 and 80 (in the Hessian series 15 was the lower limit). The results show that on the whole this method of handling the material was adequate, considering the ends to be gained and the amount of material available.

3. *Fundamental Constants.*

As a preliminary to the discussion of the correlation of brain-weight with other characters it is necessary to exhibit and discuss the constants, measuring type and variability of the characters considered. Certain problems which are of considerable interest on their own account present themselves here. Of first importance perhaps is the question as to the amount of differentiation in respect of brain-weight which exists between different sub-races of men, both in type and variability. Further, the rather extensive material worked over makes it possible to settle the problem of the relative variability of the sexes with respect to the weight of the brain. On this point Pearson (*loc. cit.*) has made a brief communication based on an analysis of four short series of English data, Bischoff's Bavarian series, and a French series. His general conclusion is that in respect of brain-weight the sexes are sensibly equally variable. Another interesting problem which merits discussion here is as to how the relative variability in brain-weight compares with the variability of other organs and characters of the body.

In Table I are exhibited the values of the Means, Standard Deviations and Coefficients of Variation together with their probable errors for each of the characters studied in this work. While, of course, the primary object of study is the weight of the brain, yet it is necessary for the correlation determinations that we have the fundamental constants for the other characters, age, stature, etc. To effect economy of space the constants for these other characters have been included in this first table. The frequency distributions from which the constants have been deduced will be found in the fundamental tables, numbered in Arabic numerals from 1 to 44, collected at the end of this paper. Regarding the calculation of the constants the following explanations should be made. In preparing the correlation tables from the raw statistical material the following values for the units of grouping were chosen, and used uniformly throughout the work, except in certain cases where it was impossible to keep to them.

Character	Standard Unit of Grouping
Brain-weight ...	50 grams
Age	5 years
Stature	3 centimetres
Skull length ...	5 millimetres
Skull breadth ...	5 millimetres
Body-weight ...	5 kilograms

These standard units were chosen after considerable study and experimenting and are believed to be the best values for the data discussed. However, it has been shown recently by other workers * that "the system of grouping adopted is within wide limits immaterial." An examination of those cases recorded in the present paper where it was necessary to use different units of grouping—notably in the Matiegka data where the brain-weights were grouped into 100 gram classes and the ages into 10 year classes—leads to the same conclusion.

The standard deviation was evaluated according to the equation $\sigma = \sqrt{\mu_2}$. In obtaining μ_2 Sheppard's correction was used throughout for the characters brain-weight, stature, skull length, skull breadth and body-weight. The uncorrected "rough" moment was used in getting the standard deviation in age, as in this case there is no approach to high contact at either end of the range.

The decimals have been retained to three places in the tabulated values in order that the significant value of the first place may be seen. It will be understood, of course, that in the computations the decimals were retained to a larger number of places for arithmetical reasons.

4. *Brain-weight Types.*

In brain-weight work hitherto it has not been possible to make accurate comparisons of the mean weight of the brain in different races. There were two reasons for this: one that the probable errors of the means were not determined, and the other that no method was available by which allowance could be made for the differences in the mean stature and age of the samples of material to be compared. Obviously if brain-weight is correlated with stature and age, we should expect to get a considerable difference between the mean brain-weight of two groups which differed widely with respect to these other characters. Inasmuch as a knowledge of the degree of correlation between these characters affords a method whereby the material can be reduced to a "standard stature-age base," it will be worth while to examine the actual differences in brain-weight types in the four racial groups here discussed.

* "Assortative Mating in Man." A Cooperative Study. *Biometrika*, Vol. II. pp. 481—498, 1903.

TABLE I. *Fundamental Constants.*

Race	Character	Mean						Standard Deviation		Coefficient of Variation	
		No.	♂	Table	No.	♀	Table	♂	♀	♂	♀
Swedes ...	Brain-weight (Total) ...	416	1400·481 ± 3·516	1	233	1252·682 ± 4·452	2	106·329 ± 2·486	100·757 ± 3·148	7·592 ± ·179	8 043 ± ·253
,,	,, (Young) ...	262	1415·267 ± 4·550	3	127	1269·488 ± 6·317	4	109·180 ± 3·217	105·543 ± 4·467	7·714 ± ·229	8·314 ± ·354
Hessians	,, (Total) ...	475	1391·737 ± 3·487	13	281	1259·875 ± 4·119	14	112·675 ± 2·466	102·368 ± 2·642	8·096 ± ·178	8·125 ± ·233
,,	,, (Young) ...	291	1405·756 ± 4·402	15	173	1280·202 ± 5·223	16	111·328 ± 3·113	101·841 ± 3·693	7·919 ± ·223	7·955 ± ·290
Bohemians	,, (Total) ...	372	1454·839 ± 3·973	27	197	1310·914 ± 4·650	28	113·608 ± 2·813	96·772 ± 3·288	7·809 ± ·197	7·382 ± ·252
,,	,, (Young) ...	266	1460·150 ± 4·731	25	133	1313·910 ± 5·453	26	114·395 ± 3·345	93·227 ± 3·855	7·835 ± ·231	7·095 ± ·295
,,	,, (Skull) ...	299	1463·712 ± 4·503	29	159	1321·698 ± 5·073	30	115·440 ± 3·159	94·840 ± 3·587	7·887 ± ·219	7·176 ± ·273
Bavarians	,, (Total, Long*)	529	1363·185 ± 3·245	37	323	1220·356 ± 3·820	38	110·644 ± 2·295	101·776 ± 2·701	8·118 ± ·122	8·340 ± ·223
,,	,, (Young, Long*)	365	1369·110 ± 3·841	39	238	1235·504 ± 3·997	40	108·776 ± 2·716	91·421 ± 2·826	7·945 ± ·200	7·399 ± ·230
,,	,, (Total, Short*)	365	1357·466 ± 3·888	35	241	1218·776 ± 4·616	36	120·128 ± 2·749	106·246 ± 3·264	8·849 ± ·223	8·717 ± ·270
Swedes ...	Stature (Total) ...	416	169·789 ± ·225	1	233	158·710 ± ·297	2	6·807 ± ·159	6·719 ± ·210	4·009 ± ·094	4·233 ± ·133
,,	,, (Young) ...	262	170·176 ± ·280	3	127	159·673 ± ·418	4	6·728 ± ·198	6·987 ± ·296	3·954 ± ·117	4·375 ± ·186
Hessians	,, (Total) ...	475	167·359 ± ·223	13	281	156·178 ± ·277	14	7·189 ± ·157	6·895 ± ·196	4·296 ± ·094	4·400 ± ·125
,,	,, (Young) ...	291	167·294 ± ·282	15	173	156·980 ± ·384	16	7·118 ± ·199	7·490 ± ·272	4·255 ± ·119	4·771 ± ·173
Bohemians	,, (Young) ...	266	169·417 ± ·303	25	133	157·650 ± ·418	26	7·324 ± ·214	7·152 ± ·296	4·323 ± ·127	4·537 ± ·188
Bavarians	,, (Total, Short*)	365	166·549 ± ·226	35	241	154·712 ± ·270	36	6·393 ± ·160	6·213 ± ·191	3·838 ± ·096	4·016 ± ·124
Swedes ...	Age (Total) ...	416	45·024 ± ·452	5	233	47·629 ± ·671	6	13·671 ± ·320	15·195 ± ·475		
,,	,, (Young) ...	262	36·317 ± ·327	7	127	35·689 ± ·480	8	7·847 ± ·231	8·027 ± ·340		
Hessians	,, (Total) ...	475	42·889 ± ·521	17	281	44·795 ± ·656	18	16·843 ± ·369	16·304 ± ·464		
,,	,, (Young) ...	291	31·572 ± ·390	19	173	34·061 ± ·497	20	9·861 ± ·276	9·685 ± ·351		
Bohemians	,, (Total) ...	372	45·699 ± ·523	27	197	41·447 ± ·792	28	14·957 ± ·370	16·482 ± ·560		
Bavarians	,, (Total, Long*)	529	43·171 ± ·396	37	323	40·333 ± ·561	38	13·495 ± ·280	14·954 ± ·397		
,,	,, (Young, Long*)	365	35·582 ± ·255	39	238	32·731 ± ·340	40	7·216 ± ·180	7·768 ± ·240		
Bohemians	Skull length (Young) ...	299	176·547 ± ·287	29	159	170·142 ± ·294	30	7·368 ± ·203	5·500 ± ·208	4·173 ± ·115	3·233 ± ·122
,,	,, breadth (Young) ...	299	149·841 ± ·260	31	159	144·953 ± ·296	32	6·654 ± ·184	5·530 ± ·209	4·441 ± ·123	3·815 ± ·145
Bavarians	Body-weight (Total, Short*)	365	49·925 ± ·376	41	241	42·894 ± ·461	42	10·646 ± ·266	10·601 ± ·326	21·324 ± ·608	24·715 ± ·804

* Cf. p. 17.

Taking the gross values given in Table I the following Table II has been prepared to facilitate comparison.

TABLE II.

Brain-weight Types.

	Hessians	Bavarians	Swedes	Bohemians
Total Males				
Mean brain-weight	1391·737	1363·185	1400·481	1454·839
Mean age	42·889	43·171	45·024	45·699
Mean stature ...	167·359	166·549	169·789	—
Young Males				
Mean brain-weight	1405·756	1369·110	1415·267	1460·150
Mean age	31·572	35·582	36·317	40·545*
Mean stature ...	167·294	—	170·176	169·417
Total Females				
Mean brain-weight	1259·875	1220·356	1252·682	1310·914
Mean age	44·795	40·333	47·639	41·447
Mean stature ...	156·688	154·712	158·710	—
Young Females				
Mean brain-weight	1280·202	1235·504	1269·488	1313·910
Mean age	34·061	32·731	35·689	35·613†
Mean stature ...	156·980	—	159·673	157·650

It is apparent at once that, considering the magnitude of the probable errors, there are significant differences between the Bavarians, Swedes, and Bohemians, with respect to brain-weight, in both sexes and age classes. Between the Swedish and Hessian means the differences are smaller, and as the following table shows, are not significant when their probable errors are taken into account:

Total Males: Swedish mean—Hessian mean = $8{\cdot}744 \pm 4{\cdot}952$.
Total Females: Hessian mean—Swedish mean = $7{\cdot}193 \pm 6{\cdot}065$.
Young Males: Swedish mean—Hessian mean = $9{\cdot}511 \pm 6{\cdot}331$.
Young Females: Hessian mean—Swedish mean = $10{\cdot}714 \pm 8{\cdot}197$.

In no case is the difference as great as even twice its probable error, hence we must conclude that the differences exhibited might, so far as the extent of material allows us to judge, be due to random sampling.

* This value was calculated by taking the first four age classes of the "total" series. It includes more individuals than the "young" series for the other characters. This was the only way in which the mean age of the "young" series in this case could be even approximated to, as the raw material given by Matiegka is incomplete in this respect. The age as given is probably a little too high, but the error cannot be great.

Is this agreement between the Hessian and Swedish peoples in the matter of brain-weight real, or only apparent and the result of compensating differences in the other correlated characters? An answer to this question can be reached in two ways; either by reducing one set to the same "stature-age base" as the other and then comparing results, or by analyzing the effects on the brain-weight of the observed stature and age differences in the two sets of material. Both of these methods depend on the use of characteristic equations, the derivation of which will be described further on in the paper. For the present I shall make practical use of such equations without further discussion of their derivation or validity. Table II shows that in the samples with which we are dealing the Hessians, both male and female, average *younger* and *shorter* than the Swedes. Reducing the Hessians to the same "stature-age base" as the Swedes, by means of the appropriate equations given on p. 63 *infra*, the following results are obtained:

TABLE III.

		Equation
The probable brain-weight of a group of Hessian ♂♂ having the same age and stature as the Swedish ♂♂ (Total)	=1396·480	(23)
Swedish brain-weight observed	=1400·481	
Swedish *heavier*. Difference ...	= 4·001	
The probable brain-weight of a group of Hessian ♀♀ having the same age and stature as the Swedish ♀♀ (Total)	=1258·232	(25)
Swedish brain-weight observed	=1252·682	
Hessian *heavier*. Difference ...	= 5·550	
The probable brain-weight of a group of Hessian ♂♂ having the same age and stature as the Young Swedish ♂♂	=1407·457	(24)
Swedish brain-weight observed	=1415·267	
Swedish *heavier*. Difference ...	= 7·810	
The probable brain-weight of a group of Hessian ♀♀ having the same age and stature as the Young Swedish ♀♀	=1283·540	(26)
Swedish brain-weight observed	=1269·488	
Hessian *heavier*. Difference ...	= 14·052	

These differences after the stature-age corrections are made are extremely small. This is particularly well shown if they are reduced to a relative basis, by expressing the differences as percentages of the observed Swedish brain-weight:

The difference between Total Swedish ♂♂ and Hessians (calculated) =0·29 % of the observed Swedish brain-weight.

The difference between Total Swedish ♀♀ and Hessians (calculated) =0·44 % of the observed Swedish brain-weight.

The difference between Young Swedish ♂♂ and Hessians (calculated) =0·55 % of the observed Swedish brain-weight.

The difference between Young Swedish ♀♀ and Hessians (calculated) =1·11 % of the observed Swedish brain-weight.

It appears then that, so far as the available material may be considered valid as representing the whole population, the conclusion is justified that the Hessian and Swedish peoples are sensibly alike in respect to their brain-weight. The agreement would be closer than that actually observed if the assumption made in the course of reasoning here followed, that the regression of brain-weight on age and stature is strictly linear, were exactly true. As will be shown later these regressions are not strictly linear but they approach linearity with sufficient closeness to serve for all practical purposes. This, together with the difference in age distribution of the "total" and "young" series, also accounts for the fact that while in the "total" series and male "young" series the gross differences between Swedes and Hessians are lowered when we reduce to a common "stature-age base," these differences are slightly increased, on the other hand, when we deal in the same way with the shorter female "young" series.

This is in agreement with the general fact that these peoples are probably the most closely related ethnically of any with which we are dealing in the present paper. The Swedes may be considered to be among the purest representatives of the original blonde, dolichocephalic Teutonic race (Ripley* and Deniker†). In the case of the Hessians some intermixture of this Teutonic with the brachycephalic, characteristically brunette Alpine type has occurred. The differentiation from the Swedes in such important characters as skull form and stature is not great however‡.

Turning now to the other racial groups discussed, viz., the Bavarians and Bohemians (Czechs), we find as would be expected, that the differences in brain-weight are greater. Using the Hessian data as a basis for comparison, the gross differences with their probable errors are exhibited in the following table:

Total Males:	Hessian mean—Bavarian mean $= 28{\cdot}552 \pm 4{\cdot}763$.
Total Females:	Hessian mean—Bavarian mean $= 39{\cdot}519 \pm 5{\cdot}618$.
Young Males:	Hessian mean—Bavarian mean $= 36{\cdot}646 \pm 5{\cdot}842$.
Young Females:	Hessian mean—Bavarian mean $= 44{\cdot}698 \pm 6{\cdot}577$.
Total Males:	Bohemian mean—Hessian mean $= 53{\cdot}102 \pm 5{\cdot}286$.
Total Females:	Bohemian mean—Hessian mean $= 51{\cdot}039 \pm 6{\cdot}212$.
Young Males:	Bohemian mean—Hessian mean $= 54{\cdot}394 \pm 6{\cdot}462$.
Young Females:	Bohemian mean—Hessian mean $= 33{\cdot}708 \pm 7{\cdot}551$.

In all cases the differences are seen to be well above what might arise from errors in statistical sampling. Reducing to a common "stature-age base" by the method followed above the following results are obtained:

* Ripley, W. Z.: *The Races of Europe*, New York, 1899, pp. xxxii, 624.

† Deniker: *The Races of Man*, London and New York, 1900, pp. xxiii, 328 and 611.

‡ This is well shown graphically in the maps indicating the distribution of stature and cephalic index in Europe, given by Ripley (*loc. cit.* pp. 96 and 53). Cf. also Deniker, *loc. cit.* pp. 328 and 329.

TABLE IV.

	Probable Brain-weight of a group of Hessians of the same *sex*, *age* and *stature* as the	Mean Brain-weight *observed* (Bavarian or Bohemian)	Difference expressed as in excess or defect of Hessian values	Difference as per cent. of observed mean (Bavarian or Bohemian)	Equation on which Hessian estimate is based
Bavarian ♂♂, Total	1388·984	1363·185	−25·799	−1·9 %	No. 23, p. 63
„ ♀♀, Total	1265·137	1220·356	−44·781	−3·7 %	„ 25, „
„ ♂♂, Young	1397·157*	1369·110	−28·047	−2·0 %	„ 24, „
„ ♀♀, Young	1277·317*	1235·504	−41·813	−3·4 %	„ 26, „
Bohemian ♂♂, Total	1394·570†	1454·839	+60·269	+4·1 %	„ 23, „
„ ♀♀, Total	1259·310†	1310·914	+51·604	+3·9 %	„ 25, „
„ ♂♂, Young	1399·527‡	1460·150	+60·623	+4·2 %	„ 24, „
„ ♀♀, Young	1278·935‡	1313·910	+34·975	+2·7 %	„ 26, „

This table brings out several points of considerable interest. In the first instance is to be noted the general effect of reducing the Hessians to the same "stature-age base" as the other races, upon the interracial differences in mean brain-weight. In the case of the Bavarians the deviations from the Hessian means are reduced slightly in both male series, and the "young" female series, when stature and age differences are eliminated. In the "total" female series the difference is increased over the original gross difference. The explanation for this discrepancy in the female "total" series is to be found in a peculiar abnormality which this series shows in its elemental frequency distribution, and which will be discussed later (p. 40). In the case of the Bohemians we get the somewhat remarkable result that a reduction to a common "stature-age base" actually increases the differences of this racial group in brain-weight as compared with the Hessians. Or in other words, in the samples with which we are dealing the stature and age differences act in a compensatory way and bring the mean brain-weights closer together than they would be if we dealt with selected samples of the populations, each sample having the same mean age and stature. The fact already noted (p. 18) that the mean brain-weights deduced from Matiegka's tables for the Bohemians are somewhat too large, may now be examined in detail. This Bohemian material was published by Matiegka in the form of correlation tables with unduly large units of grouping (cf. Tables 25 to 28, Appendix). The base unit for brain-weight was 100 gr. Now in calculating the general population mean from these tables one assumes that the individuals in each elemental frequency group centre in brain-weight at the mid-point of that group. Thus, the individuals recorded as having a brain-weight of between 1300 and 1400 grams are assumed to centre at 1350 gr. But evidently this assumption will not be true except at the middle of the whole range. For example, the brain-weights recorded

* This assumes that the mean stature of the Bavarians would be the same in the "young" group as it is in the "total" group. The error introduced by this procedure is practically negligible.

† Here again, on account of lack of data, the mean stature is assumed to be the same in "total" and "young" groups.

‡ See footnote on p. 24.

as between 1500 and 1600 will centre somewhat below 1550. Now, of course, if the distribution is exactly or approximately normal and the number of individuals is sufficiently large, the errors from this cause on one side of the general population mean will balance those on the other, and we shall still be able to get a very close value for this mean from the frequency distribution. Unfortunately, in Matiegka's series, however, the total numbers are not sufficiently large to overcome entirely this error. So we have the following differences: by direct calculation from the individual observations Matiegka finds for the mean brain-weight of the "young" (20—59) series in the male 1450·4 gr., in the female 1305·5 gr. Our corresponding values are 1460·150 gr. and 1313·910 gr. or there is an excess of 9·7 gr. and 8·4 gr. respectively. For the "total" series (20—80) I have calculated from regression values which Matiegka gives (p. 7), based on the individual observations, general population means which gives values as follows: males 1441·4 gr., females 1294·8 gr. The corresponding values from the tables are: males 1454·8 gr., females 1310·9 gr., or the excess of the table means is 13·4 gr. for the males and 16·1 for the females. These differences are small and for practical purposes negligible.

The general conclusion may be drawn that, *apart from all differences in stature, and in the mean age of the samples studied*, the Bavarian mean brain-weight is lower than the Hessian (roughly about 2 °/₀), and the Bohemian mean brain-weight is higher than the Hessian (roughly about 3·5 °/₀).

These results seem to be of some importance as indicating a method whereby brain-weight may have some scientific validity as an anthropological character. This can hardly be said to be the case when nothing more is done than to tabulate means without probable errors, and no attempt is made to get rid of disturbing stature and age effects. Interracial differences in brain-weight may mean something or nothing. With modern biometrical methods it is possible to measure exactly these differences, and when such methods are used it is the belief of the writer that brain-weight can become a really significant anthropological character. In the cases discussed here we see ethnic affinities and differences clearly reflected in the brain-weight. Of the four racial groups studied the two most closely related in origin and general anthropological characters, the Swedes and the Hessians, have mean brain-weights sensibly identical. The Bavarians and Bohemians, though close neighbours geographically, arise from totally distinct ethnic stocks ("Alpine" and Slav respectively) and associated with this we find a wide difference in the weight of the brain.

On the question of the homogeneity of brain-weight statistics, some evidence is furnished in the third and fourth columns of Table IV. If the inclusion of aged persons (50 years and over) introduced any considerable element of heterogeneity so far as brain-weight constants are concerned it would be expected that the interracial differences in the means would not be equal in the "total" and in the "young" series. As a matter of fact in the material here discussed they are sensibly equal, or perhaps better, they are not significantly different. The whole

question of the homogeneity of this material will be treated fully in connection with the discussion of the variability in brain-weight.

The interesting questions brought up by this table of means, regarding sexual differences, association of brain-weight with other characters, etc., will be discussed in later sections of the paper.

5. *Variation in Brain-weight. Homogeneity of Material.*

Variability in the weight of the brain has been subjected to exact investigation by only one worker, Pearson (*loc. cit.*), so far as is known to the writer. He deduced from his material (cf. p. 15 *supra*) coefficients of variation ranging in value from 7·93 °/₀ to 10·64 °/₀, the higher values being from admittedly heterogeneous series.

The numerical values for the variation constants and their probable errors found in the present work are given in Table I.

The question of homogeneity of material should be first discussed. For reasons which have been set forth above it would be hopeless to look for any high degree of homogeneity in any collection of human brain-weighings at present available. The best we can hope for is a fair degree of homogeneity, and reasonably the same degree in different series which are to be compared. Unfortunately there is not available here, as in the case of craniometrical investigations, series in which fair homogeneity can be inferred with high probability, so that it is not possible to make a direct estimate by comparing variabilities with such a "known base." Instead resort must be had to indirect methods. The best of such indirect methods is based on the fact that if a random sample be taken from a *homogeneous* collection of material the variation constants for the sample and the whole collection will not significantly differ. On the contrary if the material is non-homogeneous such a sampling will give different values for the constants. If the sample be *selected*, i.e. not random, the variation constants for the character selected will of course be lowered. Now in the material as treated here we have in the "young" series for each racial group a *selection* from the "total" series, but a selection based on age, not on brain-weight directly. If there were no correlation between brain-weight and age such a selection on an age basis would be, of course, a random sample so far as brain-weight is concerned. Unfortunately for the argument, there is, as will be shown later, a sensible though in general low correlation between age and brain-weight. This being the case it would be expected that such a selection as has been made in separating out the "young" from the "total" series would result in a lowering of the standard deviation and coefficient of variation in brain-weight. Now, as a matter of fact, as the following table shows, the correlation between brain-weight and age is so low that in the relatively small series under discussion, the "young" series forms practically a random sample of the "total" series, within the limits of error, with respect to brain-weight.

TABLE V.

Difference in Variation Constants between "Young" and "Total" Series.

	♂		♀	
	S. D.	C. of V.	S. D.	C. of V.
Swedes	+2·851 ± 4·066	+·122 ± ·291	+ 4·786 ± 5·465	+·271 ± ·435
Hessians	−1·347 ± 3·971	−·177 ± ·285	− ·527 ± 4·541	−·170 ± ·372
Bavarians ...	−1·888 ± 3·556	−·173 ± ·234	−10·357 ± 3·909	−·941 ± ·320
Bohemians ...	+ ·787 ± 4·371	+·026 ± ·304	− 3·545 ± 5·067	−·287 ± ·388
„ (Skull)*	+1·832 ± 4·230	+·078 ± ·295	− 1·932 ± 4·866	−·206 ± ·372

Difference + when "Young" is greater.
„ − „ „ less.

In only one case—the Bavarian females—is the difference as large as its probable error, and in this case the difference is not three times the probable error. The conclusion seems justified that the series are reasonably homogeneous in other respects than age. The age distribution is such as to exclude growth effects, and, as the differences indicate, the effects of senescence on brain-weight are so insignificant that for practical purposes the material may be considered sufficiently homogeneous to warrant further biometrical study. From the character of the differences it would appear that the most homogeneous series are, on the whole, the Swedish and the Bohemian.

A direct examination of the standard deviations and coefficients of variation confirms the conclusion. In the following table I have arranged in order according to ascending value, the variation constants for the "total" series, using as the basis for the grouping the coefficient of variation:

TABLE VI.

Comparison of Different Races in Respect to Variation in Brain-weight and Skull Capacity.

	Males			Females	
	S. D.	C. of V.		S. D.	C. of V.
Swedes	106·329	7·592	Bohemians	96·772	7·382
Bohemians	113·608	7·809	Swedes	100·757	8·043
Hessians	112·675	8·096	Hessians	102·368	8·125
Bavarians	110·664	8·118	Bavarians	101·776	8·340
English†...	124·48	9·20	English†	118·7	9·72
Skull Capacity, English‡	—	8·28	Skull Capacity, English‡	—	8·68
„ „ Germans§	—	7·74	„ „ Germans§	—	8·19

* See p. 17 and Table I. for the series here referred to.
† Pearson: *The Chances of Death*, Vol. I. p. 321.
‡ W. R. Macdonell: *Biometrika*, Vol. III. p. 221.
§ Pearson: *The Chances of Death*, Vol. I. p. 333.

All of the four series used in this paper are in good agreement, both among themselves and with the skull capacity values. They are distinctly lower than the figures from the admittedly heterogeneous, Reid, Peacock, Sims and Clendinning English series, worked over by Pearson. Taking all the evidence together we are driven to the conclusion that the series are fairly homogeneous. If any one of our series is markedly heterogeneous all the others must be equally so, which would be a very improbable result. That the series cannot be very heterogeneous in their make-up is also shown by the values of the coefficients of variation for skull capacity from two admittedly homogeneous series, the Whitechapel skulls studied by Macdonell, and Ranke's Bavarian series.

A comparison of the variability in brain-weight with that shown by other organs and characters of the human body may next be undertaken. I have arranged in the following table (VII) the coefficients of variation for a variety of characters which have been studied by biometrical workers. The arrangement is in general that of descending order of values in the male series. The attempt has been made in the table to include representatives of all the different classes of organs and characters for which we have biometric data available.

The most noticeable and remarkable fact brought out by the foregoing table is that with the exception of capacity, all skull characters are roughly only about half as variable as brain-weight. Some such a relation as this might have been predicted, on the general ground that brain-weight measurements and statistics are on many accounts rather "loose," and would *indicate* a higher variability than would exact measurements on skulls, even though it had no real existence. Such reasoning, however, takes no account of the agreement, which is really remarkably close when we recall the numerous sources of error in brain-weight returns, between skull capacity and brain-weight in their variability. All will admit that capacity is the most difficult skull character to measure accurately, yet no one would maintain that the difference in variability between cephalic index, for example, and skull capacity was entirely, or even in any considerable part, due to the element of error in the measurement of the latter. The agreement in variability between skull capacity and brain-weight is, of course, to be expected on theoretical grounds. That it should turn out in practice to be so close is a first-rate guarantee of the general trustworthiness of brain-weight statistics.

As to the explanation of the great variability in brain-weight and skull capacity as compared with the other skull characters we may tentatively reason about the matter in the following way. If the list of organs and characters given in the foregoing table be examined a natural division into three groups almost immediately suggests itself. First, we have at the bottom of the list the "bone" measurements, in general including all those characters which depend primarily for their values on the dimensions of various parts of the skeleton. These characters give values for the coefficient of variation up to from 5 to 7, certain of the mandibular variabilities exceeding this limit. Next comes the group giving values for the coefficients of from 7 to 10, with the limits fairly sharply marked off.

TABLE VII. *Coefficients of Variation for Man.*

	♂	♀
Weight of Spleen (General Hospital Population)*	50·58	—
" " " (Healthy)†	38·21	—
Dermal Sensitivity‡	35·70	45·70
Weight of Heart (General Hospital Population)*	32·39	—
Keenness of Sight‡	28·68	32·21
Weight of Kidneys (General Hospital Population)*	24·63	—
Weight of Body (Bavarians)	21·32	24·715
Weight of Liver (General Hospital Population)*	21·12	—
Swiftness of Blow‡	19·4	17·1
Weight of Heart (Healthy)†	17·71	—
" " Kidneys (Healthy)†	16·80	—
Breathing Capacity‡	16·6	20·4
Strength of Pull‡	15·0	19·3
Weight of Liver (Healthy)†	14·80	—
Height of Mandible (English, both sexes)§	11·73	11·73
Weight of Body (English)‡	10·37	13·37
Skull Capacity (Etruscan)‖	9·58	8·54
Brain-weight (French)‡	**9·16**	**9·14**
Skull Capacity (Modern Italian)‖	8·34	8·99
" " (English)¶	8·28	8·68
" " (Egyptian Mummies)‖	8·13	8·29
Brain-weight (Bavarian)	**8·118**	**8·340**
" " *(Hessian)*	**8·096**	**8·125**
" " *(Bohemian)*	**7·809**	**7·382**
Skull Capacity (Modern German)‖	7·74	8·19
" " (Naqada)‖	7·72	6·92
Brain-weight (Swedish)	**7·592**	**8·043**
Skull Capacity (Parisian French)‖	7·36	7·10
" " (Aino)‖	7·07	6·90
Mandible, Distance between Foramina mentalia (English, both sexes)§	6·23	6·23
Length of Forearm**	5·24	5·21
" " Femur (French)‡	5·05	5·04
" " Tibia " ‡	4·975	5·365
" " Humerus (French)‡	4·89	5·61
" " Radius " ‡	4·87	5·23
Skull, Height to Breadth Index (English)¶	4·86	4·16
" Breadth to Height " " ¶	4·83	4·17
Length of Finger (English Criminals)††	4·74	—
Skull, Ratio of Height to Horizontal Length (English)¶	4·61	4·10
Length of Foot (English)††	4·59	—
Skull, Cephalic Index for Horizontal Length (English)¶	4·38	3·99
Length of Cubit (English Criminals)††	4·36	—
Skull, Least Breadth of Forehead (English)¶	4·29	4·55
" Height (English)¶	4·21	3·96
" Length of Base (English)¶	4·07	4·11
" Cephalic Index for Greatest Length (English)¶	3·95	4·03
Stature (English)**	3·99	3·83
Skull, Ratio of Height to Greatest Length (English)¶	3·80	4·21
" Greatest Breadth (English)¶	3·75	3·54
" Auricular Height (English)¶	3·73	4·12
" Face Breadth (English Criminals)††	3·707	—
Skull, Cross Circumference (English)¶	3·70	3·97
" Sagittal " " ¶	3·63	3·90
Head Breadth (English Criminals)††	3·333	—
Skull, Length (English)¶	3·31	3·45
Head Length (English Criminals)††	3·154	—
Skull, Horizontal Circumference (English)¶	2·87	2·92

* Greenwood, M.: *Biometrika*, Vol. III. p. 66.

† *Ibid.* p. 67.

‡ Pearson, K.: *The Chances of Death*, Vol. I. pp. 293—377.

§ Macdonell, W. R.: *Biometrika*, Vol. III. p. 225.

‖ *Ibid.* p. 221 (after Pearson).

¶ *Ibid.* p. 222.

** Pearson and Lee: *Biometrika*, Vol. II. p. 370.

†† Macdonell, W. R.: *Biometrika*, Vol. I. p. 202.

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Finally we have all the characters giving values above 10. Now as one passes from one end of the series to the other a definite biological relationship is plain. At the upper end of the series (coefficient of variation 10 and over) the organs and characters tabulated are such as depend in a very considerable degree for their values as determined by measurement, on the general *metabolic condition of the organism* as a whole at the time the measurements are made or immediately before. This will be freely admitted, I think, for such characters as "Dermal sensitivity," "Keenness of sight," "Weight of body," "Swiftness of blow," "Strength of pull," "Breathing capacity" and the like. The same thing, though less apparent perhaps, is, I believe, true when the characters are the weights of viscera. Indeed Greenwood's own results show this to be the case, when we get such different results for means, variabilities and correlation accordingly as we deal with the "general hospital population," "healthy" organs, or the same viscera in diseased conditions of different characters. Wynn* has only recently shown that in rabbits there is a considerable degree of probability that continued administration of digitalis will in a short time raise the mean weight of the heart appreciably, while at the same time the body weight is lowered. It is to be regretted that his series of experiments included so (statistically) few individuals.

Furthermore, it seems reasonable to assume that in this upper group the thing measured in the majority, if not in all cases, is not the thing natural selection has acted upon directly, allowing that it has acted at all. I think it may be fairly assumed that so far as natural selection has acted at all on these organs and characters, the selection has been in the direction of *ability to function properly* so as best to conserve the physiological economy of the organism as a whole, rather than in the direction of absolute size of organ or character. The physiologically balanced functioning with reference to the needs of the organism as a whole is the important thing in such organs as the spleen, pancreas, liver, etc. Absolute size of organ can hardly be a very close direct measure of ability to function well. So then, I am inclined to attribute the high variability observed in those organs and characters falling together in the uppermost part of the table to these two factors: (*a*) the value obtained by measurement depends to a considerable degree on the general metabolic condition of the individual at the time, and (*b*) the thing measured is not the thing with which natural selection, so far as it has acted at all, has had directly to do.

On the other hand, if we turn to the lowest group in the above table (coefficients of variation under 5—7 for the males) we find the two factors just mentioned almost exactly reversed. Here the characters are either of the skeleton directly, or in the case of the anthropometrical data are characters which closely depend for their measured values on the size of portions of the skeleton. Obviously the general metabolic condition of the organism has, within wide limits, little effect on the length of the femur or the skull, for example, and if there is an effect it manifests itself very slowly. Further it is to be presumed that the absolute

* *Journal of the American Medical Association*, Vol. XLIII. pp. 164, 165, 1904.

size is of greater direct selective value in the case of parts of the skeleton than in the case of organs like the liver, etc. In other words, size is much more directly related to proper functioning in the former than in the latter case. The skull may at first sight appear to form an exception here. It must be kept in mind, however, that the skull serves two very important—perhaps equally important—functions; viz., (1) the enclosing and protecting of the brain, and (2) the serving as a basis for the attachment of the complexes of muscles which actuate the masticatory apparatus and support and move the head. With reference to each of these factors natural selection may act on the skull. So far as the second factor at least is concerned, size of skull (in a broad sense) will be of selective value. So that here again, since size and function are interrelated, we should expect to find the results of selective action reflected in the size measurements of characters. In general, I think it can safely be maintained that the low variability shown in the group of characters under discussion is what might be expected to result from the operation of the factors just mentioned.

In the middle group (coefficients of variation ranging from 7 to 10 in the male) including brain-weight and skull capacity we seem to have also an intermediate condition, with respect to the two factors which have been mentioned as among the causes which contribute to the observed variability. In the first place, there can be no doubt in face of the evidence both from the biometrical and anatomical standpoints, that the cranial capacity is quite highly correlated with volume of the brain*. Admitting this, the discussion may be restricted to the brain. Now it has been known from the time of the earliest collection of brain-weight statistics that the weight of the brain is influenced to a certain extent by the general metabolic condition of the individual preceding death. I am inclined to think however that the extent of this influence has been over-estimated. My primary reason for this view comes from the analysis of the statistics themselves. If the metabolic condition of the individual preceding death influenced brain-weight to a marked degree, one would hardly expect to find the agreement shown in the constants of variation and correlation tabulated in this paper, when different groups of individuals with different distribution of ante-mortem affections were compared. We might expect by chance to get two groups in agreement, but on this supposition the odds would be great against getting four groups to agree so closely as they actually do. Furthermore, there is a certain amount of evidence from other sources indicating that the influence of ante-mortem general metabolic conditions on brain-weight, while always present as one factor, does not produce so marked an effect on the weight of the brain as on the size measurements of some of the organs in our most variable group. For example, Mühlmann† studied microscopically the brains of 24 individuals of different ages from birth to 90 years to determine the amount of pigment degeneration in the ganglion cells at the different ages. In the adults the causes of death included such divergent con-

* On the anatomical side, cf. Symington, *Nature*, Vol. 68, pp. 539—544.

† *Verhandl. d. deutsch. Pathol. Gesellsch.* Bd. III. pp. 148—157.

ditions as endocarditis, pneumonia cruposa, and nephritis chronica. Yet the differences found in the appearance of the brain in different cases were clearly associated with differences in age, and not with causes of death. On this whole question more data are needed. Exact measurements of the correlation between skull capacity and brain-weight are much to be desired. We also lack definite biometrical evidence regarding the effect of morbid conditions on the weight of the brain. The evidence at present available seems to lead to the conclusion here indicated.

On the other hand, it can hardly be maintained that natural selection has acted on the size of the brain to the degree that it has on skeletal structures. The brain, in this respect, comes nearer to the organs included in our most variable group. So that, on the one hand with a less marked dependence of the measurement on the previous metabolic condition of the organism than is found in the most variable group of organs and characters, and on the other hand, with a less stringent selection with reference to size as compared with the least variable group, an intermediate condition in variability for the brain is to be expected.

Of course, too much stress cannot be laid on this argument as to the relative variability of the characters of the human body in view of the meagreness of the data at present available. The data at hand, however, plainly suggest some such an explanation, and it is only as a tentative suggestion worthy of being kept in mind as further data are available that the matter has been developed here. To summarise; when a series of the various organs and characters of the body whose variation has been measured are arranged in the order of their relative variability, those characters falling at the most variable end of the series are those whose measured value is affected markedly by the previous general metabolic condition of the organism, and in which ability to function properly is not closely related to size of organ, and in which natural selection, if it has acted at all on the character, has not acted directly upon the thing measured. The opposite end of the series includes the least variable organs and characters, in which both the above mentioned conditions are reversed. Brain-weight and skull capacity occupy an intermediate condition both with reference to variability and the conditioning factors discussed.

Attention may be turned next to the question of the relative variability of the different races in respect to brain-weight. In the following scheme the races are arranged in descending order of mean brain-weight in the left-hand columns, and of variability (measured by the coefficient of variation) in the right-hand column. The lists are based on "total" series.

From these lists it would appear that as a general rule the higher the mean brain-weight is, the lower will be the variability. But it must be kept in mind that with our present series none of the differences in the variability columns is significant. Taking the extremes, Bavarians and Swedes for the males, and Bavarians and Bohemians for the females, the differences between the coefficients of variability with their probable errors are respectively $\cdot526 \pm \cdot217$ and $\cdot958 \pm \cdot337$.

In these cases the difference is less than three times its probable error and cannot be considered as certainly significant. The lists are introduced merely to show the

♂		♀	
Mean	C. of V.	Mean	C. of V.
Bohemians	Bavarians	Bohemians	Bavarians
Swedes	Hessians	Hessians	Hessians
Hessians	Bohemians	Swedes	Swedes
Bavarians	Swedes	Bavarians	Bohemians

general trend of the results. It is possible that with much larger series of brain-weight statistics and consequently reduced probable errors some such relation as that just stated might be definitely proven.

For the sake of comparison and reference I include here tabular arrangements of the means and variabilities for the characters stature and age, made in the same way as that for brain-weight.

Stature.

♂		♀	
Mean	C. of V.	Mean	C. of V.
Swedes	Bohemians	Swedes	Bohemians
Bohemians	Hessians	Bohemians	Hessians
Hessians	Swedes	Hessians	Swedes
Bavarians	Bavarians	Bavarians	Bavarians

Age.

♂		♀	
Mean	S. D.	Mean	S. D.
Bohemians	Hessians	Swedes	Bohemians
Swedes	Bohemians	Hessians	Hessians
Bavarians	Swedes	Bavarians	Swedes
Hessians	Bavarians	Bohemians	Bavarians

Comparing the different characters the means seem to be entirely chaotic, but there is more regularity as regards the variabilities. The relative variabilities in stature and age for the different samples run parallel, with the exception of the Hessians and Bohemians in the male series. The samples which show the greatest variability in age of individuals included, also show the greatest variability in stature, and *vice versâ*. This relation is, of course, to be expected, but the relation between

the brain-weight series and the other two is rather curious. There is no parallelism between the variability columns of the brain-weight series and the other two, but the variability columns of the stature series together with the females of the age series exactly agree (with the exception of the transposition of the Hessians and Swedes) with the columns of *means* of brain-weight. In other words in those series showing the greatest variability in stature (and in the females of age also) we have the highest mean brain-weight, and *vice versâ*. No reason for this curious parallelism is apparent, and it may be purely accidental, but it seems worth noting.

The questions of the relative variability of the sexes in brain-weight and the racial differences as affected by sex will be discussed in a later section of the paper.

6. *On the Nature of the Frequency Distribution.*

In this section the variability in brain-weight will be discussed analytically, according to the methods and nomenclature of Pearson's fundamental memoir on Skew Variation* and its supplement†. My purpose in considering these curves analytically is not primarily that data may be furnished so that the material may be fitted with appropriate curves, but rather in order that definite knowledge may be had as to whether the variation in this character obeys the "normal" law of the deviation of errors. Both Miss Fawcett‡ and Macdonell§ have reached the conclusion that, for practical purposes at least, the majority of skull characters may be considered to conform to this law in their variation. It is of prime importance to determine in how far the same is true of the weight of the brain. Since this was my chief object in analyzing the data, I shall not at this time deal graphically with the curves, but instead shall merely present the chief analytical constants arranged in tabular form. Furthermore, from considerations of the time involved in computation, I have not determined the analytical constants for all the brain-weight frequency distributions given in this paper. Instead, after examining all the data, I decided to confine myself to the "total" series for both sexes of the four races.

The analytical constants for the selected series are exhibited in Table VIII. In the second column is given the number of brains on which the calculation in each case is based. The third column gives the unit in terms of which the second, third and fourth moments about the mean (μ_2, μ_3, and μ_4) are calculated: the next two columns give β_1 and $\sqrt{\beta_1}$, and the two following, β_2 and $3-\beta_2$. Following this are given in order the "criterion" ($\kappa_1 = 2\beta_2 - 3\beta_1 - 6$)‖, the mean, mode and skewness. The skewness was calculated from the moments directly by the formula

$$Sk = \tfrac{1}{2}\frac{\sqrt{\beta_1}(\beta_2+3)}{5\beta_2 - 6\beta_1 - 9},\text{¶}$$

and from this the distance from mean to mode was obtained by multiplying by σ.

* *Phil. Trans.* Vol. 186, A, pp. 343—414.

† *Ibid.* Vol. 197, A, pp. 443—459.

‡ *Biometrika*, Vol. I. p. 443.

§ *Ibid.* Vol. III. p. 227.

‖ *Phil. Trans.* Vol. 197, A, p. 444.

¶ Pearson, K.: "On the Mathematical Theory of Errors of Judgment and on the Personal Equation," *Phil. Trans.* Vol. 198, A, pp. 235—299. Page 277.

TABLE VIII.

Analytical Constants of Curves.

Brain-weight.

Race and Series		No.	Unit	μ_2	μ_3	μ_4	β_1	$\sqrt{\beta_1}$	β_2	$3-\beta_2$	Criterion	Mean	Mode	Skewness
Swede	"Total" ♂	416	50 g.	4·5223	1·6292	57·18995	·0287	·1694	2·7964	·2036	− ·4934	1400·481	1389·627	·1021
„	„ ♀	233	50 g.	4·0608	1·8441	51·1703	·0508	·2254	3·1031	− ·1031	·0539	1252·682	1241·526	·1107
Hessian	„ ♂	475	50 g.	5·0783	− 4·1458	86·8638	·1312	·3623	3·3682	− ·3682	·3428	1391·737	1373·311	·1635
„	„ ♀	281	50 g.	4·1917	− ·2601	49·8514	·0092	·0958	2·8372	·1628	− ·3531	1259·875	1254·294	·0545
Bohemian	„ ♂	372	100 g.	1·2907	·5908	5·2300	·1623	·4029	3·1396	− ·1396	− ·2078	1454·839	1430·292	·2161
„	„ ♀	197	100 g.	·9365	·1033	2·5144	·01298	·1139	2·8671	·1329	− ·3048	1310·914	1304·762	·0636
Bavarian	„ ♂	529	50 g.	4·8986	2·4919	79·2516	·0528	·2298	3·3026	− ·3026	·4467	1363·185	1352·047	·1006
„	„ ♀	323	50 g.	4·1433	− ·9391	77·7815	·0124	·1114	4·5408	−1·5408	3·0444	1220·356	1217·221	·0308
„	"Young" ♀	238	50 g.	3·3431	3·0329	45·6227	·2462	·4962	4·0820	−1·0820	2·4255	1235·504	1219·334	·1769

Considering first the *skewness*, it is seen to be in all the series positive, or the mean is greater than the mode. Further we note that in all cases the value is low. Whether the values can be considered significant however can only be determined by an examination of the probable errors. The formula for the probable error of the skewness is $\cdot 67449\sqrt{\frac{3}{2n}}$ *. The limiting values for this probable error for values of n ranging from 197 to 529 as in the present case are respectively ·0589 and ·0359. Having regard to the number of cases on which the calculations are based it appears that in six out of the nine cases tabulated the skewness can be regarded as certainly or probably insignificant. In all of these six cases the skewness is less than thrice its probable error, in two cases it about equals its probable error, and in one is less. The remaining three cases out of the total (skewness, ·1635, ·2161 and ·1769) are very probably or certainly significant. In general we may safely conclude, I think, that, *in the case of the weight of the brain, the distance from the mean to the mode will be very small. If the mean and mode do not coincide the mean will be greater than the mode.* This agrees with Miss Fawcett's† conclusion for the most important skull characters in the Naqada race. Macdonell‡ finds, however, that in the case of the English, considering the same skull characters, "if we were to draw the curves, the mean would be found in half the number of the curves to be less, and in the other half to be greater than the mode." In neither brain-weight nor skull series does there appear to be any definite preponderance in the value of the skewness of one sex over the other.

We may turn now to the other constants, which are of most significance in determining whether the distribution may be considered normal within the limits of error; viz., $\sqrt{\beta_1}$, β_2, and the criterion. The probable error of $\sqrt{\beta_1}\left(=\cdot 67449\sqrt{\frac{6}{n}}\right)$ ranges in value for our series between ·0718 ($n=529$), and ·1177 ($n=197$); that of $\beta_2\left(=\cdot 67449\sqrt{\frac{24}{n}}\right)$ between ·1437 ($n=529$) and ·2354 ($n=197$); and that of the criterion $\left(=\cdot 67449\sqrt{\frac{96}{n}}\right)$ between ·2873 ($n=529$) and ·4708 ($n=197$). Considering the probable errors of $\sqrt{\beta_1}$, it is seen at once that of the eight "total" series three give certainly insignificant values (·0958, ·1139, ·1113) for $\sqrt{\beta_1}$; two others give values which are probably insignificant (·1694, ·2254). One (·2298) is probably significant; and the two remaining values (·3623 and ·4029) are certainly significant. The Bavarian "young" ♀ series gives a certainly significant value for $\sqrt{\beta_1}$. Taking next the deviation of β_2 from 3 in comparison with the probable error of β_2 we see that in four cases ($3-\beta_2=-\cdot 1031$, ·1628, $-\cdot 1396$, ·1329) β_2 differs from 3 by an insignificant amount. In one case (·2036) the difference is less than twice the probable error and hence may be considered

* The formulae for the probable errors of the analytical constants are given on p. 278 of Pearson's memoir on "The Mathematical Theory of Errors of Judgment," *loc. cit. supra.*

† *Loc. cit.* p. 443.

‡ *Biometrika*, Vol. III. p. 227.

as very probably insignificant. In two cases (·3682 and ·3026) the difference is less than thrice the probable error and hence may be considered possibly, or even perhaps, probably insignificant. The Bavarian ♀ series, both "total" and "young," give certainly significant values for $3-\beta_2$. Considering finally the criterion it is seen that in all cases except the two Bavarian ♀ the criterion differs from zero by a certainly or very probably insignificant amount. These two Bavarian ♀ series differed so greatly from the normal curve in most of the analytical constants that it was thought desirable to determine their position precisely by means of another constant κ_2*. For the "total" series I found $\kappa_2 = ·0032$, and for the "young" series $\kappa_2 = ·0816$. By the scheme given by Pearson (*loc. cit.* p. 445) we see that the "total" series, when the probable errors of the constants are considered, comes very close to the condition demanding a curve of Type II ($\kappa_2 = 0$, $\beta_1 = 0$, β_2 not $= 3$). The "young" series clearly demands a curve of Type IV ($\kappa_2 > 0$ and < 1). The deviation of these Bavarian female curves from the normal type I believe to be due to an undue accumulation of individuals in one brain-weight class; viz., that from 1250—1300 gr. It seems altogether probable that some of the individuals which should have gone into the next higher class (represented in the "total" series by a frequency of only 26 as against 69 in the class next lower) have by some error been entered in Bischoff's lists with too low brain-weights. What the source of error was it is, of course, impossible now to determine. The abnormality of the Bavarian females has already been noted in the discussion of the means and variabilities. Leaving these two series out of account I think that on the whole we may safely conclude, as Miss Fawcett and Macdonell (*loc. cit.* p. 443, and p. 227 resp.) have for skull characters, that:

With series of brain weighings such as are considered in this paper we shall reach for most practical purposes adequate graphical representations of the frequency by using the normal curve of deviation: $y = y_0 e^{-x^2/2\sigma^2}$.

It should always be kept in mind, however, that our series, both on the brain-weight and skull sides, are too small to fix absolutely the normality or non-normality of the variation in these characters. Some of the distributions certainly differ from normality. The conclusion stated above is to be considered simply as a *practical* result, rather than as a theoretical generalization.

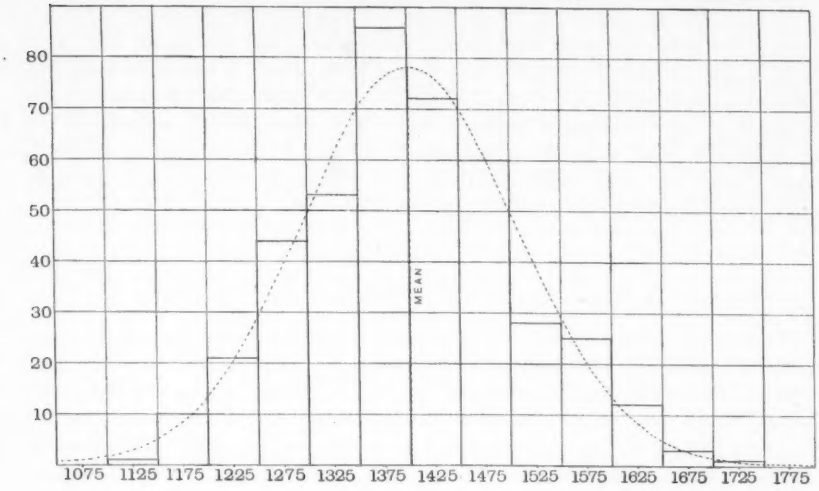
This result seems to be of considerable importance as indicating the worth of brain-weight statistics. It shows that such statistics justify careful study and analysis, and that, contrary to the statements of certain recent writers on the subject, there is no general fallacy inherent in the data themselves which renders abortive any attempt to reach through them the truth regarding the mass relations of the brain.

In order to test exactly how well the normal curve represents the data in a single case I have fitted the Swedish male "total" series with a normal curve. The frequency histogram and its fitted curve are shown graphically in the

* $\kappa_2 = \dfrac{\beta_1(\beta_2+3)^2}{4(\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)}$, Pearson, *Phil. Trans.* Vol. 197, A, p. 444.

accompanying diagram. The unit of x is 50 grams. The scale of frequency at the left of the diagram gives y.

DIAGRAM I. Showing the distribution of variation in the brain-weight of adult Swedish Males.



The general equation $y = y_0 e^{-x^2/2\sigma^2}$, where in this case $y_0 = \dfrac{416}{\sigma\sqrt{2\pi}}$ and $\sigma = 2{\cdot}1266$, becomes $y = 78{\cdot}0401\, e^{-{\cdot}01106x^2}$, with the origin at 1400·481. Applying Pearson's test of the goodness of fit*, and comparing *areas* instead of mid-ordinates, as is necessary where the number of frequency groups is so small, we get the results shown in the accompanying table. The table gives the observed frequencies (m_r'),

Grams of brain-weight	Observed	Calculated	$\frac{(m_r - m_r')^2}{m_r}$
Under 1100	0	·981	·981
1100—1150	1	2·9	1·24
1150—1200	10	8·5	·26
1200—1250	21	20·3	·02
1250—1300	44	39·0	·64
1300—1350	53	60·4	·91
1350—1400	86	75·2	1·55
1400—1450	72	75·3	·14
1450—1500	60	60·8	·01
1500—1550	28	39·4	3·29
1550—1600	25	20·6	·94
1600—1650	12	8·7	1·25
1650—1700	3	2·9	·003
1700—1750	1	·8	·05
1750 and over	0	·036	·036
Totals	416	415·817	11·320

* *Phil. Mag.* Vol. L. pp. 157—175.

the calculated frequencies (m_r), and the ratio of the squared difference between the two to the calculated $\left(\frac{(m_r - m_r')^2}{m_r}\right)$.

Here n' the number of frequency groups is 15 and $\chi^2 = 11{\cdot}32$. From Elderton's Tables* the value of P may be obtained. With $n' = 15$ and $\chi^2 = 11$ the value of P as given in the table is ·686036. Or, expressed in words, if the brain-weight of Swedish males obeyed the "normal" distribution of frequencies we should expect to get a greater divergence between theory and observation in roughly 65 out of every 100 trials when the trials were based on random samples of 416 individuals each. In other words, the fit may be considered very fair, and certainly entirely satisfactory for all practical purposes.

7. *The Correlation of Brain-weight with other Characters.*

In the introductory portion of this paper it was stated that one of the chief objects for which the work was undertaken was to measure exactly the degree and nature of the association between brain-weight and other characters in man. All workers on the subject have realized the importance of such determinations and numerous attempts have been made to arrive at them. The methods used, however, have not been such as to lead to definite and sound conclusions. These methods have in the main consisted in tabulating the mean brain-weights for various groupings of the other characters under consideration. In this way it can of course be determined whether there is any shift of the mean as the other character changes. Further than this one cannot go. Besides, in this method there lurk numerous pitfalls unless one uses it with a clear understanding of some of the fundamentals of statistical science. Different results can be obtained as the material is differently grouped. Hence, it is not strange that we find conservative anatomists making only very general statements as to the correlation of brain-weight with other characters, and, in those cases where an attempt at greater precision of statement is made, considerable difference of opinion as to what conclusions shall be drawn from the data. What are wanted in the case are not simply inspections and general "appreciations" of tabular lists of brain-weighings, or of ratios and indices, or even of correlation tables or regression lines, but instead definitely determined coefficients of correlation with their probable errors. With these we can get in given cases precise statements of the degree of correlation between different characters.

It would of course be highly desirable to measure the correlation between the weight of the brain and a great variety of other organs and characters of the body. Such characters will at once suggest themselves to anyone interested in the problems of correlation. For instance, aside from age, stature and body-weight for which material is available, there are the whole range of skull characters; the

* *Biometrika*, Vol. I. p. 161.

weights of the various viscera; the weight and volume of the spinal cord; characters of the muscular system, etc. A study of the correlation between any or all of these characters and the weight of the brain would be most interesting. But, unfortunately, material for such study cannot be had. Brain-weight statistics usually provide data for but few other characters; in the majority of cases only sex, age and stature. In two of the series used in this work, viz., the Swedish and Hessian, this was the case. For these two groups I have determined all possible correlations for both sexes, and for two age groupings ("young" and "total"). This gives as the pairs of characters, (*a*) brain-weight and stature, (*b*) brain-weight and age, (*c*) stature and age. Bischoff's Bavarian material furnished in addition to the above, data on the body-weight. The following correlations have been determined for this material: (*a*) brain-weight and age ("total" and "young"), (*b*) brain-weight and stature ("total"), (*c*) brain-weight and body-weight ("total"), (*d*) body-weight and stature ("total"). In the case of the Bohemian material data were furnished from which the correlation between skull length and skull breadth could be determined so that for this group I have been able to calculate coefficients of correlation for the following pairs of characters: (*a*) brain-weight and stature ("young"), (*b*) brain-weight and age ("total"), (*c*) brain-weight and skull length ("young"), (*d*) brain-weight and skull breadth ("young"), (*e*) skull length and skull breadth ("young"). In Table IX are exhibited the coefficients of correlation together with their probable errors. The correlation tables from which the coefficients were calculated are given in the appendix to this paper, Tables 1 to 44. References are given in Table IX telling for each coefficient the original table from which it was calculated. The coefficients of correlation were determined from the usual Bravais formula according to which the coefficient of correlation

$$r = \frac{S(xy)}{N\sigma_1\sigma_2},$$

where x and y are deviations from the means of the two correlated characters and σ_1 and σ_2 are the respective standard deviations; the usual* method of evaluating r from the above equation was used.

The most striking fact of a general nature shown by this table is the generally low degree of correlation which exists between the weight of the brain and other characters. The coefficients run noticeably lower even than those of skull characters (cf. Macdonell's Table V†) and very markedly below those between different characters of the long bones‡. On the other hand our brain-weight correlations give values of the same general order of magnitude as those found by Greenwood§ for various abdominal and thoracic viscera. Attention may be called

* Yule: *Jour. Roy. Stat. Soc.* Vol. LX. Part IV. pp. 1—44 (Reprint).

† *Biometrika*, Vol. III. p. 232.

‡ Lee and Pearson: *Phil. Trans.* Vol. 196, A, pp. 228, 229.

§ *Loc. cit.* p. 19 above.

TABLE IX.

Brain-weight. Correlations.

Character and Series	Swedes ♂ No.		Table	Swedes ♀ No.		Table	Hessians ♂ No.		Table	Hessians ♀ No.		Table
Brain-weight and Stature (Total) ...	416	·1830 ± ·0320	1	233	·3490 ± ·0388	2	475	·1823 ± ·0299	13	281	·1828 ± ·0389	14
" " " " (Young) ...	262	·1796 ± ·0403	3	127	·3390 ± ·0530	4	291	·1741 ± ·0383	15	173	·1809 ± ·0496	16
Brain-weight and Age (Total)	416	− ·2493 ± ·0310	5	233	− ·2336 ± ·0418	6	475	− ·1673 ± ·0300	17	281	− ·3598 ± ·0350	18
" " " " (Young)	262	− ·1705 ± ·0405	7	127	− ·1512 ± ·0585	8	291	− ·0750 ± ·0393	19	173	− ·1650 ± ·0499	20
Stature and Age (Total)	416	− ·1620 ± ·0322	9	233	− ·2279 ± ·0419	10	475	·0481 ± ·0308	21	281	− ·0915 ± ·0399	22
" " " (Young)...	262	− ·2549 ± ·0390	11	127	− ·2120 ± ·0572	12	291	·2102 ± ·0378	23	173	− ·0474 ± ·0512	24

Character and Series	Bohemians ♂ No.		Table	Bohemians ♀ No.		Table	Bavarians ♂ No.		Table	Bavarians ♀ No.		Table
Brain-weight and Stature (Total) ...							365	·1664 ± ·0343	35	241	·2236 ± ·0413	36
" " " " (Young) ...	266	·2034 ± ·0397	25	133	·2168 ± ·0557	26						
Brain-weight and Age (Total)	372	− ·2045 ± ·0335	27	197	− ·2558 ± ·0449	28	529	− ·1225 ± ·0290	37	323	− ·2405 ± ·0354	38
" " " " (Young)							365	− ·0100 ± ·0353	39	238	·0114 ± ·0412	40
Brain-weight and Skull Length (Young)	299	·5482 ± ·0273	29	159	·3604 ± ·0465	30						
" " " Skull Breadth (Young)	299	·4655 ± ·0306	31	159	·5041 ± ·0399	32						
Brain-weight and Body-weight (Total) ...							365	·1671 ± ·0343	41	241	·2260 ± ·0412	42
Skull Length and Skull Breadth (Young)	299	·4006 ± ·0327	33	159	·1954 ± ·0514	34						
Body-weight and Stature (Total) ...							365	·3962 ± ·0298	43	241	·4334 ± ·0352	44

here to the general uniformity of the correlations for the same characters in different series. All the brain-weight and stature correlations are seen to be positive, while with a single exception (Bavarian female "young" series) all the correlations of brain-weight with age are negative. The coefficients are generally lower for the "young" than for the "total" series, as is to be expected. In the case of the correlation of brain-weight with age, some of the coefficients for the "young" series are evidently insignificant in comparison with their probable errors. The correlations between stature and age are negative with the exception of the Hessian male series. The positive sign in these series arises from the fact that in this material the age class 15 to 20 was included. All growth in stature has not stopped at age 20, and as a consequence there are included in the tables (21 and 23) five individuals of unusually small stature and low age. These serve to change the sign of the coefficients. Their greater effect in the "young" series is apparent. The detailed discussion of the various brain-weight correlations and the regressions based on them I propose to take up in separate sections of the paper now to follow.

8. *Brain-weight and Sex.*

All brain-weight statistics show that the brain of the male is absolutely heavier than that of the female. In the series here discussed the absolute differences in mean brain-weight between males and females are as follows:

	Male mean—Female mean	
	"Total" series	"Young" series
Swedes ...	147·8	145·8
Bohemians ...	143·9	146·2
Bavarians ...	142·8	133·6
Hessians ...	131·9	125·6

From this table the following points are to be noted:

(*a*) Considering the size of the probable errors involved it is evident at once that the *absolute* difference is sensibly the same for all four races. Taking the extremes of the "total" column the difference between the Swede and Hessian sex differences is 15·9 with a probable error of ± 7·9, or in other words the difference is almost exactly twice its probable error and cannot be considered certainly significant. The "young" series points to the same conclusion. So then, Weisbach's law for stature that the greatest sex differences occur in those races having the highest mean stature, does not appear to hold for brain-weight, so far as absolute differences are concerned.

(*b*) The absolute sex differences are sensibly the same for both age groupings. We may conclude then, that whatever changes occur in the weight of the brain with advancing age act in such a way as to leave the absolute sex difference unchanged.

These absolute differences may be converted into relative differences by expressing them as percentages of the ♂ and ♀ means. When so expressed they take the following form:

Sex Differences in Relation to Means.

Race	"Total" series		"Young" series	
	Percentage of ♂ mean	Percentage of ♀ mean	Percentage of ♂ mean	Percentage of ♀ mean
Swedes ...	10·6 %	11·8 %	10·3 %	11·5 %
Bavarians ...	10·5 %	11·7 %	9·8 %	10·8 %
Bohemians ...	9·9 %	10·97 %	10·0 %	11·1 %
Hessians ...	9·5 %	10·5 %	8·9 %	9·8 %

The relative differences confirm the conclusions reached from the examination of absolute differences. The result that the changes which take place in the weight of the brain during adult life affect both sexes to an equal degree relatively seems to be of some importance. It implies that the factors which bring about the gradual lowering in the weight of the brain with advancing age are organic in a strict sense, and not dependent on environmental conditions.

The same thing is shown if the relative differences expressed in another way, namely by taking the ratio of male to female mean, are examined. For our series these ratios are as follows:

Sex Ratios. Means. Brain-weight.

Race	"Total" series	"Young" series
Swedes ...	1·118	1·115
Bavarians ...	1·117	1·108
Bohemians ...	1·110	1·111
Hessians ...	1·105	1·098

The sex ratios for a number of other characters are given in the following table for comparison. The values in this table are taken from a table in Pearson's *Chances of Death**.

* Vol. I. p. 374.

Sex Ratios. Means. Various Physical Characters.

Organ or Character	Sex Ratio
Body weight, Babies	1·034
„ „ Children	1·038
„ „ Adults	1·193
Weight of Vital Organs	1·130
Stature, Children	1·007
„ Adults	1·077
Height, Sitting	1·032
Long Bones	1·086
Chest Girth	1·024
Squeeze of Hands	1·207
Keenness of Sight and Touch ...	1·061
Skull Capacity	1·124
„ Circumference	1·042
Cephalic Index	·997
Head Index	·995
Profile Angle	·994
Alveolar Angle	·994
Nose and Palate	1·013

We may turn next to the question of the relative variability of the sexes in respect to brain-weight. Pearson* found that in the case of the English, the female showed "slightly more" variability than males; the French data which he worked over gave sensibly equal variabilities for the sexes, as did also Bischoff's Bavarian data. In the following table are shown the differences between the male and female coefficients of variation with the probable errors of the differences. The values tabulated are the male minus the female constant in each case.

Relative Variability of the Sexes. Male – Female.

Race	Coefficient of Variation	
	"Total" series	"Young" series
Bohemians ...	·427 ± ·320	·740 ± ·375
Hessians ...	– ·029 ± ·293	– ·036 ± ·366
Bavarians ...	– ·222 ± ·254	·546 ± ·305
Swedes ...	– ·451 ± ·310	– ·600 ± ·422

In three cases out of the eight the male has a larger coefficient of variation, but in no instance can the differences between male and female coefficients be considered significant when compared with the probable errors. In only one case (Bohemian "young" series) does the difference approach closely to a value even twice as great as its probable error. Therefore we must conclude that, so far as the series here considered are concerned, there is no significant difference between

* *Chances of Death*, Vol. I. pp. 321, 322.

males and females in respect to variability in the weight of the brain. In view of the general reliability, from a statistical standpoint, which has been previously demonstrated to exist in the material on which this work is based, it seems not unreasonable to predict that it will be found that equal variability of the sexes in brain-weight holds generally.

Another problem which presents itself in this connection is as to how much of the observed difference between males and females in brain-weight is to be accounted for by the fact that the male body is in general larger than the female. In other words, would a group of females of the same stature and age, say, as a given group of males have the same brain-weight? It is possible to reach a general solution of this problem by the use of characteristic equations based on the correlations and regressions. I propose now to discuss this question by this method. Of course, the prediction may be made by characteristic equations in either direction; i.e., we may predict the probable brain-weight of a group of males having other characters the same as in a given group of females, or we can predict the probable brain-weight of a group of females having the characters the same as in a given group of males. In the former case characteristic equations based on the male data would be used, in the latter the equations would be those deduced from female data. We may consider first the characters, age, and stature. In the following tables are given (*a*) the probable brain-weights of a group of individuals of the sex indicated, as calculated from the characteristic equation referred to in the last column of the table; (*b*) the observed brain-weights of a group of the opposite sex having for the other characters the mean values used in the characteristic equation in calculating (*a*); (*c*) the difference between (*a*) and (*b*); (*d*) a reference to the equation from which (*a*) is calculated.

TABLE X.

Probable Female Brain-weight with Stature and Age equal to Observed Male.

Race	Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Hessian (Total) ...	1287·963	1391·737	103·774	25
„ (Young) ...	1308·941	1405·756	96·815	26
Swede (Total) ...	1307·340	1400·481	93·141	21
„ (Young) ...	1319·783	1415·267	95·484	22

The mean difference here is 97·304. The mean observed difference between male and female without any allowance for stature and age is 137·8 for the same races. So then the differences in stature and age between the males and females in our Swedish and Hessian samples account for only 40·5 gr. or about 29 per cent. of the observed difference in mean brain-weight between the sexes.

Let us now proceed to the reversed prediction, and find the probable brain-weights of a group of males having the same stature and age as the means for

these characters in the Hessian and Swedish females. The results are shown in Table XI, which is arranged in the same manner as the preceding table.

TABLE XI.

Probable Male Brain-weight with Stature and Age equal to Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Hessian (Total) ...	1357·591	1259·875	97·716	23
„ (Young) ...	1369·431	1280·202	89·229	24
Swede (Total) ...	1370·543	1252·682	117·861	19
„ (Young) ...	1391·624	1269·488	122·136	20

The mean difference here is 106·736, or in other words we reach essentially the same result as before. The general conclusion up to this point then is that there is a difference in the mean brain-weight between the sexes of roughly 100 gr. after allowance has been made for stature and age differences by taking the mean brain-weight of an array of individuals of one sex having for type of stature and age the respective means of these characters observed in the other sex. Differences in stature and age account for less than one-third of the observed sex difference in brain-weight.

We may next determine what part of the observed sex difference in brain-weight is to be accounted for by differences in skull length and breadth. Here only one race, the Bohemian, can be used on account of lack of material. Let us examine the effect on the mean brain-weight of selecting a group of individuals of one sex having the same mean skull length and skull breadth as the opposite sex. Making the designated selection first from the males and then from the females we have the results shown in Table XII.

TABLE XII.

Probable Male Brain-weight with Mean Skull Length and Skull Breadth equal to the Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Bohemian ...	1395·631	1310·914	84·717	35

Probable Female Brain-weight with Mean Skull Length and Skull Breadth equal to the Observed Male.

Race	Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Bohemian ...	1389·736	1454·839	65·103	36

The mean of the two differences is 74·910, while the observed difference between male and female mean brain-weights in the Bohemian "skull" series is 142·014, or in other words by selecting from either sex individuals with mean skull length and skull breadth equal to those found in the opposite sex, we are able to reduce the sex difference in mean brain-weight by about 67 grams, or 47 °/₀. This result is of considerable interest. We can come nearer to the brain-weight of one sex by selecting skull length and breadth to the means of that sex than we can by selecting stature and age, but still not a great deal nearer. We should on general grounds expect that aside from skull capacity, skull length and skull breadth would be the two characters whose selection would indirectly modify brain-weight the most. Admitting this to be the case there still remains a considerable difference to be accounted for. It is interesting to note that the sexes are brought nearer together in brain-weight when males are selected than when the selection is from the females. This is explained by the fact that the brain-weight is somewhat more highly correlated with the skull characters in the males in our series.

Finally, we may examine the effect of a selection of stature and body-weight on the sex difference in brain-weight. The material here comes from the Bavarian series, and the prediction is made in the same way as in the other cases; viz., by selecting males with mean stature and body-weight equal to the female means and *vice versâ*, and determining the mean brain-weight of the selected groups. The results are shown in Table XIII.

TABLE XIII.

Probable Male Brain-weight with Mean Stature and Body-weight equal to the Observed Female.

Race	Predicted male brain-weight	Observed female brain-weight	Difference	Equation
Bavarian ...	1321·496	1218·776	102·720	29

Probable Female Brain-weight with Mean Stature and Body-weight equal to the Observed Male.

Race	Predicted female brain-weight	Observed male brain-weight	Difference	Equation
Bavarian ...	1261·298	1357·466	96·168	30

Here the better result is obtained by selecting from the females on account of the higher correlation between brain-weight and body-weight and brain-weight and stature in this sex. The mean of the two differences is 99·444, and the observed sex difference in the case is 138·690 gr. So then the differences between

the sexes in body-weight and stature account for 39·246 gr. or 28 °/₀ of the observed difference in mean brain-weights. In other words, by selecting individuals on the basis of body-weight and stature alone we shall produce about the same degree of change in the mean brain-weight as by selecting on the basis of stature and age alone.

Taking all the results together we reach the general conclusion that the difference between the sexes in mean brain-weight is only in part to be accounted for by differences in other bodily characters. While in general it is true that a small body has a brain of low weight, yet in order for the observed difference in mean brain-weight in men and women to be due to this factor alone either the women ought to have very much smaller bodies than they actually possess, or the men ought to have larger bodies, or both sexes ought to be different from what they actually are in size of body in the directions indicated. Of course it is very easy to say by way of speculation that the smaller brain of woman is due to the fact that the female human organism is subjected to less strenuous demands along the lines of motor and intellectual activity than the male. But such speculation leads to nothing on account of our lack of definite scientific evidence as to the degree of correlation between the weight of the brain and amount and intensity of psychic activity.

One other question remains to be considered in connection with the relation of brain-weight to sex. Does either sex show any considerably higher degree of correlation between brain-weight and other characters than the other? Table IX furnishes data on this point. With the exception of the Swedish brain-weight and age correlations and the brain-weight and skull-length correlations it is seen that the female coefficients are uniformly larger than the male. The differences are in many of the cases not significant in comparison with their probable errors but the general tendency towards higher correlations in the females is clearly evident. The meaning of this tendency is in this case difficult to conjecture. It falls in line with the result which has been pointed out by Lee and Pearson in several papers*, viz., that for physical characters generally women usually show a higher degree of correlation than men, in civilized races.

9. *Brain-weight, Age and Stature.*

The coefficients of correlation between brain-weight and age and stature respectively have already been given in Table IX, but before proceeding further in the discussion of their significance it is necessary to determine whether the regressions are linear. One might on general grounds expect the brain-weight and age correlations to be skew, since this is generally the case with growth correlations where one of the variables is age and the other the absolute magnitude of some organ or character. If in any case the correlation turns out to be skew, it of course greatly complicates the problem.

The method of analysis which I have followed in determining the degree of

* E.g., "On the Relative Variability and Correlation in Civilized and Uncivilized Races," *Roy. Soc. Proc.* Vol. LXI. pp. 343—357.

approach to linearity of the regressions is that given by Pearson*. The two constants which are of the greatest significance here are, (*a*) the mean square deviation of the means of the arrays from the regression line, Σ_M^2, and (*b*) a constant η, called the correlation ratio, giving the mean reduction in variability of an array as compared with the whole population. Evidently

$$\Sigma_M^2 = \sigma_M^2 - r^2\sigma^2 \quad \text{.....(i)},$$

and

$$\eta = \frac{\sigma_M}{\sigma} \quad \text{.....(ii)},$$

whence by simple substitution $\Sigma_M^2 = (\eta^2 - r^2)\sigma^2$ .....(iii),

where in these equations σ is, as usual, the standard deviation of the variates about the mean for the whole population, r is the coefficient of correlation between the two variables concerned, and σ_M is the standard deviation of the means of the arrays about the mean of these means. The deviation of Σ_M from zero, and of η from r, measure the deviation of the system from linearity†.

I propose to discuss the relation of brain-weight first to age alone, then to stature alone, and finally to both age and stature together.

In Table XIV are exhibited the values of Σ_M and η for the correlations between brain-weight and age discussed in this paper. In calculating σ_M from which to obtain η according to the relation given above, in this and all other cases the means of the arrays were weighted with the number of cases on which they were based. This procedure of course gave the mean of the means of the arrays the same value as the general population mean calculated from the elemental frequency distribution.

TABLE XIV.

Analytical Constants for Linearity of Regression. Brain-weight and Age.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Swedish (Total) ...	$-\cdot2493 \pm \cdot0310$	$-\cdot2336 \pm \cdot0418$	·2876	·2770	$\cdot1434\,\sigma$	$\cdot1489\,\sigma$
„ (Young)...	$-\cdot1705 \pm \cdot0405$	$-\cdot1512 \pm \cdot0585$	·2251	·2143	$\cdot1469\,\sigma$	$\cdot1519\,\sigma$
Hessian (Total) ...	$-\cdot1673 \pm \cdot0300$	$-\cdot3598 \pm \cdot0350$	·2002	·3864	$\cdot10996\,\sigma$	$\cdot1409\,\sigma$
„ (Young)...	$-\cdot0750 \pm \cdot0393$	$-\cdot1650 \pm \cdot0499$	·1411	·1961	$\cdot1195\,\sigma$	$\cdot1060\,\sigma$
Bavarian (Total) ...	$-\cdot1225 \pm \cdot0290$	$-\cdot2405 \pm \cdot0354$	·1962	·3481	$\cdot1533\,\sigma$	$\cdot2517\,\sigma$
„ (Young)...	$-\cdot0100 \pm \cdot0353$	$\cdot0114 \pm \cdot0412$	·0676	·1958	$\cdot0669\,\sigma$	$\cdot1955\,\sigma$
Bohemian (Total) ...	$-\cdot2045 \pm \cdot0335$	$-\cdot2558 \pm \cdot0449$	·2441	·3033	$\cdot1333\,\sigma$	$\cdot1628\,\sigma$

* *Roy. Soc. Proc.* Vol. LXXI. pp. 303—313, especially the footnote, pp. 303, 304. Since the above was written a very full treatment of the whole subject of skew correlation and non-linear regression by Prof. Pearson has appeared as "Mathematical Contributions to the Theory of Evolution, XIV." *Drapers' Company Research Memoirs*, Biometric Series II. 1905.

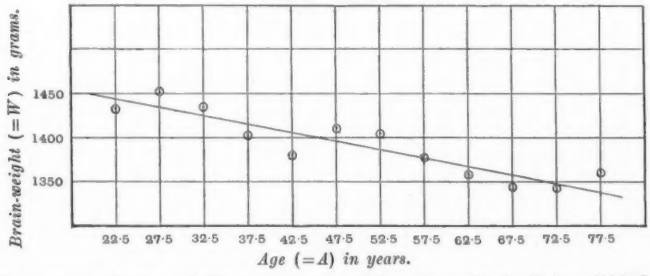
† Of course the deviation of η from r must be considered numerically simply, because η is necessarily a positive quantity from (ii) above, since neither σ_M nor σ can be negative. If the difference between η and r were taken with reference to sign in cases where r is minus, an altogether false notion of the degree of departure of the regression from strict linearity would be obtained. The degree of this departure will always, of course, be immediately given by Σ_M whatever the sign of r.

It is seen at once that η differs from r and Σ_M from zero in all cases. This, of course, implies departure of the system from linearity. But evidently deviations of a system from linearity may be due to either one or both of two causes. Either, on the one hand, the system may be truly non-linear, in which event the means of the arrays will be fitted better by some curve than by a straight line, or, on the other hand, the points fixed by the means of the arrays may not lie exactly on a straight line and still no curve will represent the relationship between the two variables concerned better than a straight line so drawn that the mean square of the deviations of the points from the line is a minimum. In the first case we have true non-linearity of the regression, while in the second the deviation from linearity is due to the errors of random sampling, and it might reasonably be expected that if the whole population could be studied the regression would become strictly linear. Now evidently in both these cases η will differ from r and Σ_M from zero, so that recourse must be had to some further method in order to determine into which class a given case falls. Two such methods immediately suggest themselves: one, to examine the probable errors involved, the other, to inspect the fitted regression line.

An examination of Table XIV makes it immediately evident that the differences between η and r (without regard to sign) give values of the same order of magnitude as the probable errors of r. In only one case is this difference as great as three times the probable error of r, and in the great majority of cases it does not approach such a value*. So then it seems probable that in our series the regression of brain-weight on age is linear within the errors arising from random sampling. The approach to linearity is sensibly the same both for the whole period of adult life and for the younger half of this period.

In order to bring out the facts graphically I have had prepared a series of diagrams showing the regression lines for brain-weight on age. For the sake of economizing space, and since there is essential agreement between the different series with respect to η, it was decided not to publish all the regression diagrams. I have chosen for representation here the regression lines for the Swedish and the Hessian data. These are shown in Diagrams II to IX.

DIAGRAM II. Probable Brain-weight for given Age.
Swedish ♂ Total.



[* The proper test is the probable error of $\eta \sim r$, which we hope will shortly be published. ED.]

DIAGRAM III. Probable Brain-weight for given Age.

Swedish ♀ Total.

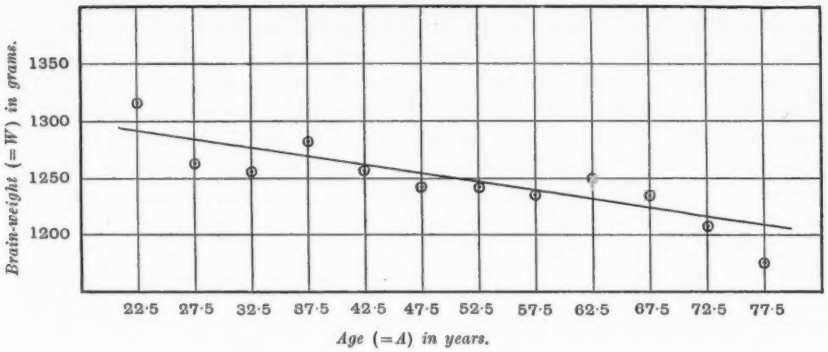


DIAGRAM IV. Probable Brain-weight for given Age.

Hessian ♂ Total.

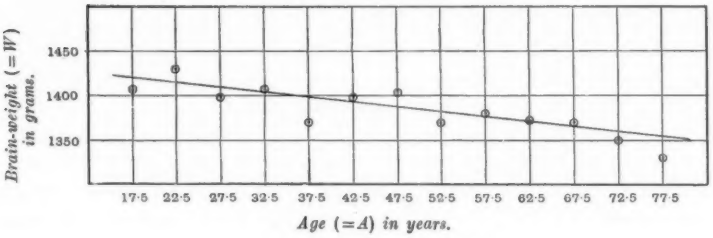


DIAGRAM V. Probable Brain-weight with given Age.

Hessian ♀ Total.

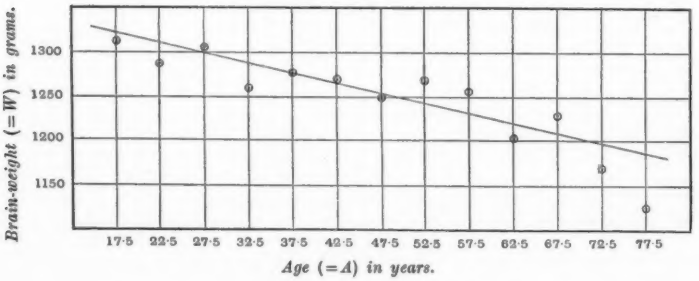


DIAGRAM VI. Probable Brain-weight with given Age.
Swedish ♂ Young.

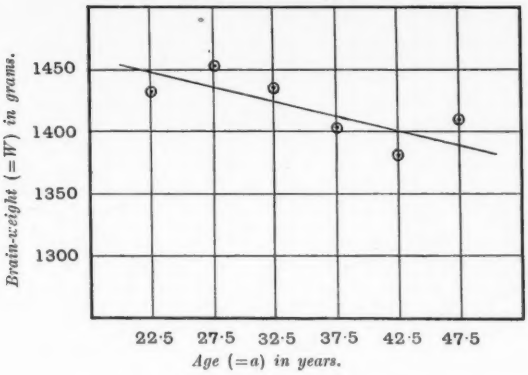


DIAGRAM VII. Probable Brain-weight with given Age.
Swedish ♀ Young.

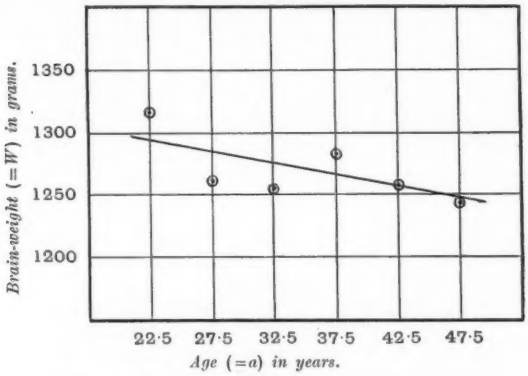


DIAGRAM VIII. Probable Brain-weight for given Age.
Hessian ♂ Young.

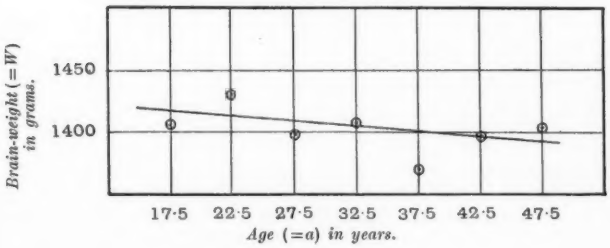
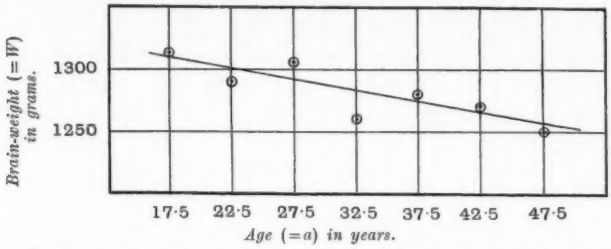


DIAGRAM IX. Probable Brain-weight for given Age.
Hessian ♀ Young.



These diagrams show clearly, I think, that the regression of brain-weight on age in these cases is sensibly linear within the ages 15 to 80, so far as can be judged on the material available. The regression diagrams for the other two racial groups show the same relation. Obviously no simple curve will represent the systems of points shown in these diagrams II to IX better than the straight line does. So then, until we have very much larger samples of material to work with, we shall be justified in assuming for practical purposes linearity of regression between brain-weight and age, and in considering the deviations of the means of the arrays from the regression line to be due to errors incident to sampling in lots of less than 500 individuals.

The equations to the regression lines for these diagrams may now be given. For convenience in practical use I have put these equations in the form of "characteristic" equations. The significance of the letters used is as follows: W denotes the probable brain-weight in grams of an array of individuals of age type A or a. The ages included by A are 15 or 20 to 80; by a 15 or 20 to 50, or in other words the equations containing A are deduced from the "total" series and those containing a from the "young" series. Σ is the standard deviation of the array having the probable W as its mean.

(1)	Swedish	♂	$W = 1487{\cdot}783 - 1{\cdot}939\,A$,	$\Sigma = 102{\cdot}972$
(2)	„	♂	$W = 1501{\cdot}411 - 2{\cdot}372\,a$,	$\Sigma = 107{\cdot}582$
(3)	„	♀	$W = 1326{\cdot}475 - 1{\cdot}549\,A$,	$\Sigma = 97{\cdot}969$
(4)	„	♀	$W = 1340{\cdot}438 - 1{\cdot}988\,a$,	$\Sigma = 104{\cdot}329$
(5)	Hessian	♂	$W = 1439{\cdot}734 - 1{\cdot}119\,A$,	$\Sigma = 111{\cdot}087$
(6)	„	♂	$W = 1432{\cdot}497 - {\cdot}847\,a$,	$\Sigma = 111{\cdot}014$
(7)	„	♀	$W = 1361{\cdot}067 - 2{\cdot}259\,A$,	$\Sigma = 95{\cdot}512$
(8)	„	♀	$W = 1339{\cdot}298 - 1{\cdot}735\,a$,	$\Sigma = 100{\cdot}445$

The foregoing constants bring out a number of interesting and important facts regarding the relation of brain-weight to age. In the first place it is clear that diminution in brain-weight with advancing years takes place very slowly. In 10 years the Swedish males lose 19·39 grs. and the Hessian males 11·19 grs.*

* In order to economize space I have not tabulated the regression coefficients separately. It will be understood of course by anyone wishing to refer to or use these regression coefficients that the number prefixing a letter in the second member of any of the characteristic equations 1 to 36 is the regression coefficient (either gross or partial) of brain-weight on the character indicated by the letter.

These are comparatively speaking slight changes, amounting respectively to 1·3 °/₀ and ·8 °/₀ of the mean brain-weight for the whole adult period. In the same length of time (10 years) males lose an average of ·34″ in stature or approximately ·5 °/₀ of the mean stature *. The diminution in brain-weight seems thus to be relatively somewhat more rapid than the diminution in stature.

The fact that the regression is linear implies that this diminution of brain-weight with advancing age so far as can be judged from the material here discussed is sensibly equally distributed over the whole adult period. To be sure the slope of the lines is in all but one case (the Swedes) less when the individuals between 20 and 50 are treated separately, but the difference is a small one when the probable error is considered (cf. Table IX or XIV). Splitting the material up in this way of course greatly reduces the already statistically small series, so that the probable errors of the coefficients of correlation for the "young" series become large. It is significant, however, that with a single exception (the otherwise abnormal Bavarian female series) all the coefficients of correlation between brain-weight and age for the "young" series are negative. The evidence presented by each of the four series here discussed, which are representative and have been shown to be statistically trustworthy, and which if not as large as might be desired by the statistician are among the largest ever collected by the neurologist, is mutually accordant, and leads to but one conclusion: namely, that *after age* 15—20 *there is a steady though very gradual diminution in the weight of the brain with advancing age.* This conclusion I have emphasized because it is not in agreement with that reached by some eminent neurological authorities. I would further emphasize the fact that the conclusion is by no means final in the mind of the biometrician. It simply marks the point to which the material at present available leads. Very much larger series of brain-weighings are needed before the precise form of the regression can be finally determined. But the fact that four distinct series of fair length taken by different observers on different races lead to such closely accordant results as those shown in Table XIV may be taken as very strong presumptive evidence of the essential correctness of the conclusion reached.

It is noteworthy that there is no tendency towards more rapid diminution on brain-weight after age 50 to 60. The rate of diminution shows no such marked changes at this time as has been maintained to occur by many of the workers on the subject.

With the exception of the Swedish series the females in all cases show a higher correlation between brain-weight and age than the males. This difference seems in the case of the Hessians and Bavarians (total series) to be probably significant. According to Powys' (*loc. cit.*) figures the slope of the regression line between stature and age is slightly steeper for the females. It is difficult to assign any reason for this greater correlation between brain-weight and age in

* Powys, A. O.: "Data for the Problem of Evolution in Man. Anthropometric Data from Australia," *Biometrika*, Vol. I. pp. 30—49.

the female. It has been shown above that there is in general no significant difference between the sexes in respect to variability in brain-weight, so that the usual formula of "low variation and high correlation" connoting stability of type does not hold in this case. It is possible that we have here an expression in a particular case of a greater general "evenness" in the females of the bodily changes accompanying increasing age, which in turn might be due to the generally more even environmental conditions to which women are subjected. It is noteworthy in this connection that the correlation in respect to duration of life is generally higher between pairs of female relatives than between pairs of male relatives*.

The linearity of the regression of brain-weight on age is of interest as possibly indicating a fundamental difference in the modes of action of the biological processes of growth on the one hand and senescence on the other. When growth in absolute magnitude of a character is plotted on a base line of age the result is usually a curved line (cf. for example the 12—25 portion of Powys' stature curve, *loc. cit.*), which implies, of course, that the amount of increment in the character in question is not the same for each unit of time. On the other hand in the case of decrease of brain-weight and stature with advancing age the decrement seems to be practically uniform for each unit of time. If these relations should prove to be generally true they would furnish a very interesting field for further study and analysis.

Turning now to the relation of brain-weight to stature, we have in Table XV the values of r, η and Σ_M, for all the brain-weight and stature correlations.

TABLE XV.

Linearity of Regression. Brain-weight and Stature.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Swedish (Total) ...	·1830 ± ·0320	·3490 ± ·0388	·2439	·3847	·1612 σ	·1618 σ
„ (Young)...	·1796 ± ·0403	·3390 ± ·0530	·2837	·4738	·2196 σ	·3310 σ
Hessian (Total) ...	·1823 ± ·0299	·1828 ± ·0389	·2864	·3215	·2209 σ	·2645 σ
„ (Young)...	·1741 ± ·0383	·1809 ± ·0496	·2714	·3807	·2082 σ	·3350 σ
Bavarian (Total) ...	·1664 ± ·0343	·2236 ± ·0413	·2262	·3270	·1532 σ	·2386 σ
Bohemian (Young) ...	·2034 ± ·0397	·2168 ± ·0557	·2419	·3591	·1309 σ	·2863 σ

Again, the values of η and Σ_M are seen to diverge considerably from r and 0 respectively. It is clear that on the whole the regression of brain-weight on stature approaches less closely to linearity than does the regression of brain-weight on age. This is, I think, a somewhat remarkable result, and one not likely to have been foreseen.

* Beeton and Pearson: *Biometrika*, Vol. I. p. 60.

In order to determine whether the departure of the regression from strict linearity is due simply to errors of sampling or to a fundamentally different law of relation between the two variables it is again necessary to have recourse to regression diagrams. I have selected eight diagrams which are fairly representative.

DIAGRAM X. Probable Brain-weight for given Stature.
Swedish ♂ Total.

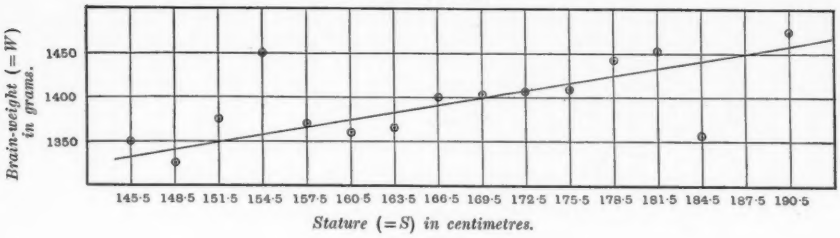


DIAGRAM XI. Probable Brain-weight for given Stature.
Swedish ♀ Total.

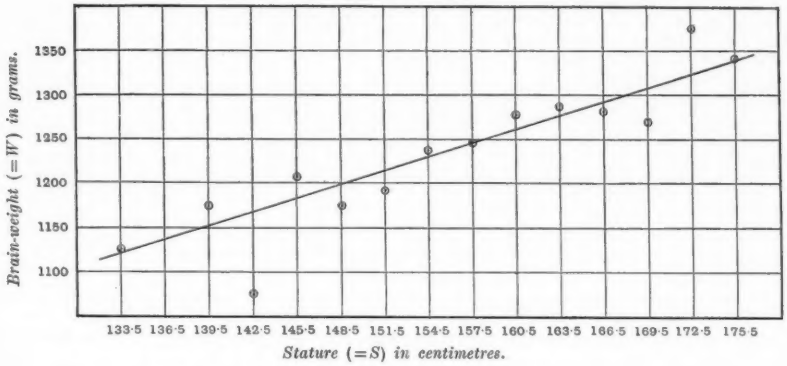


DIAGRAM XII. Probable Brain-weight for given Stature.
Swedish ♂ Young.

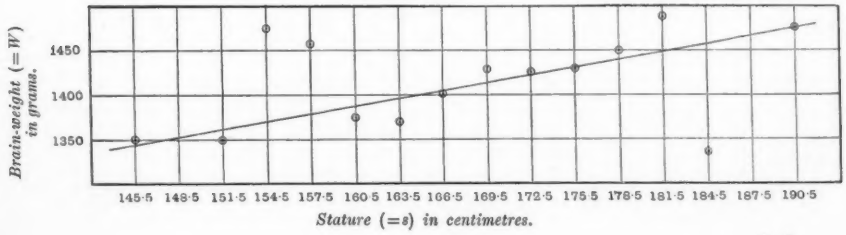


DIAGRAM XIII. Probable Brain-weight for given Stature.
Swedish ♀ Young.

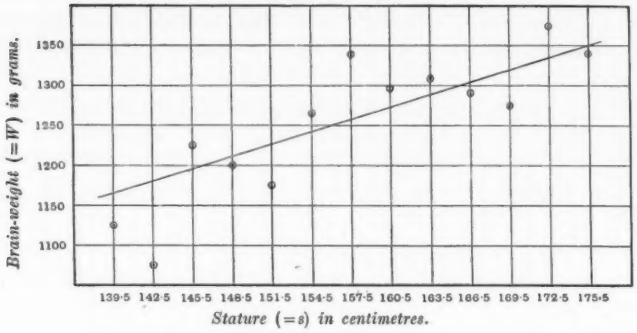


DIAGRAM XIV. Probable Brain-weight for given Stature.
Hessian ♂ Total.

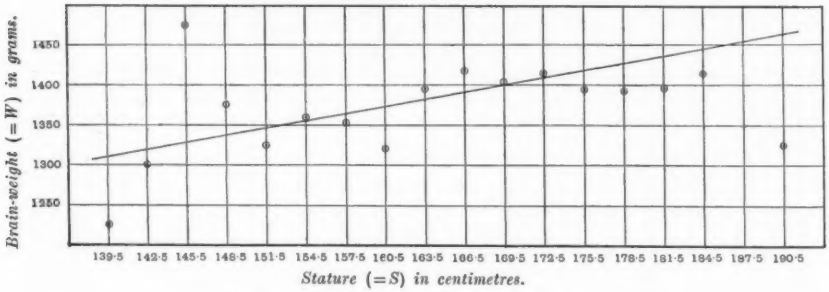


DIAGRAM XV. Probable Brain-weight for given Stature.
Hessian ♀ Total.

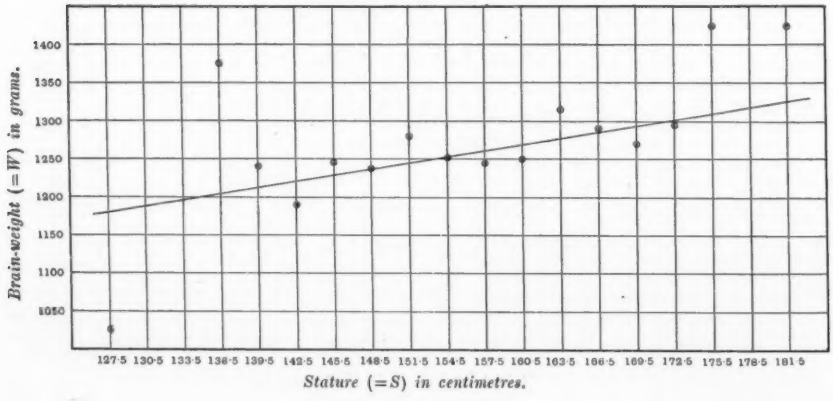


DIAGRAM XVI. Probable Brain-weight for given Stature.
Hessian ♂ Young.

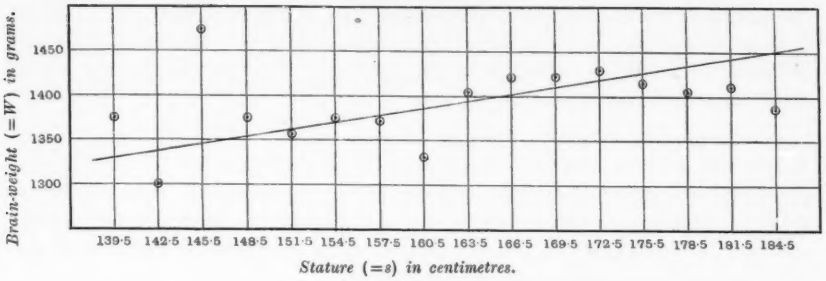
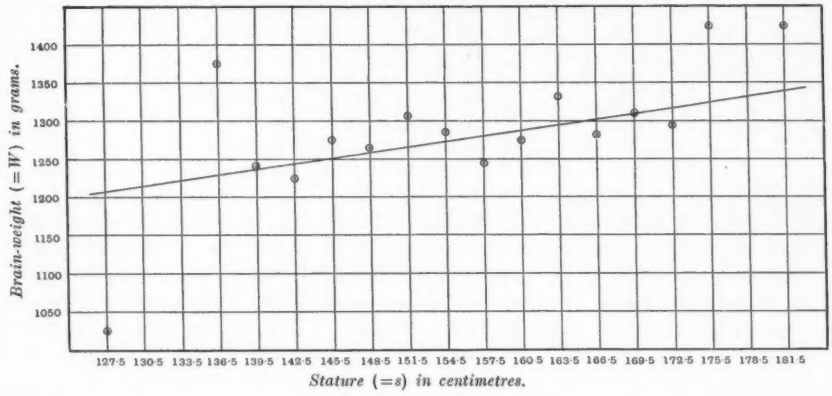


DIAGRAM XVII. Probable Brain-weight for given Stature.
Hessian ♀ Young.



It is evident from these diagrams that a straight line represents the regression relation between brain-weight and stature better than any simple curve would. So that we are justified in concluding, as in the case of age, that until we have very much larger collections of data than are at present available we can do no better than proceed on the assumption that the regression of brain-weight on stature is linear. In fact, in these samples there is no evidence that there is any tendency towards anything but a linear relation between the two variables. The deviations of the means of the arrays from the regression line are only such as would reasonably be expected to arise when we deal with relatively small samples. In connection with all of the regression diagrams in this paper it should be kept in mind that the outlying points are based on single individuals, or at most only on comparatively few individuals. Hence deviations of these outlying points from the regression line have very little or no significance as regards the general trend of the results. In fact it would really be better if

the outlying points were left out entirely in the graphical representations. The non-biometrical reader should understand that each plotted point carries weight in proportion to the fraction which the number of individuals in the array on which it is based is of the total number of individuals. With this caution in mind the substantial linearity of the regression through the bulk of the observations, which alone are of importance, becomes evident.

The regression equations showing numerically the relation between brain-weight and stature, may now be given. In these equations W denotes the probable brain-weight of an array of individuals of mean stature S or s in centimetres. As before, the equations involving S are based on the "total" series, and those involving s on the "young" series. Σ is the standard deviation of the array having the probable mean brain-weight W.

(9)	Swedish	♂	$W=915{\cdot}054+2{\cdot}859\,S,$	$\Sigma=104{\cdot}533$
(10)	„	♂	$W=919{\cdot}374+2{\cdot}914\,s,$	$\Sigma=107{\cdot}405$
(11)	„	♀	$W=421{\cdot}994+5{\cdot}234\,S,$	$\Sigma=\ 94{\cdot}421$
(12)	„	♀	$W=451{\cdot}643+5{\cdot}121\,s,$	$\Sigma=\ 99{\cdot}294$
(13)	Hessian	♂	$W=913{\cdot}592+2{\cdot}857\,S,$	$\Sigma=110{\cdot}787$
(14)	„	♂	$W=950{\cdot}214+2{\cdot}723\,s,$	$\Sigma=109{\cdot}628$
(15)	„	♀	$W=834{\cdot}624+2{\cdot}714\,S,$	$\Sigma=100{\cdot}643$
(16)	„	♀	$W=894{\cdot}031+2{\cdot}460\,s,$	$\Sigma=100{\cdot}161$
(17)	Bavarian	♂	$W=836{\cdot}667+3{\cdot}127\,S,$	$\Sigma=118{\cdot}452$
(18)	„	♀	$W=627{\cdot}157+3{\cdot}824\,s,$	$\Sigma=103{\cdot}562$

We may now proceed to the general conclusions to be drawn from the data given above regarding the relation of brain-weight to stature. In the first place it is to be noted that the correlation coefficients between these two variables are for each series of the same general order of magnitude as those between brain-weight and age, though of course positive instead of negative, as they are in that case. Or it may be concluded that during adult life, at least, brain-weight is only a very little more closely related to general size of body, in so far as we may take stature as a measure of this, than to age. When we turn to the regression coefficients, however, it is seen that a unit of stature connotes a somewhat larger change of brain-weight than a unit of age. Thus, in the Swedish males, an increase of 10 cm. in stature connotes an increase in brain-weight of 28·59 gr., while an increase of 10 years in age connotes a decrease of 19·39 gr. in the brain-weight.

As in the correlation of brain-weight with age, the females have uniformly a higher degree of correlation between stature and brain-weight than the males. I am inclined to think that the same general sort of explanation may be given in this case as in that of the brain-weight and age correlations. The correlation between brain-weight and stature and the regression of brain-weight on stature are sensibly the same for the early adult period of life as for the total adult period. Without any question, I think, this correlation is to be regarded in its origin as a growth correlation, the relation between the two variables being

practically fixed at the end of the growth period. After that time the relation undergoes no marked change throughout the remainder of life, since both stature and brain-weight regress on age at not far from the same rate.

Finally, the combined effect of stature and age on the weight of the brain may be examined. This relationship can best be examined through the medium of multiple regression equations of the form:

$$x_1 = \beta_{12}\, x_2 + \beta_{13}\, x_3,$$

in which β_{12} and β_{13} are the partial regression coefficients of x_1 on its associated variables x_2 and x_3, it being understood that x_1, x_2 and x_3 here stand for deviations from the means of the respective variables. For our material these equations, when reduced to absolute magnitudes, take the following forms, the significance of the letters being the same as in the regression equations given before.

(19)	Swedish	♂	$W = 1091{\cdot}021 + 2{\cdot}288\,S - 1{\cdot}755\,A$,	$\Sigma = 101{\cdot}819$
(20)	„	♂	$W = 1080{\cdot}715 + 2{\cdot}362\,s - 1{\cdot}856\,a$,	$\Sigma = 106{\cdot}477$
(21)	„	♀	$W = 561{\cdot}433 + 4{\cdot}679\,S - 1{\cdot}078\,A$,	$\Sigma = 93{\cdot}066$
(22)	„	♀	$W = 533{\cdot}407 + 4{\cdot}854\,s - 1{\cdot}092\,a$,	$\Sigma = 98{\cdot}923$
(23)	Hessian	♂	$W = 942{\cdot}154 + 2{\cdot}989\,S - 1{\cdot}181\,A$,	$\Sigma = 108{\cdot}992$
(24)	„	♂	$W = 926{\cdot}586 + 3{\cdot}107\,s - 1{\cdot}318\,a$,	$\Sigma = 108{\cdot}889$
(25)	„	♀	$W = 1005{\cdot}607 + 2{\cdot}244\,S - 2{\cdot}173\,A$,	$\Sigma = 94{\cdot}261$
(26)	„	♀	$W = 974{\cdot}382 + 2{\cdot}345\,s - 1{\cdot}829\,a$,	$\Sigma = 98{\cdot}883$

These equations give the probable mean brain-weight of an array of adult individuals of either sex, having any given age (in years) and mean stature (in centimetres). They are, I believe, the first multiple regression equations relating brain-weight to stature, age and sex, to be published. They are of interest from several points of view. Examples of their practical use have been given earlier in the paper (pp. 25 and 48) in connection with the discussion of racial differences in mean brain-weight and later in connection with the sex differences. They afford a means whereby it is possible to make scientific comparisons of the mean brain-weight of different races, since by their use the probable brain-weight of a group of Swedes or Hessians having the same mean stature and age as the sample of the race with which comparison is to be made can be ascertained. We can in effect reduce both races to the same stature-age base and then examine the brain-weight differences. It would seem that these equations ought to prove very useful to anthropologists, who frequently wish to make comparisons of this kind. They are based on the two sets of brain-weighings which I believe to be on the whole the most reliable and trustworthy at present available. These equations also make it possible to determine the effect of indirect selection of stature and age on brain-weight.

As a practical example of this last use of these equations let us consider the following problem: By how much would it be necessary to modify the stature of a group of Swedish males, having a mean age of 45 years, in order that they might have the same mean brain-weight as a group of Bohemian males of the same mean

age? The mean brain-weight of the Bohemian males with a mean age of 45·699 is 1454·839 gr. From equation (19) we have then

$$1454{\cdot}839 = 1091{\cdot}021 + 2{\cdot}288\,S - (1{\cdot}755 \times 45{\cdot}699)$$

in which S is the mean stature of the selected group of Swedes. Solving we find

$$S = 194{\cdot}06,$$

or, in other words, our group of Swedes would have a *mean* height of 6 ft. 4·4 in.! Now as a matter of fact the Swedish males with a mean age of 45·02 years have a mean stature of 169·79 cm. In order then to get from the adult male Swedish population a group having the same mean brain-weight as the adult male Bohemian population, by a selection of stature alone, it would be necessary to raise the mean stature of the Swedes 24·3 cm. or roughly 9½ inches. Of course it is not for a moment to be supposed that evolution in brain-weight proceeds by the indirect selection of stature alone. Indeed when it is remembered that the mean stature of Bohemian males is only 169·4 cm. the example just cited shows how little the indirect selection of stature has to do with the matter. In the discussion of evolutionary problems, however, it is important to know just how much or how little effect each factor involved is capable of producing. It is in this direction of measuring the relative effect of different combinations of factors that regression equations such as those given above have great value.

It is apparent from equations (19) to (26) that stature and age in a measure compensate one another so far as their effect on brain-weight is concerned. The influence of stature is however in all cases greater. It is of some interest to know in the particular cases how great an increase in stature compensates a given increase in age and *vice versâ*. In order to show this I have prepared the following table.

An increase of 10 cm. in stature is compensated in its effect on brain-weight by an increase in age in the

Swedish males of	13·0 years	Total series only.
Swedish females of	43·4 „	
Hessian males of	25·3 „	
Hessian females of	10·3 „	

10. *Brain-weight and Body-weight.*

In only one of the four series of brain-weighings studied (the Bavarian) was body-weight tabulated in the returns. Recent workers on the subject have left this character out of their data on the general ground that it is of very doubtful significance or reliability in post-mortem returns from a general hospital population. Certainly it is true that it is influenced to a very marked degree by the

conditions preceding death. Yet as Donaldson* has pointed out if we could get this character free of disturbing ante-mortem effects it would furnish the best measure of the general "size" of the body. Since "size" of the body is one of the things to which brain-weight should first of all be related I decided to include in this study a consideration of the correlation of brain-weight with body-weight. I was fully aware of the weakness of the material and expected much worse results than those actually obtained. A glance at Table I will recall the facts as to the distribution of the frequency of body-weight. The means are of course lower than they would be for the living population of the same race and age. This is to be expected. The variation is large but there is very good accord between males and females in this respect. Turning now to the correlations I have arranged in Table XVI the values of r, η and Σ_M for the correlation between brain-weight and body-weight.

TABLE XVI.

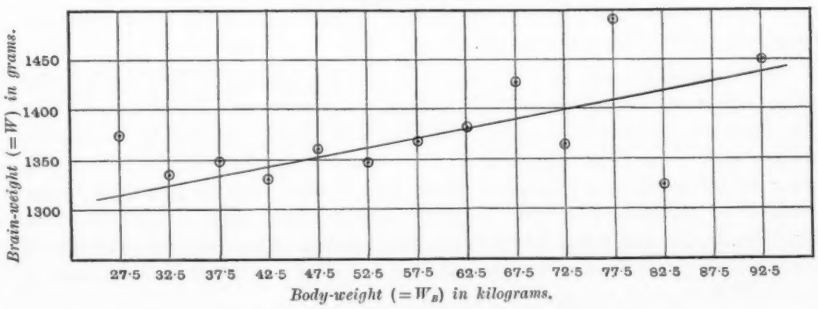
Linearity of Regression. Brain-weight on Body-weight.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Bavarian (Total)	·1671 ± ·0343	·2260 ± ·0412	·1845	·3189	·0782 σ	·2250 σ

The result for the males is very satisfactory, indicating a quite reasonable approach to linearity of regression. The female series does not give so good a result, but it will be recalled that throughout the work this Bavarian female series has been found to be somewhat abnormal. The regression lines are shown in Diagrams XVIII and XIX.

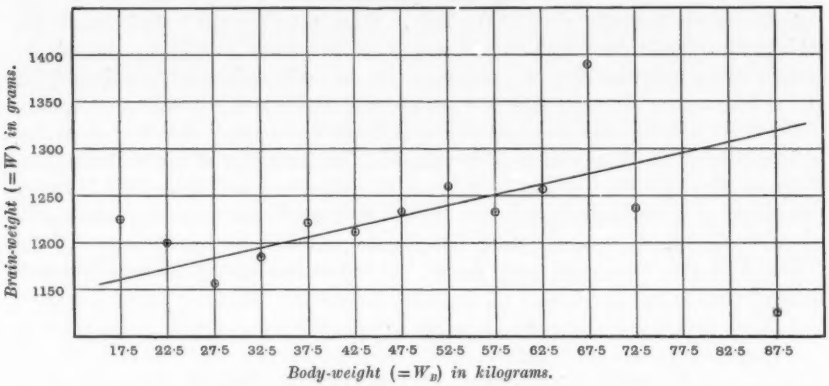
DIAGRAM XVIII. Probable Brain-weight for given Body-weight.

Bavarian ♂ Total.



* H. H. Donaldson: *The Growth of the Brain.*

DIAGRAM XIX. Probable Brain-weight for given Body-weight.
Bavarian ♀ Total.



Through the great bulk of the observations the regression is evidently linear*. The male series comes very close to linearity.

The equations to the regression lines may now be given. In addition to the equation showing the regression of brain-weight on body-weight I have included multiple regression equations giving the probable brain-weight for given types of body-weight and stature together. The significance of the letters is as follows: W denotes the probable brain-weight of an array having a mean body-weight in kilograms W_B, or body-weight W_B and stature S. Σ is the standard deviation of the array.

(27)	Bavarian	♂	$W=1263{\cdot}308+1{\cdot}886\,W_B$,	$\Sigma=118{\cdot}439$
(28)	„	♀	$W=1121{\cdot}621+2{\cdot}265\,W_B$,	$\Sigma=103{\cdot}495$
(29)	„	♂	$W=917{\cdot}748+2{\cdot}234\,S+1{\cdot}355\,W_B$,	$\Sigma=117{\cdot}711$
(30)	„	♀	$W=741{\cdot}078+2{\cdot}646\,S+1{\cdot}593\,W_B$,	$\Sigma=102{\cdot}432$

The correlation of brain-weight with body-weight is of about the same degree as the correlation of the former with the stature, but the regression approaches more closely to linearity in the case of body-weight. As in the other cases the female shows the higher correlation.

The absolute increase of brain-weight with every increase of 10 kilos. in body-weight is for the males 18·86 gr. and for the females 22·65 gr. Equations (29) and (30) show that in each case stature contributes absolutely more to an increase in brain-weight than does body-weight. The exact proportions are as follows: in order to bring about a change in brain-weight equal to that produced by an increase of stature of 10 cm. the body-weight would have to be increased 16·5 kilos. in the males, and 16·6 kilos. in the females. The greater dependence of the brain-weight on bodily characters in the female as compared with the male is very well shown in equations (29) and (30).

* Cf. pp. 61, 62.

11. *Brain-weight and Skull Characters.*

As was to be expected the correlation of brain-weight with the skull length and breadth gives the highest values for the coefficients of any of the correlations dealt with. In fact one would predict on general grounds, I think, a somewhat closer degree of correlation between these characters than is actually found in the Bohemian statistics. It is likely, however, that the values here obtained (Table XVII) are lower than they should be on account of the methods used in measuring skull length and breadth. As has been fully described above (p. 18) these measurements were taken on the horizontal cut surfaces after the dome of the skull had been removed by sawing around it. This is clearly not a very precise way in which to get skull measurements. In order to show so far as may be what has been the effect of this procedure I have prepared the following tables comparing the various constants of the Bohemian skull length and breadth series with other well-known cranial series. Taking first the means we have:

Race	Skull Length		Skull Breadth	
	♂	♀	♂	♀
Bohemians	176·547	170·142	149·841	144·953
Whitechapel English*...	189·06	180·36	140·67	134·68
Altbayerisch†	180·58	173·45	150·47	143·98
Würtemburger† ...	179·48	172·74	147·88	142·90

The means give reasonably accordant values considering that the Bohemians are a markedly brachycephalic race. Turning next to the coefficients of variation we have:

Race	Skull Length		Skull Breadth	
	♂	♀	♂	♀
Bohemians	4·173	3·233	4·441	3·815
Whitechapel English‡...	3·31	3·45	3·75	3·54
Bavarian §	3·37	3·57	3·89	3·39
French‖	3·97	3·65	4·21	3·67
Naqada¶...	3·17	3·14	3·29	3·45
Aino§	3·20	3·08	2·76	2·68

* Macdonell, *Biometrika*, Vol. III. pp. 208, 209.
† Alice Lee, *Phil. Trans.* Vol. 196, A, pp. 225—264.
‡ Macdonell, *loc. cit.* p. 220.
§ Lee, *loc. cit.*
‖ Unpublished reduction of Measurements in Broca's MS. by C. D. Fawcett.
¶ Miss Fawcett, *Biometrika*, Vol. I. pp. 438 and 456.

The Bohemian females are fairly accordant in variability with the other series in both length and breadth, but the males are apparently somewhat more variable. This greater variability in the Bohemian male series is due I believe not to less homogeneous material but to the manner in which the measurements are made. The coefficients of correlation between length and breadth may next be considered.

Race	♂	♀
Bohemians	·4006	·1954
Whitechapel English* ...	·240	·350
French†	·089	– ·042
German‡	·286	·488
English Criminals § ...	·402	—
English Middle Classes § ...	·345	—
Aino‡	·432	·376
Naqada†	·344	·143

The results here are somewhat chaotic, but clearly the difference in degree of correlation between Bohemian males and females is too great to be entirely normal. It is probable that the male coefficient is somewhat too high and the female too low. The agreement between the Bohemian male coefficient and Macdonell's value for measurements on the heads of living English criminals is striking. We should expect measurements made as the Bohemian were, to agree better with measurements on the living head than with those from the skull.

The practical result to which we come is that conclusions from the Bohemian data as to the correlation between brain-weight and skull length and breadth must be only tentative until they can be tested by other less objectionable material.

We may now examine in more detail the results as to the correlation of brain-weight with these skull characters. In Table XVII are given the values of r, η, and Σ_M.

TABLE XVII.

Linearity of Regression. Brain-weight and Skull Characters.

Race and Series	r		η		Σ_M	
	♂	♀	♂	♀	♂	♀
Bohemians, Skull Length	·5482 ± ·0273	·3604 ± ·0465	·5660	·4032	·1408 σ	·1808 σ
" " Breadth	·4655 ± ·0306	·5041 ± ·0399	·5498	·5368	·2926 σ	·1845 σ

The four regression diagrams concerned are Diagrams XX to XXIII.

* Macdonell, *loc. cit.* p. 233.

† C. D. Fawcett, *loc. cit.*

‡ Lee, *loc. cit.*

§ Macdonell, *Biometrika*, Vol. I. pp. 181 and 188, "Measurements on living head and not skull."

DIAGRAM XX. Probable Brain-weight for given Skull Length.

Bohemian ♂ Young. 20—59 years.

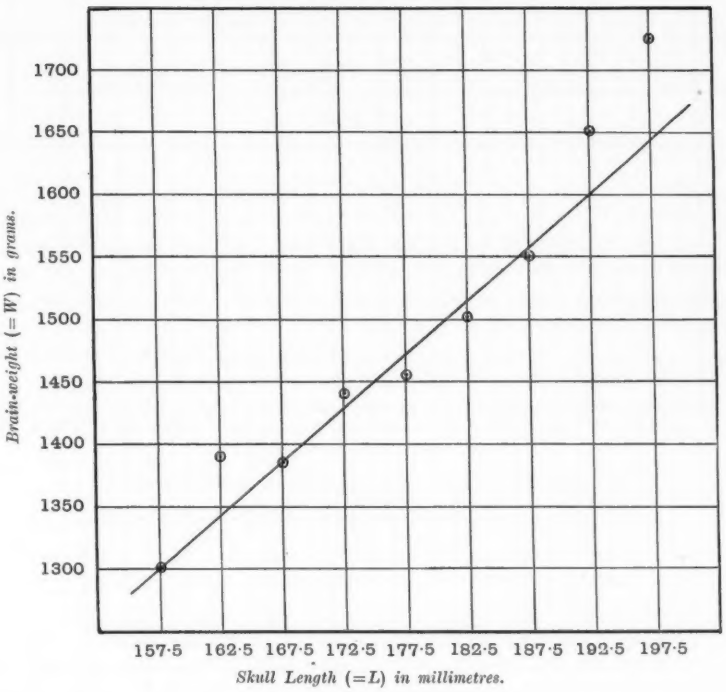


DIAGRAM XXI. Probable Brain-weight for given Skull Breadth.
Bohemian ♂ Young. 20—59 years.

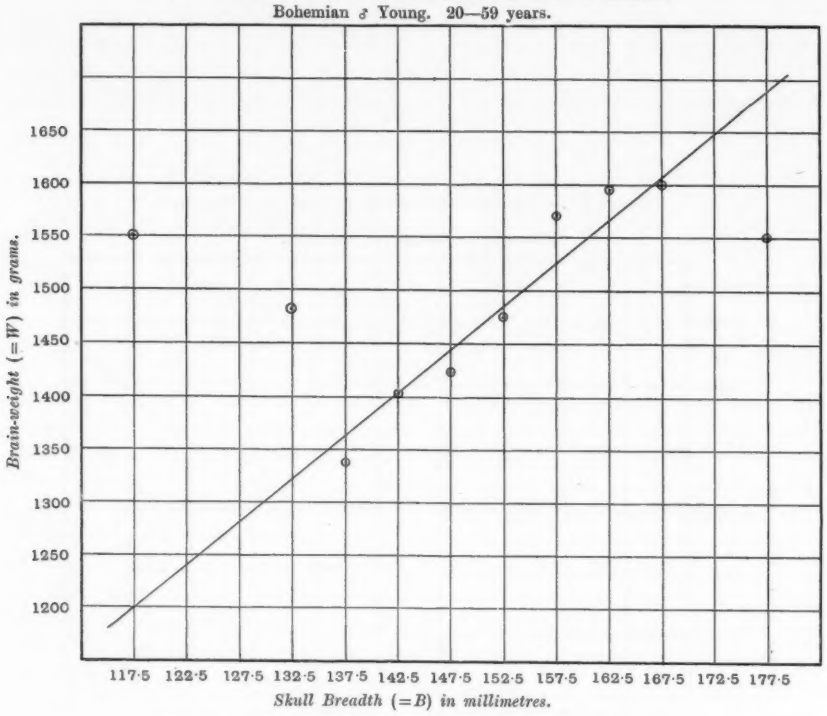


DIAGRAM XXII. Probable Brain-weight for given Skull Length.
Bohemian ♀ Young. 20—59 years.

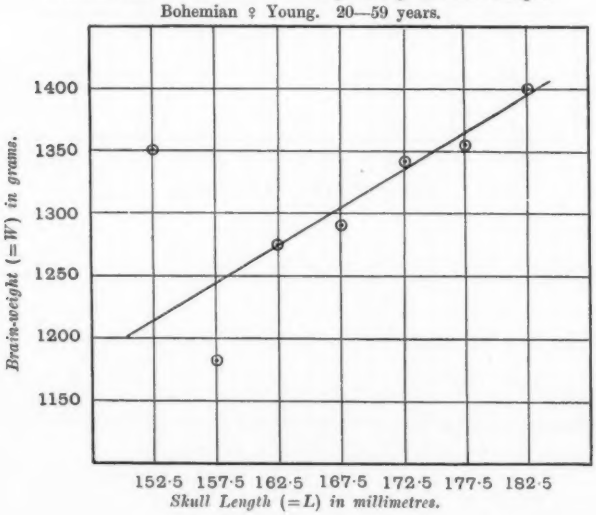
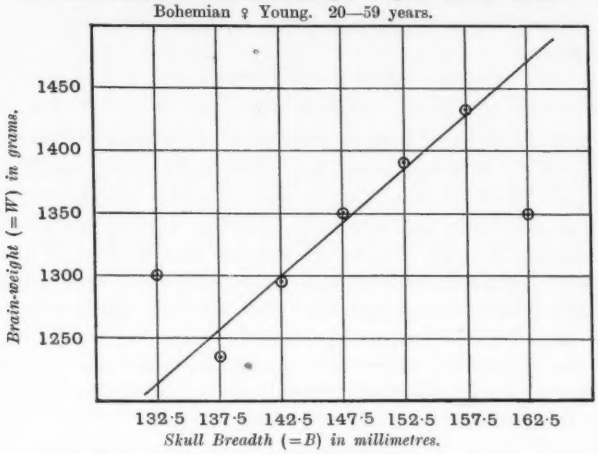


DIAGRAM XXIII. Probable Brain-weight for given Skull Breadth.
Bohemian ♀ Young. 20—59 years.



Through the bulk of the observations the regression is very evidently linear. Diagram XXI shows that the considerable deviations of η from r and Σ_M from zero in the case of the male brain-weight and skull breadth regression are really due to the inclusion in the material of five individuals which are to be regarded as either erroneously measured or recorded, or else abnormal or extremely rare normal cases (cf. Table 31, Appendix).

The regression equations follow. In these equations W denotes the probable brain-weight in grams of an array of individuals having a mean skull length L and skull breadth B in millimetres.

(31)	Bohemian	♂	$W = 8{\cdot}589\,L - 52{\cdot}650,$	$\Sigma = 96{\cdot}548$
(32)	„	♂	$W = 8{\cdot}076\,B + 253{\cdot}596,$	$\Sigma = 102{\cdot}170$
(33)	„	♀	$W = 6{\cdot}215\,L + 264{\cdot}265,$	$\Sigma = 88{\cdot}466$
(34)	„	♀	$W = 8{\cdot}646\,B + 68{\cdot}434,$	$\Sigma = 81{\cdot}908$
(35)	„	♂	$W = 6{\cdot}751\,L + 5{\cdot}082\,B - 489{\cdot}649,$	$\Sigma = 99{\cdot}745$
(36)	„	♀	$W = 4{\cdot}696\,L + 7{\cdot}766\,B - 602{\cdot}994,$	$\Sigma = 77{\cdot}893$

One rather curious point may be noted here in passing. As the figures stand a smaller probable error will be made in estimating the mean brain-weight of males from a knowledge of skull length alone (equation 31) than if both length and breadth of skull are used. This apparent paradox arises for the following reason*. The standard deviation of the array in the regression of x_1 on its associated variables x_2 and x_3 is given by

$$\Sigma = \sigma_1 \sqrt{1 - R_1^2},$$

where

$$R_1^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}.$$

When the regression is of x_1 on x_2 alone the standard deviation of the array is

$$\Sigma = \sigma_1 \sqrt{1 - r_{12}^2}.$$

* Cf. Yule, *loc. cit.*

Now the condition under which we shall make a smaller standard error in estimating x_1 from the two variables x_2 and x_3 than in estimating it from x_2 only, is evidently that

$$(1 - R_1^2) < (1 - r_{12}^2)$$

or that

$$R_1^2 > r_{12}^2.$$

Now in the case noted above $R_1^2 < r_{12}^2$, the figures being $\cdot 2534 < \cdot 3005$.

Returning to the main line of the discussion, the results as to the correlation of brain-weight with skull characters are not altogether satisfactory, as they are apparently not entirely in accord with themselves. One of the points here on which information is most desired is whether brain-weight is more closely correlated with length or breadth of skull. According to our results brain-weight is more closely correlated with length of skull in the male and breadth of skull in the female. This seems a somewhat anomalous result as there is no apparent reason why one should not expect the sexes to show the same relation in the matter. However, Macdonell's (*loc. cit.*) figures for the correlation of skull capacity with length and breadth show a similar relation but in the opposite direction. He finds for the Whitechapel English skulls that the correlation between capacity and length is lower than the correlation between capacity and breadth in the males, and higher in the females. Other skull series do not, however, show this reversed relationship between the sexes. With the data at present available one cannot reach any general conclusion as to the relative closeness of correlation between brain-weight and skull length and breadth. It is very much to be hoped that future collectors of brain-weight statistics will include in their returns such head measurements as length, breadth, height and circumference taken before the cranial cavity is opened, and by standard anthropometrical methods.

A comparison of the values of the coefficients of correlation between brain-weight and skull length and breadth and between skull capacity and these characters is of interest. I have adapted the following table from Macdonell's* Table XIII.

Race	Brain-weight and Skull Length		Brain-weight and Skull Breadth	
	♂	♀	♂	♀
Bohemians ...	$\cdot 5482 \pm \cdot 0273$	$\cdot 3604 \pm \cdot 0465$	$\cdot 4655 \pm \cdot 0306$	$\cdot 5041 \pm \cdot 0399$
	Skull Capacity and Length		Skull Capacity and Breadth	
English* ...	$\cdot 597 \pm \cdot 051$	$\cdot 691 \pm \cdot 040$	$\cdot 631 \pm \cdot 048$	$\cdot 646 \pm \cdot 044$
Naqada † ...	$\cdot 501 \pm \cdot 054$	$\cdot 599 \pm \cdot 039$	$\cdot 434 \pm \cdot 058$	$\cdot 532 \pm \cdot 044$
German ‡ ...	$\cdot 515 \pm \cdot 050$	$\cdot 687 \pm \cdot 037$	$\cdot 672 \pm \cdot 037$	$\cdot 706 \pm \cdot 034$
Aino‡ ...	$\cdot 893 \pm \cdot 016$	$\cdot 663 \pm \cdot 053$	$\cdot 561 \pm \cdot 053$	$\cdot 502 \pm \cdot 070$

* Macdonell, *loc. cit.* p. 235. † Fawcett, *loc. cit.* ‡ Lee, *loc. cit.*

The brain-weight correlations with two exceptions are smaller than the corresponding skull capacity correlations. A portion at least of this difference is probably to be accounted for on the ground that brain-weight fluctuates as a result of causes which do not at all affect skull capacity. Possibly the whole of the difference is to be accounted for in this way, but until there is available a more reliable collection of material on the brain-weight side the question cannot be definitely settled. Until such material is available we shall be justified, I think, in assuming that the values of the coefficients for the correlation between brain-weight and skull characters given by the male series probably represent more nearly the normal relation than do those of the female series.

Making all due allowance for the shortcomings of the material the following points are clearly brought out: (*a*) Brain-weight is positively correlated with both skull length and skull breadth. (*b*) The correlations here are higher than any of the other correlations of brain-weight with bodily characters. (*c*) The regression of brain-weight on these skull characters is approximately linear. (*d*) Indirect selection of skull characters is capable of modifying brain-weight to a considerable extent.

The use of equations (31) to (36) in selection problems has been illustrated above (p. 49). One other example may be given because it bears very directly on a question of perennial interest in certain quarters. The problem to which I have reference may be stated for our present purpose in this form: Does dolichocephaly imply a greater brain-weight than brachycephaly or *vice versâ**? Mere inspection of equations (35) and (36) shows that neither dolichocephaly nor brachycephaly will have a great advantage over the other in respect to brain-weight, but the matter can be made somewhat more *anschaulich* by taking a particular example. Consequently I have chosen the following figures. In Macdonell's Tables I and II (*loc. cit.*) are given the values for cephalic index, skull-length and skull-breadth for a dolichocephalic race (the Whitechapel English series) on the one hand and a brachycephalic race (the Altbayerisch) on the other. Those values are as follows:

	♂		♀
English ...	Cephalic index = 74·34	English ...	Cephalic index = 74·73
Altbayerisch ...	" " = 83·20	Altbayerisch ...	" " = 83·10
English ...	Length = 189·06	English ...	Length = 180·36
" ...	Breadth = 140·67	" ...	Breadth = 134·68
Altbayerisch ...	Length = 180·58	Altbayerisch ...	Length = 173·45
" ...	Breadth = 150·47	" ...	Breadth = 143·98

* Evidently the real essence of the question, so far as it has what might be called "humanistic" interest, rests on the assumption of a reasonably high positive correlation between brain-weight and intellectual ability. "Brains" not brain-weights are the things! Up to this point I have carefully avoided any discussion of the validity of such an assumption as that mentioned, deeming it entirely irrelevant to the main purpose of the paper. The point is discussed in the concluding section.

Now using equations (35) and (36) for males and females respectively we have for the probable mean brain-weights of arrays having the types of length and breadth of the English and Altbayerisch series respectively, the following values:

♂	♀
English B.W. =1501·580 Altbayerisch B.W. =1494·135	English B.W. =1289·901 Altbayerisch B.W. =1329·676

In the males the dolichocephalic and brachycephalic arrays have practically equal brain-weights, while in the females the brachycephalic array gives a brain-weight about 40 grs. higher than the dolichocephalic. But, as has been pointed out above, the male equations probably come much nearer representing the normal relations than do the female. On the whole, then, there seems to be little or no evidence that either dolichocephaly or brachycephaly is associated with large brain-weight.

It should perhaps be noted that no significance is to be attached to the absolute values of the brain-weights calculated from the English and Altbayerisch skull types, as indicating the probable brain-weight of those races. The differences only are of significance. The measurements of these skulls were taken only for the sake of convenience; hypothetical dolichocephalic and brachycephalic skull means would have served the purpose as well. The brain-weights obtained are of course the probable mean brain-weight of arrays of *Bohemians* having the specified skull characters. Intra-racial regression equations can of course only be used for inter-racial prediction when the regression of all characters on one another is the same in both races, which is a condition never likely to be exactly fulfilled in practice. Of the races and characters discussed in this paper the Hessians and Swedes in brain-weight, stature, and age come nearest to fulfilling this condition.

12. *The Weight of the Cerebrum.*

Up to this point the discussion has had to do entirely with the variation and correlation of the whole brain or encephalon. In the manuscript material placed at my disposal by Prof. Donaldson (*vide supra*, p. 15) data were given from which it was possible to determine the variation in the weight of the cerebral hemispheres, and the correlation of this character with stature and age for English males. The results obtained from this material may now be examined. It should be stated that the records of weight were in ounces, and of stature in inches in the original material. The units of grouping and the distribution of the frequencies may be seen in Tables 45 and 46 of the Appendix. After the constants had been calculated in ounces and inches they were transferred to the metric system in order to facilitate comparison with the other parts of the paper. The values of the constants are exhibited in Table XVIII.

TABLE XVIII.

Variation in the Weight of the Cerebrum. English Males.

Boyd-Marshall Data.

Character	Mean			S. D.	C. of V.
	No.		Table		
Weight of cerebrum	308	1184·940 ± 4·859 gr.	45	126·438 ± 3·436 gr.	10·670 ± ·293
Stature	308	171·602 ± ·268 cm.	45	6·976 ± ·190 cm.	4·065 ± ·111
Age	308	49·481 ± ·645 yr.	46	16·770 ± ·456 yr.	—

For the correlation of weight of cerebrum with age I find from Table 46

$$r = -\cdot1412 \pm \cdot0377,$$

and for the correlation of the weight of cerebrum with stature from Table 45

$$r = \cdot1202 \pm \cdot0379.$$

These values bring out the following points. The cerebral hemispheres are markedly more variable in weight as judged by the coefficient of variation than is the entire encephalon. This greater variation may denote a really greater variability of this part of the brain, or it may be due to the variable element which enters as a result of the separating of these organs from the rest of the brain. It seems to me likely that it is in part at least real, for the following reasons: first, the cerebral hemispheres have attained relatively enormous development late in their phylogenetic history; and again from the functional standpoint the cerebrum is the most variable part of the brain.

The correlations of the weight of the cerebrum with age and stature are both somewhat lower than the corresponding values for the other series of males falling within the ages 20 to 80. The differences can hardly be considered as significant however in view of the probable errors.

For the coefficient of regression of brain-weight on age I find $b_1 = -1\cdot065$, and on stature $b_1 = 2\cdot178$. By comparing these with the regression coefficients for the entire encephalon on the same characters in the series previously discussed it becomes evident at once to what a great extent changes in the total brain-weight are due to changes in the weight of the cerebrum. Unfortunately it is not possible to get relative figures here on account of lack of knowledge of the regression of the total brain-weight on stature and age for the English series. Of course the weight of the cerebrum forms a large part (ca. 87 per cent. for the English series) of the total brain-weight. The problem here is to determine whether the changes in the weight of the cerebrum cause less or more than their proportional part (say 87 per cent.) of the changes in total brain-weight due to stature and age changes. I have tried a number of approximate methods

of getting at this question with the data available, but none of them led to any result sufficiently well grounded to warrant taking space for its publication. Putting all the results so obtained together they seemed to indicate the following conclusion, which is of course subject to modification when further material is available: viz., that changes in stature affect all parts of the brain equally (i.e., in the proportion of their absolute masses) while changes in age have a relatively greater effect on the cerebral hemispheres than on the remainder of the brain (cerebellum, pons, and medulla).

The final conclusions regarding variation and correlation in the weight of the cerebrum may be summed up as follows: the cerebrum is somewhat more variable in weight than is the entire encephalon, and this character is slightly less closely correlated with age and stature. The values of the variation and correlation constants are, however, as is to be expected, of the same general order of magnitude as those for the total brain-weight.

13. *Concluding Remarks.*

In concluding this paper I wish to call attention to what seem to me to be some of the broader aspects of the work. There are at least two general results of the work which alone justify its being done, I think. The first and most important is that the paper contributes reduced material to the existing collection of biometric data on man. In the nature of the case such a collection grows slowly, but every increase in it means a definite, although it may be small, step in advance in our knowledge regarding the fundamental problems of anthropology. I regard as the second most important result the fact that it may now fairly be said that the essential trustworthiness of the most important of the existing collections of brain-weight statistics has been demonstrated, and that consequently reasoning by statistical methods on the problems involved is not of necessity lacking in validity. The agreement in the statistical constants from four series of data so divergent in their origin as those treated in this paper cannot reasonably be held to be fortuitous. It can mean, I think, but one thing, namely, that the same kind of general lawfulness underlies the variation and correlation of brain-weight and the variation and correlation in other characters of the organism With this conclusion presumably no student of brain-weight would disagree, but some of the most eminent students have disagreed with the converse proposition that it is possible to gain a knowledge of the nature of this lawfulness by statistical methods applied to large masses of material when brain-weight is the thing concerned. This sceptical attitude owes its origin, I think, to the nature of the statistical methods which have hitherto been applied to the problem. Brain-weight statisticians have erred in two directions in the handling of their material. On the one hand, following the much-to-be-condemned practice in other fields of anthropology, entirely heterogeneous material has been grouped together to attain large total numbers. The culminating example here is, perhaps, Topinard's series of the brain-weights of 13,000 Europeans. Secondly, certain brain-weight workers

have attempted to apply to the individual results gained by somewhat doubtful statistical analysis of mass data, and *vice versâ*, what is much worse, have argued that the relations found to hold in a single individual might be considered to hold for the mass. As an example here might be cited Marshall's* attempt to correct the brain-weights of De Morgan, Thackeray, and others for stature and age. The results detailed in this paper will indicate to the biometrician at least how dangerous both of these methods of procedure are with such material as brain-weight statistics. Of course both disregard fundamental canons of general statistical theory, but the chance of grave error becomes very much greater when the character under consideration shows on the one hand definite and clearly marked types in different races, and on the other hand low correlations†. With the application of adequate statistical methods it is possible to reach definite and significant results regarding the weight of the brain. When we have biometric constants for a considerable number of long series of brain-weight statistics from different races we shall be able to advance our knowledge greatly with reference to the general problems of the evolution of man's brain.

While the discussion of such problems can at the present time admittedly lead to only tentative results, still I think it will be worth while to examine for a moment the bearing of our results on a single evolution problem. Progressive evolution through natural selection may be brought about in any character either by the direct selection of that particular character or by the indirect selection of other characters correlated with it. As we proceed in the analysis of the problem the proportional effect of each of these two sets of factors must be quantitatively determined. A first approximation to such a determination has been here made

* *Loc. cit., supra*, p. 18.

† In this connection Weigner's recent paper "Ein Beitrag zur Bedeutung des Gehirngewichts beim Menschen," *Anatomische Hefte*, Bd. 23, pp. 69—109 is decidedly open to criticism. But when one, after very briefly analyzing the weighings of 69 male and 66 female brains (*sic*) from individuals ranging in age from 1½ to 82 years, by the most superficial of statistical methods, feels justified in drawing the following sweeping conclusions *inter alia*, what can biometry offer that will be in any way effective? Weigner's three principal conclusions as to fact are (*loc. cit.* p. 108):

1. "Das Gehirngewicht steht in keinem direkten Zusammenhang mit dem Alter, und wenn auch die angegebenen Zahlen sich zu Gunsten dieses Zusammenhanges verwerten einen, verlieren dieselben an der Bedeutung, da durch dieselben nicht der Beweis geführt werden kann, wie sich Gehirngewicht bei einem und demselben Individuum in verschiedenen Altersperioden verhält;

2. zwischen dem Gehirngewichte und der Körperlänge lässt sich keine bestimmte Proportion feststellen;

3. die Durchschnittszahl hat bloss einen relativen Wert, und zwar deshalb, weil die das Gehirngewicht repräsentierenden Zahlen durch die Wägung eines zufällig uns gerade zur Verfügung stehenden Materials gewonnen sind und das aus demselben bestimmte arithmetische Mittel keine konstante Grösse, sondern eine frei bewegliche Reihe von Zahlen uns darstellt."

It should be said that the material on which these conclusions are based is apparently within the limits of its probable errors entirely normal, and shows an approximately linear regression of brain-weight on age during adult life. This being the case the very positive conclusions seem to take on the character of extreme dogmatism. If 1. were true, by the same token no life insurance company could fix upon a premium rate for the insurance of lives of individuals, aged 21 say, which would enable it to carry on its business justly and fairly, because no life insurance company knows when any one individual aged 21 will die. Yet life insurance companies seem to be struggling on!

for brain-weight and "size of body." It has been shown (p. 66) that of a given change in mean brain-weight between 25 and 30 per cent. can be brought about by a selection of "size of body," measuring this both by stature and weight of body. As it is fair to assume that natural selection has acted on "size of body" we thus reach some idea of how much of an effect such selection has had on the weight of the brain.

A vital point at issue is as to what are the underlying causes of brain-weight differences. We have seen that there are definite racial types in brain-weight apart from differences in the other bodily characters which could be studied in this connection. To take a concrete example, we find that groups of male and female Bohemians having the same mean length and breadth of skull have probable mean brain-weights differing by about 69 grams, the female mean being lower than the male, of course. To what is this difference to be ascribed? Evidently there are a considerable number of possible factors which may enter into the matter. In the first place these males and females would still differ in stature by a certain amount so that a part of the remaining brain-weight difference would be accounted for in this way, but probably less than 25 per cent. Selecting too the same skull size would undoubtedly reduce the gross sexual difference in stature to some extent. The mean age of the arrays is essentially the same, so that nothing is gained from this source. To account for the still remaining difference there are at least the following possible factors:

1. The specific gravity of the female brain may be lower.

2. A unit increase in skull length and breadth may be associated with a greater change in skull height in the male than in the female.

3. The walls of the female skull may be thicker than in the male.

4. The female brain may fill the skull cavity less closely than does the male. (The opposite has been stated to be the case. Cf. Donaldson, *loc. cit.* p. 118.)

5. The ventricles of the brain may be proportionately larger in the female than in the male brain.

With the material at present available it is impossible to determine which of these possible factors really play the important rôle. It is my purpose merely to call attention to the kind of problem that confronts one at this point in the analysis of brain-weight constants. It at least will serve to show some of the directions in which more definite information is desired.

Donaldson* has well said: "The search for correlation between the size and form of the brain and the degree of the intelligence has interested all who have worked on this organ, and although it might be designated as the psychologist's standpoint, it has, from the very first, been in some measure before the minds of all." This point has so far been avoided in this paper for the reason that it seemed to me desirable to settle the question of the purely physical variations and

* *Loc. cit.* p. 85.

correlations before making any attempt on the much more difficult matter of brain-weight and intelligence correlation. I have, however, one point to add to the discussion of this already much debated subject. It follows rather simply from the theory of multiple correlation and regression that if the correlation coefficients r_{12} and r_{13} measuring the correlation between characters x_1 and x_2, and x_1 and x_3, are known, then it is possible to determine the limits between which the value of the coefficient of correlation r_{23} between x_2 and x_3 must lie. Without going into the details of the theory involved, which is given in convenient form by Yule*, it may be said that since

$$(r_{23} - r_{12}r_{13})^2 \text{ is not } > 1 + r_{12}^2 r_{13}^2 - r_{12}^2 - r_{13}^2,$$

r_{23} must lie between the limits

$$r_{12}r_{13} \pm (\sqrt{1 + r_{12}^2 r_{13}^2 - r_{12}^2 - r_{13}^2}).$$

Now, very fortunately, Pearson and his co-workers† at University College have determined for Cambridge undergraduates the correlation between "intelligence" as denoted by place in degree examinations and certain physical characters whose correlation with brain-weight we also know. Evidently then we have the material for getting at an approximation of the limits of the correlation between brain-weight and intelligence except that the two correlations from which the limits of the third are to be determined are not from the same material. This of course introduces an element of error, but as the coefficients of correlation of brain-weight with other characters seem to be fairly constant from race to race, a very serious error will probably not be made in assuming that the values of these correlations would not differ greatly in the English from what they are for continental races or from one another in two different groups of English. Leaving for the moment this question of possible error, let us examine the figures, first taking this problem: Assuming that Cambridge undergraduates have the weight of the cerebrum correlated with stature to roughly the same degree as is shown in Boyd's English data, what will be the limits of the correlation between weight of cerebrum and intelligence? The necessary data are as follows:

$r_{12} = -\cdot0056$ correlation coefficient between stature and intelligence.

$r_{13} = \cdot12$ " " " " " weight of cerebrum.

The limits of r_{23}, or of the coefficient of correlation between brain-weight and intelligence, are roughly $+\cdot98$ and $-\cdot99$. Now r_{12} is really insignificant, the conclusion the authors draw being that (*loc. cit.* p. 107), "stature is not correlated with place in degree examinations." Putting $r_{12} = 0$ the limits of r_{23} are $\pm\sqrt{1 - r_{13}^2}$. Or the practical conclusion we reach is that, so far as the known correlations of weight of cerebrum and intelligence respectively with stature give evidence, the correlation between weight of cerebrum and intelligence may have any value

* *Loc. cit.*

† Lee, A. Lewenz, M.A., and Pearson, K.: "On the Correlation of the Mental and Physical Characters in Man, Part II." *Roy. Soc. Proc.* Vol. LXXI. pp. 106—114, 1902.

between 0 and ± 1. There is no evidence from this source as to whether the correlation is positive or negative. Suppose we take still another case, stating the problem this time in this way: Assuming that the Cambridge undergraduates have brain-weight correlated with skull length to roughly the same degree as is shown by Bohemian males, what will be the limits of the correlation between brain-weight and intelligence? The necessary coefficients are:

$r_{12} = \cdot 0861$ correlation coefficient between skull length and intelligence.

$r_{13} = \cdot 5482$ „ „ „ „ „ and brain-weight.

The limits of r_{23} or the correlation between brain-weight and intelligence, are $+\cdot 7$ and $-\cdot 6$, or practically we reach much the same result as before. Now the Bohemian coefficient, as we have seen, probably is in error in the direction of being too small, or the probability is that the limits of r_{23} are still wider than the figures indicate. As to the reliability of these limits this may be said: in order to fix correctly the sign of the correlation between brain-weight and intelligence, both the coefficients between intelligence and physical characters and brain-weight and the same physical characters would have to be greater than $\sqrt{\cdot 5}$ or $\cdot 707$. Now I think it is reasonably evident from what we already know, that for civilized man at least there is no likelihood whatever of ever finding values as high as $\cdot 707$ for the coefficients concerned. Or in other words the degree of the correlation between brain-weight and intelligence is indeterminate, with the probability that it is sensibly equal to zero. That is, brain-weight and intelligence in the sense of mental capacity are probably not sensibly correlated. All the inferential evidence when subjected to careful scrutiny leads to the same conclusion, I think.

In closing I may perhaps be permitted to point out what seem to me to be the most promising lines for future investigation of the problems regarding the weight of the brain. In the first instance it seems to me highly desirable to transfer the problem for a time at least from man, where in the nature of the case all the fundamental records must be vitiated from the fact that we cannot weigh the brains of any considerable number of normal men instantly killed. As a consequence of this impairment of the data resulting from complex ante-mortem conditions the *absolute* values of certain of the biometric constants concerned cannot in the case of man be proven to be sensibly identical with what we should consider necessarily normal values. To be sure, such a sensible identity cannot, on the other hand, be disproven, but a state in which one can neither prove nor disprove is scientifically not altogether satisfactory. By working with animals it is possible to weigh any number of brains of instantly killed normal individuals. From records so obtained the values of the important biometric constants like the coefficients of correlation can be satisfactorily determined. In the hands of Prof. Donaldson this line of work is yielding results of the greatest significance and value. For continuing the work on the human side the primary need is for more and larger collections of brain-weights in which close attention is paid to the racial homogeneity of the material and to the recording of other subsidiary facts

as well as brain-weight. In comparison with the present series of brain-weighings a series which should record the following facts would be almost ideal.

1. Brain-weight, by the standard method.
2. Race. In this the more essential detail given the better.
3. Country of residence during adult life.
4. Occupation (as an index of social status).
5. Immediate cause of death.
6. Chronic diseases of adult life.
7. Stature.
8. Body-weight.
9. Head length.
10. Head breadth.
11. Head height.
12. Maximum horizontal circumference of head.

(9–12: Measurements all made before skull is opened, and with hair removed at points of contact.)

13. Age.
14. Sex.

Such a list at first sight appears formidable from the practical standpoint, but evidently the things which would give the greatest difficulty are 2 to 6 inclusive. The head measurements could be made with little trouble, and a relatively small expenditure of time. This list is presented with the hope that any future workers who may be about to undertake the great labour involved in obtaining a large mass of human brain-weight statistics will at least consider the points raised. The few additional facts would take but little more time in the collection, and they would greatly enhance the value of the completed series. I would especially call attention to the need for larger series of brain-weight statistics than those we now have. In order to do really "close" statistical work on the subject it is desirable that we have larger arrays of individuals of given age and stature types in order that the regressions may be smoothed and the brain-weights in more absolutely homogeneous material may be investigated.

Finally it should be said that the present paper is, from the biometrical standpoint, only a "first" study of the problems of brain-weight correlation. Much yet remains to be done, and as will have been apparent to the reader could be done with the present material. One thing especially which might be done is to separate the material into a third "old" group, comprising the individuals falling in age between 50 and 80, and treat this group separately. I very much doubt, however, in view of the apparent substantial linearity of the regression of brain-weight on age through the whole of adult life, and considering the statistically small number of individuals which in each case would have been available, whether the value of the results so obtained would have been in any way commensurate with the labour involved. Even a biometrician must stop somewhere. When we have much larger collections of brain-weight statistics to work with an old age division of the material will be at least experimentally justified.

14. *Summary.*

Some of the more important general conclusions of a study of five series of brain-weighings, representing Swedish, Hessian, Bavarian, Bohemian, and English sub-races of man and including altogether the weights of 2100 adult male and 1034 adult female brains, may be summarised as follows:

1. There are definite racial types in brain-weight. The differences between racial groups in this character are only in part to be accounted for by differences in other characters of the body.

2. The series studied exhibit a very fair degree of homogeneity.

3. In respect to variability the characters, brain-weight and skull capacity are sensibly equal. The coefficients of variation for brain-weight are intermediate in value between those which have been determined for skeletal characters in man on the one hand, and those for the weights of the viscera and for various physiological characters on the other hand.

4. Having regard to the size of the present series we conclude that variation in weight of the brain may for practical purposes be considered to follow the "normal" law of the distribution of errors. If the mean and the mode do not exactly coincide the mean will be slightly greater than the mode in brain-weight frequency distributions.

5. The correlation of brain-weight with age, stature and body-weight, is in all cases low.

6. The correlation of brain-weight with skull length and skull breadth is, in comparison with the other characters studied, fairly high, and is for both skull length and breadth positive.

7. The sexual differences in mean brain-weight are practically constant in all the races studied, whether considered absolutely or relatively.

8. Only a part of the sexual difference is to be accounted for by differences between the sexes in other bodily characters.

9. The sexes are equally variable in respect to brain-weight.

10. The weight of the brain tends to be more highly correlated with other characters in the female than in the male.

11. The correlation of brain-weight with age is negative. The regression of brain-weight on age is, so far as can be determined from our present series, linear throughout the period of life comprised between the ages 20 to 80. In other words, there is a steady decline in the weight of the brain with advancing age, beginning at about the twentieth year and continuing throughout adult life.

12. The correlation of brain-weight with stature and with body-weight is positive and in each case the regression is linear.

13. The weight of the cerebrum follows the same laws of variation and correlation as does the weight of the entire brain, but exhibits a somewhat greater relative variability.

14. The results as to variation and correlation in brain-weight are closely accordant for all the races studied.

15. The indirect selection of bodily characters can modify the mean weight of the brain to a considerable extent. The amount of this effect has been measured in certain cases.

16. There is no evidence that brain-weight is sensibly correlated with intellectual ability. The limits of this correlation have been shown to be not closer than 0 and $\pm$ ·6.

17. So far as can be determined from present material dolichocephaly has associated with it no advantage in mean brain-weight over brachycephaly and *vice versâ*.

18. There is great need for further large and homogeneous collections of brain-weight statistics. When these are available for a considerable number of races it will be possible to pass from intra-racial to inter-racial problems.

APPENDIX OF MEASUREMENTS.

Correlation Tables.

The fundamental correlation tables on which the results of this paper are based have been placed together here at the end for convenience in reference. The tables are numbered consecutively in Arabic numerals, and each bears its own explanation. A word may be said as to the plan of arrangement. The racial order is

Swedes	Tables	1	to	12	inclusive.
Hessians	„	13	„	24	„
Bohemians	„	25	„	34	„
Bavarians	„	35	„	44	„
English	„	45 and		46	„

Under each race the order of arrangement is in each case for the tables involving brain-weight as one of the variables to precede the subsidiary tables involving only the related characters. The tables of males precede those for females, and the "total" series precede the "young."

TABLE 1.

Swedes. Brain-weight and Stature. "Total" Series. Males.

Stature (centimetres).

Brain-weight (grams).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
1100—1149	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
1150—1199	—	—	—	—	—	2	3	1	—	2	2	—	—	—	—	—	10
1200—1249	—	—	1	—	—	4	2	5	5	1	1	1	—	1	—	—	21
1250—1299	—	1	—	—	4	3	5	8	8	6	3	3	—	3	—	—	44
1300—1349	1	—	—	—	2	4	4	7	11	9	7	6	2	—	—	—	53
1350—1399	1	1	—	—	1	7	17	21	11	13	6	5	3	—	—	—	86
1400—1449	—	—	1	1	—	2	9	10	13	13	8	11	2	2	—	—	72
1450—1499	—	—	1	1	1	4	6	8	13	8	9	5	1	2	—	1	60
1500—1549	—	—	—	—	1	—	1	2	4	5	7	4	4	—	—	—	28
1550—1599	—	—	—	—	1	3	—	6	1	4	2	6	2	—	—	—	25
1600—1649	—	—	—	—	—	—	—	3	4	4	—	1	—	—	—	—	12
1650—1699	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	3
1700—1749	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	2	2	3	2	10	29	47	72	71	66	45	44	14	8	—	1	416

TABLE 2.

Swedes. Brain-weight and Stature. "Total" Series. Females.

Stature (centimetres).

Brain-weight (grams).	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	Totals
900— 949	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1000—1049	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
1050—1099	—	—	—	1	—	3	—	1	1	1	1	1	—	—	—	9
1100—1149	1	—	1	—	2	3	2	6	4	4	2	2	1	—	—	28
1150—1199	—	—	—	—	1	2	3	8	7	1	3	3	2	—	—	30
1200—1249	—	—	1	—	2	3	6	9	11	3	12	4	2	—	—	53
1250—1299	—	—	—	—	—	3	1	8	5	9	3	7	2	—	2	40
1300—1349	—	—	—	—	—	—	—	4	8	6	6	5	1	—	—	30
1350—1399	—	—	—	—	1	—	1	5	3	3	5	2	2	1	—	23
1400—1449	—	—	—	—	—	—	—	1	1	1	3	3	1	—	—	10
1450—1499	—	—	—	—	—	—	—	1	—	2	2	—	—	—	1	6
1500—1549	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
1550—1599	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	—	2	1	6	14	14	44	40	30	38	28	11	1	3	233

TABLE 3.

Swedes. Brain-weight and Stature. "Young" Series. Males.

20—50 *years.*

Stature (centimetres).

Brain-weight (grams).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
1100—1149	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
1150—1199	—	—	—	—	—	1	2	—	—	1	—	—	—	—	—	—	4
1200—1249	—	—	1	—	—	2	1	4	2	1	1	—	—	1	—	—	13
1250—1299	—	—	—	—	1	1	2	5	3	2	—	2	—	2	—	—	18
1300—1349	1	—	—	—	—	4	4	5	7	3	6	5	—	—	—	—	35
1350—1399	1	—	—	—	—	4	11	12	6	7	6	4	2	—	—	—	53
1400—1449	—	—	—	—	—	1	3	6	8	10	5	8	1	1	—	—	43
1450—1499	—	—	1	1	—	2	5	7	10	3	5	4	—	1	—	1	40
1500—1549	—	—	—	—	1	—	1	1	3	3	7	2	3	—	—	—	21
1550—1599	—	—	—	—	1	3	—	3	1	3	2	4	2	—	—	—	19
1600—1649	—	—	—	—	—	—	—	2	4	4	—	1	—	—	—	—	11
1650—1699	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	3
1700—1749	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	2	—	2	1	3	18	29	46	45	38	32	32	8	5	—	1	262

TABLE 4.

Swedes. Brain-weight and Stature. "Young" Series. Females.

20—50 *years.*

Stature (centimetres).

Brain-weight (grams).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	Totals
900— 949	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1000—1049	—	—	—	—	—	1	—	—	—	—	—	—	—	1
1050—1099	—	1	—	1	—	—	—	—	—	—	—	—	—	2
1100—1149	1	—	1	1	—	2	2	2	1	2	—	—	—	12
1150—1199	—	—	1	—	2	3	5	—	1	2	2	—	—	16
1200—1249	—	—	2	2	1	2	5	3	6	1	2	—	—	24
1250—1299	—	—	—	2	—	5	2	5	3	5	2	—	2	26
1300—1349	—	—	—	—	—	2	3	4	4	2	1	—	—	16
1350—1399	—	—	1	—	1	3	2	1	3	1	1	1	—	14
1400—1449	—	—	—	—	—	1	—	1	3	2	1	—	—	8
1450—1499	—	—	—	—	—	1	—	2	2	—	—	—	1	6
1500—1549	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1550—1599	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	1	5	6	5	20	19	18	23	16	9	1	3	127

TABLE 5.

Swedes. Brain-weight and Age. "Total" Series. Males.

Brain-weight (grams).

Age (years).	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
20—24	—	—	—	1	6	5	2	7	2	3	1	—	—	27
25—29	—	—	—	1	4	10	8	4	2	2	3	2	1	37
30—34	—	1	2	—	5	13	8	6	5	3	4	1	—	48
35—39	1	2	1	6	4	8	9	9	4	4	1	—	—	49
40—44	—	1	9	6	10	9	10	5	5	4	2	—	—	61
45—49	—	—	1	4	6	8	6	9	3	3	—	—	—	40
50—54	—	—	1	7	4	15	7	9	4	2	1	—	—	50
55—59	—	1	3	4	9	6	7	6	1	2	—	—	—	39
60—64	—	4	1	7	2	4	8	3	1	2	—	—	—	32
65—69	—	1	2	4	3	2	3	2	1	—	—	—	—	18
70—74	—	—	—	2	—	4	—	—	—	—	—	—	—	6
75—79	—	—	1	2	—	2	4	—	—	—	—	—	—	9
Totals	1	10	21	44	53	86	72	60	28	25	12	3	1	416

TABLE 6.

Swedes. Brain-weight and Age. "Total" Series. Females.

Brain-weight (grams).

Age (years).	900—949	950—999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	—	—	—	1	2	6	5	—	2	2	—	—	18
25—29	—	—	—	—	1	7	3	—	1	3	1	—	—	1	17
30—34	1	—	—	1	3	—	2	8	—	3	—	2	—	—	20
35—39	—	—	—	—	2	4	5	4	5	2	2	2	—	—	26
40—44	—	—	1	—	4	3	7	5	3	5	2	—	—	—	30
45—49	—	—	—	1	2	1	5	3	2	1	1	—	—	—	16
50—54	—	—	—	2	4	1	6	1	2	3	2	—	—	—	21
55—59	—	—	—	2	4	5	6	1	6	4	—	—	—	—	28
60—64	—	—	—	—	1	5	8	5	4	2	—	—	—	—	25
65—69	—	—	—	1	1	1	3	4	2	—	—	—	—	—	12
70—74	—	—	—	1	4	2	4	2	—	—	—	—	1	—	14
75—80	—	—	—	1	2	—	2	1	—	—	—	—	—	—	6
Totals	1	—	1	9	28	30	53	40	30	23	10	6	1	1	233

TABLE 7.

Swedes. Brain-weight and Age. "Young" Series. Males.

Brain-weight (grams).

Age (years).	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
20—24	—	—	—	1	6	5	2	7	2	3	1	—	—	27
25—29	—	—	—	1	4	10	8	4	2	2	3	2	1	37
30—34	—	1	2	—	5	13	8	6	5	3	4	1	—	48
35—39	1	2	1	6	4	8	9	9	4	4	1	—	—	49
40—44	—	1	9	6	10	9	10	5	5	4	2	—	—	61
45—49	—	—	1	4	6	8	6	9	3	3	—	—	—	40
Totals	1	4	13	18	35	53	43	40	21	19	11	3	1	262

TABLE 8.

Swedes. Brain-weight and Age. "Young" Series. Females.

Brain-weight (grams).

Age (years).	900— 999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	Totals
20—24	—	—	—	—	—	1	2	6	5	—	2	2	—	—	18
25—29	—	—	—	—	1	7	3	—	1	3	1	—	—	1	17
30—34	1	—	—	1	3	—	2	8	—	3	—	2	—	—	20
35—39	—	—	—	—	2	4	5	4	5	2	2	2	—	—	26
40—44	—	—	1	—	4	3	7	5	3	5	2	—	—	—	30
45—49	—	—	—	1	2	1	5	3	2	1	1	—	—	—	16
Totals	1	—	1	2	12	16	24	26	16	14	8	6	—	1	127

TABLE 9.

Swedes. Stature and Age. "Total" Series. Males.

Stature (centimetres).

Age (years).	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	*183—185*	*186—188*	*189—191*	Totals
20—24	—	—	—	—	—	—	3	1	5	4	7	3	2	1	—	1	27
25—29	—	—	—	—	—	1	1	7	4	6	6	10	1	1	—	—	37
30—34	—	—	1	—	1	2	5	9	12	6	6	4	—	2	—	—	48
35—39	1	—	—	1	—	5	4	12	9	6	3	6	2	—	—	—	49
40—44	—	—	1	—	1	6	8	12	9	11	5	5	3	—	—	—	61
45—49	1	—	—	—	1	4	8	5	6	5	5	4	—	1	—	—	40
50—54	—	2	—	—	1	2	9	8	10	6	3	4	3	2	—	—	50
55—59	—	—	—	—	2	2	3	7	6	9	5	2	2	1	—	—	39
60—64	—	—	1	—	3	3	4	7	3	8	1	1	1	—	—	—	32
65—69	—	—	—	—	1	2	1	1	5	3	2	3	—	—	—	—	18
70—74	—	—	—	—	—	1	—	1	1	1	1	1	—	—	—	—	6
75—79	—	—	—	1	—	1	1	2	1	1	1	1	—	—	—	—	9
Totals	2	2	3	2	10	29	47	72	71	66	45	44	14	8	—	1	416

TABLE 10.

Swedes. Stature and Age. "Total" Series. Females.

Stature (centimetres).

Age (years).	*132—134*	*135—137*	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	Totals
20—24	—	—	—	—	—	—	—	1	3	4	6	2	1	—	1	18
25—29	—	—	1	—	—	—	—	3	4	1	2	2	4	—	—	17
30—34	—	—	—	—	—	3	2	3	1	2	—	7	—	1	1	20
35—39	—	—	—	—	—	2	2	4	3	6	6	2	1	—	—	26
40—44	—	—	—	—	4	1	—	5	5	4	5	3	2	—	1	30
45—49	—	—	—	1	1	—	1	4	3	1	4	—	1	—	—	16
50—54	—	—	—	—	—	2	2	4	2	3	3	3	2	—	—	21
55—59	1	—	—	—	—	3	3	3	5	6	4	3	—	—	—	28
60—64	—	—	1	—	—	—	2	6	7	—	4	5	—	—	—	25
65—69	—	—	—	—	1	—	1	6	1	1	2	—	—	—	—	12
70—74	—	—	—	—	—	3	1	3	3	1	2	1	—	—	—	14
75—79	—	—	—	—	—	—	—	2	3	1	—	—	—	—	—	6
Totals	1	—	2	1	6	14	14	44	40	30	38	28	11	1	3	233

TABLE 11.

Swedes. Stature and Age. "Young" Series. Males.

Stature (centimetres).

Age (years).	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	*183—185*	*186—188*	*189—191*	Totals
20—24	—	—	—	—	—	—	3	1	5	4	7	3	2	1	—	1	27
25—29	—	—	—	—	—	1	1	7	4	6	6	10	1	1	—	—	37
30—34	—	—	1	—	1	2	5	9	12	6	6	4	—	2	—	—	48
35—39	1	—	—	1	—	5	4	12	9	6	3	6	2	—	—	—	49
40—44	—	—	1	—	1	6	8	12	9	11	5	5	3	—	—	—	61
45—49	1	—	—	—	1	4	8	5	6	5	5	4	—	1	—	—	40
Totals	2	—	2	1	3	18	29	46	45	38	32	32	8	5	—	1	262

TABLE 12.

Swedes. Stature and Age. "Young" Series. Females.

Stature (centimetres).

Age (years).	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	Totals
20—24	—	—	—	—	—	1	3	4	6	2	1	—	1	18
25—29	1	—	—	—	—	3	4	1	2	2	4	—	—	17
30—34	—	—	—	3	2	3	1	2	—	7	—	1	1	20
35—39	—	—	—	2	2	4	3	6	6	2	1	—	—	26
40—44	—	—	4	1	—	5	5	4	5	3	2	—	1	30
45—49	—	1	1	—	1	4	3	1	4	—	1	—	—	16
Totals	1	1	5	6	5	20	19	18	23	16	9	1	3	127

TABLE 13.

Hessians. Brain-weight and Stature. "Total" Series. Males.

Stature (centimetres).

Brain-weight (grams).	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	*183—185*	*186—188*	*189—191*	Totals
1000—1049	—	—	—	—	—	—	—	2	—	—	1	—	—	1	—	—	—	—	4
1050—1099	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	3
1100—1149	—	1	—	—	—	—	—	3	—	—	1	1	—	—	1	—	—	—	7
1150—1199	—	—	—	—	—	—	2	1	1	2	—	2	—	—	—	—	—	—	8
1200—1249	—	—	—	—	1	—	1	3	3	3	1	4	1	1	1	—	—	—	19
1250—1299	—	—	—	—	1	4	6	7	5	8	6	5	5	1	—	1	—	—	49
1300—1349	—	—	—	1	2	4	8	10	14	8	18	8	6	5	—	—	—	1	85
1350—1399	1	—	—	—	1	—	4	6	9	10	11	14	7	3	1	—	—	—	67
1400—1449	—	—	—	1	1	1	4	5	12	13	19	16	8	2	—	3	—	—	85
1450—1499	—	1	1	—	—	—	2	2	9	11	7	17	2	5	2	—	—	—	59
1500—1549	—	—	—	—	—	3	2	4	7	10	14	9	3	3	1	1	—	—	57
1550—1599	—	—	—	—	—	1	1	1	3	4	3	8	3	1	1	—	—	—	26
1600—1649	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	3
1650—1699	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
1700—1749	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	2	2	1	2	6	14	30	45	63	73	82	85	35	22	7	5	—	1	475

TABLE 14.

Hessians. Brain-weight and Stature. "Total" Series. Females.

Stature (centimetres).

Brain-weight (grams).	*126—128*	*129—131*	*132—134*	*135—137*	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	Totals
950— 999	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1000—1049	1	—	—	—	—	—	—	1	—	1	2	1	—	—	—	—	—	—	—	6
1050—1099	—	—	—	—	1	—	1	1	1	1	4	3	—	—	—	—	—	—	—	12
1100—1149	—	—	—	—	—	1	—	2	3	3	7	3	1	—	1	—	—	—	—	21
1150—1199	—	—	—	—	—	2	—	5	5	9	3	5	1	1	—	—	—	—	—	31
1200—1249	—	—	—	—	1	1	4	5	3	9	19	10	6	2	1	1	—	—	—	62
1250—1299	—	—	—	—	—	1	—	3	4	9	14	9	5	3	5	3	—	—	—	56
1300—1349	—	—	—	—	—	—	1	1	6	8	6	7	4	2	1	—	—	—	—	36
1350—1399	—	—	—	1	—	1	—	3	6	4	5	4	1	1	1	—	—	—	—	27
1400—1449	—	—	—	—	1	—	1	2	4	1	4	3	3	1	—	1	1	—	1	23
1450—1499	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
1500—1549	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
Totals	1	—	—	1	3	7	7	23	32	46	65	45	25	10	9	5	1	—	1	281

TABLE 15.

Hessians. Brain-weight and Stature. "Young" Series. Males.

15—50 *years.*

Stature (centimetres).

Brain-weight (grams).	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	*183—185*	Totals
1000—1049	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
1500—1099	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
1100—1149	—	1	—	—	—	—	—	1	—	—	1	1	—	—	1	—	5
1150—1199	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
1200—1249	—	—	—	—	—	—	1	1	1	2	—	2	—	—	—	—	7
1250—1299	—	—	—	—	1	2	4	3	2	7	2	2	3	1	—	1	28
1300—1349	—	—	—	1	—	3	3	6	10	5	10	6	4	4	—	—	52
1350—1399	1	—	—	—	1	—	3	4	4	7	8	11	2	—	—	—	41
1400—1449	—	—	—	1	1	1	2	2	7	5	9	8	3	1	—	3	43
1450—1499	—	1	1	—	—	—	2	2	7	9	6	12	2	2	2	—	46
1500—1549	—	—	—	—	—	2	1	4	3	7	11	6	2	1	—	—	37
1550—1599	—	—	—	—	—	—	1	—	2	4	2	8	3	1	1	—	22
1600—1649	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	3
1650—1699	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	2	1	2	3	8	17	26	36	50	50	58	19	10	4	4	291

TABLE 16.

Hessians. Brain-weight and Stature. "Young" Series. Females.

15—50 *years.*

Stature (centimetres).

Brain-weight (grams).	*126—128*	*129—131*	*132—134*	*135—137*	*138—140*	*141—143*	*144—146*	*147—149*	*150—152*	*153—155*	*156—158*	*159—161*	*162—164*	*165—167*	*168—170*	*171—173*	*174—176*	*177—179*	*180—182*	Totals
1000—1049	1	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	3
1050—1099	—	—	—	—	1	—	—	—	1	—	2	2	—	—	—	—	—	—	—	6
1100—1149	—	—	—	—	—	1	—	—	1	—	3	2	—	—	—	—	—	—	—	7
1150—1199	—	—	—	—	—	1	—	3	1	4	3	1	1	1	—	—	—	—	—	15
1200—1249	—	—	—	—	1	1	3	2	2	4	14	4	6	1	—	1	—	—	—	39
1250—1299	—	—	—	—	—	—	—	2	3	5	11	6	2	2	2	3	—	—	—	36
1300—1349	—	—	—	—	—	—	—	1	4	5	1	5	3	2	1	—	—	—	—	22
1350—1399	—	—	—	1	—	1	—	2	4	2	3	3	1	1	1	—	—	—	—	19
1400—1449	—	—	—	—	1	—	1	2	4	1	2	3	3	—	—	1	1	—	1	20
1450—1499	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
1500—1549	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
Totals	1	—	—	1	3	4	4	13	20	22	41	26	20	7	4	5	1	—	1	173

TABLE 17.

Hessians. Brain-weight and Age. "Total" Series. Males.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	1700—1749	Totals
15—19	—	—	1	—	—	2	8	4	10	5	4	2	—	—	—	36
20—24	—	1	1	1	—	2	7	9	6	15	10	5	1	—	—	58
25—29	—	—	1	—	1	6	14	6	6	8	7	1	1	1	—	52
30—34	—	—	1	—	5	2	4	5	9	5	6	3	—	1	—	41
35—39	—	—	—	1	—	6	5	2	6	1	1	2	—	—	—	24
40—44	1	—	—	—	1	4	6	14	1	6	5	4	—	—	—	42
45—49	1	—	1	—	—	6	8	1	5	6	4	5	1	—	—	38
50—54	1	1	—	2	6	6	8	2	17	3	8	—	—	—	1	55
55—59	—	—	1	2	—	5	6	10	8	3	5	1	—	—	—	41
60—64	1	1	—	—	2	1	5	6	12	2	3	—	—	—	—	33
65—69	—	—	—	2	1	4	8	4	3	4	3	1	—	—	—	30
70—74	—	—	1	—	3	2	4	2	1	1	1	2	—	—	—	17
75—79	—	—	—	—	—	3	2	2	1	—	—	—	—	—	—	8
Totals	4	3	7	8	19	49	85	67	85	59	57	26	3	2	1	475

TABLE 18.

Hessians. Brain-weight and Age. "Total" Series. Females.

Brain-weight (grams).

Age (years).	950— 999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	Totals
15—19	—	—	1	—	2	4	2	—	—	7	1	—	17
20—24	—	—	1	—	1	5	9	1	2	1	1	1	22
25—29	—	—	1	1	2	4	3	6	4	4	—	1	26
30—34	—	—	—	3	2	6	2	—	3	1	—	1	18
35—39	—	—	3	—	4	5	5	6	6	2	1	—	32
40—44	—	1	—	2	1	6	10	7	2	2	—	—	31
45—49	—	2	—	1	3	9	5	2	2	3	—	—	27
50—54	—	—	—	2	3	8	4	4	5	1	—	—	27
55—59	—	—	2	3	2	5	3	5	3	2	—	—	25
60—64	—	—	2	3	2	3	5	1	—	—	—	—	16
65—69	—	—	1	—	4	4	5	2	—	—	—	—	16
70—74	1	2	1	5	3	3	3	2	—	—	—	—	20
75—79	—	1	—	1	2	—	—	—	—	—	—	—	4
Totals	1	6	12	21	31	62	56	36	27	23	3	3	281

TABLE 19.

Hessians. Brain-weight and Age. "Young" Series. Males.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
15—19	—	—	1	—	—	2	8	4	10	5	4	2	—	—	36
20—24	—	1	1	1	—	2	7	4	6	15	10	5	1	—	58
25—29	—	—	1	—	1	6	14	6	6	8	7	1	1	1	52
30—34	—	—	1	—	5	2	4	5	9	5	6	3	—	1	41
35—39	—	—	—	1	—	6	5	2	6	1	1	2	—	—	24
40—44	1	—	—	—	1	4	6	14	1	6	5	4	—	—	42
45—49	1	—	1	—	—	6	8	1	5	6	4	5	1	—	38
Totals	2	1	5	2	7	28	52	41	43	46	37	22	3	2	291

TABLE 20.

Hessians. Brain-weight and Age. "Young" Series. Females.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	Totals
15—19	—	1	—	2	4	2	—	—	7	1	—	17
20—24	—	1	—	1	5	9	1	2	1	1	1	22
25—29	—	1	1	2	4	3	6	4	4	—	1	26
30—34	—	—	3	2	6	2	—	3	1	—	1	18
35—39	—	3	—	4	5	5	6	6	2	1	—	32
40—44	1	—	2	1	6	10	7	2	2	—	—	31
45—49	2	—	1	3	9	5	2	2	3	—	—	27
Totals	3	6	7	15	39	36	22	19	20	3	3	173

TABLE 21.

Hessians. Stature and Age. "Total" Series. Males.

Stature (centimetres).

Age (years).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	189—191	Totals
15—19	1	2	1	2	1	4	4	4	4	3	5	2	2	1	—	—	—	—	36
20—24	—	—	—	—	—	1	—	6	8	9	18	10	2	4	—	—	—	—	58
25—29	—	—	—	—	—	—	6	7	8	8	8	11	3	1	—	—	—	—	52
30—34	—	—	—	—	1	1	—	2	6	9	2	12	3	1	2	2	—	—	41
35—39	—	—	—	—	—	—	2	—	3	5	4	6	2	1	—	1	—	—	24
40—44	—	—	—	—	1	1	4	5	3	6	7	9	3	1	1	1	—	—	42
45—49	—	—	—	—	—	1	1	2	4	10	6	8	4	1	1	—	—	—	38
50—54	1	—	—	—	1	1	3	3	8	11	4	11	6	4	1	1	—	—	55
55—59	—	—	—	—	—	—	3	3	4	4	11	5	5	4	1	—	—	1	41
60—64	—	—	—	—	—	1	1	4	4	2	9	7	3	2	—	—	—	—	33
65—69	—	—	—	—	1	3	3	2	6	3	4	3	2	2	1	—	—	—	30
70—74	—	—	—	—	1	1	1	5	4	2	3	—	—	—	—	—	—	—	17
75—79	—	—	—	—	—	—	2	2	1	1	1	1	—	—	—	—	—	—	8
Totals	2	2	1	2	6	14	30	45	63	73	82	85	35	22	7	5	—	1	475

TABLE 22.

Hessians. Stature and Age. "Total" Series. Females.

Stature (centimetres).

Age (years).	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—183	Totals
15—19	—	—	—	—	2	—	—	3	3	1	3	1	2	—	1	1	—	—	—	17
20—24	—	—	—	—	—	—	—	—	—	6	7	3	4	—	1	—	1	—	—	22
25—29	—	—	—	1	—	1	—	—	7	2	5	2	5	1	1	1	—	—	—	26
30—34	—	—	—	—	—	1	—	1	2	2	5	2	2	1	—	2	—	—	—	18
35—39	—	—	—	—	1	2	1	4	1	4	7	7	4	—	—	1	—	—	—	32
40—44	—	—	—	—	—	—	1	4	3	3	7	7	1	3	1	—	—	—	1	31
45—49	1	—	—	—	—	—	2	1	4	4	7	4	2	2	—	—	—	—	—	27
50—54	—	—	—	—	—	—	—	5	4	4	8	4	—	—	2	—	—	—	—	27
55—59	—	—	—	—	—	—	2	—	—	6	6	6	1	2	2	—	—	—	—	25
60—64	—	—	—	—	—	—	—	1	1	5	3	3	2	—	1	—	—	—	—	16
65—69	—	—	—	—	—	1	1	—	3	3	5	2	—	1	—	—	—	—	—	16
70—74	—	—	—	—	—	2	—	3	4	4	2	3	2	—	—	—	—	—	—	20
75—79	—	—	—	—	—	—	—	1	—	2	—	1	—	—	—	—	—	—	—	4
Totals	1	—	—	1	3	7	7	23	32	46	65	45	25	10	9	5	1	—	1	281

TABLE 23.

Hessians. Stature and Age. "Young" Series. Males.

15—50 *years.*

Stature (centimetres).

Age (years).	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	Totals
15—19	1	2	1	2	1	4	4	4	4	3	5	2	2	1	—	—	36
20—24	—	—	—	—	—	1	—	6	8	9	18	10	2	4	—	—	58
25—29	—	—	—	—	—	—	6	7	8	8	8	11	3	1	—	—	52
30—34	—	—	—	—	1	1	—	2	6	9	2	12	3	1	2	2	41
35—39	—	—	—	—	—	—	2	—	3	5	4	6	2	1	—	1	24
40—44	—	—	—	—	1	1	4	5	3	6	7	9	3	1	1	1	42
45—49	—	—	—	—	—	1	1	2	4	10	6	8	4	1	1	—	38
Totals	1	2	1	2	3	8	17	26	36	50	50	58	19	10	4	4	291

TABLE 24.

Hessians. Stature and Age. "Young" Series. Females.

15—50 *years.*

Stature (centimetres).

Age (years).	126—128	129—131	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	Totals
15—19	—	—	—	—	2	—	—	3	3	1	3	1	2	—	1	1	—	—	—	17
20—24	—	—	—	—	—	—	—	—	—	6	7	3	4	—	1	—	1	—	—	22
25—29	—	—	—	1	—	1	—	—	7	2	5	2	5	1	1	1	—	—	—	26
30—34	—	—	—	—	—	1	—	1	2	2	5	2	2	1	—	2	—	—	—	18
35—39	—	—	—	—	1	2	1	4	1	4	7	7	4	—	—	1	—	—	—	32
40—44	—	—	—	—	—	—	1	4	3	3	7	7	1	3	1	—	—	—	1	31
45—49	1	—	—	—	—	—	2	1	4	4	7	4	2	2	—	—	—	—	—	27
Totals	1	—	—	1	3	4	4	13	20	22	41	26	20	7	4	5	1	—	1	173

TABLE 25.

Bohemians. Brain-weight and Stature.
"Young" Series. Males. 20—59 years.

Brain-weight (grams).

Stature (centimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	Totals
150—154	—	1	3	—	1	—	—	—	5
155—159	1	1	4	4	4	1	—	—	15
160—164	—	5	15	22	9	2	1	—	54
165—169	—	3	15	24	17	7	—	—	66
170—174	—	5	17	21	21	2	2	1	69
175—179	—	3	7	9	10	5	2	—	36
180—184	—	1	—	7	4	2	—	1	15
185—189	—	—	2	—	2	—	—	—	4
190—194	—	—	—	—	—	1	—	—	1
Totals	1	19	63	88	68	20	5	2	266

TABLE 26.

Bohemians. Brain-weight and Stature.
"Young" Series. Females. 20—59 years.

Brain-weight (grams).

Stature (centimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
130—134	—	—	1	—	—	—	1
135—139	—	—	—	1	—	—	1
140—144	—	—	—	2	—	—	2
145—149	—	2	7	5	2	—	16
150—154	1	—	11	6	4	—	22
155—159	2	4	19	14	2	1	42
160—164	—	3	6	14	7	2	32
165—169	—	—	2	5	4	—	11
170—174	—	—	2	2	1	—	5
175—179	—	—	—	—	1	—	1
Totals	3	9	48	49	21	3	133

TABLE 27.

Bohemians. Brain-weight and Age.
"Total" Series. Males.

Brain-weight (grams).

Age (years).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	Totals
20—29	—	4	12	28	21	9	2	1	77
30—39	—	5	12	17	14	6	3	1	58
40—49	1	10	24	27	23	6	—	—	91
50—59	—	3	19	31	19	4	1	—	77
60—69	—	4	17	11	9	4	1	—	46
70—79	—	1	14	8	—	—	—	—	23
Totals	1	27	98	122	86	29	7	2	372

TABLE 28.

Bohemians. Brain-weight and Age.
"Total" Series. Females.

Brain-weight (grams).

Age (years).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
20—29	1	2	28	26	13	2	72
30—39	—	2	9	14	7	3	35
40—49	1	4	8	10	7	—	30
50—59	1	2	14	6	2	1	26
60—69	—	4	10	5	—	—	19
70—79	—	5	4	4	2	—	15
Totals	3	19	73	65	31	6	197

TABLE 29.

Bohemians. Brain-weight and Skull Length. Males. 20—59 *years.*

Brain-weight (grams).

Skull length (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	1800—1899	Totals
155—159	—	—	1	1	—	—	—	—	—	2
160—164	—	—	2	6	4	2	—	—	—	14
165—169	1	—	9	10	18	3	1	—	—	42
170—174	—	—	5	19	28	11	4	1	—	68
175—179	—	—	4	19	29	23	4	—	—	79
180—184	—	—	—	10	19	23	8	1	—	61
185—189	—	—	—	1	2	12	4	—	—	19
190—194	—	—	—	—	1	2	3	4	—	10
195—199	—	—	—	—	—	1	1	—	2	4
Totals	1	—	21	66	101	77	25	6	2	299

TABLE 30.

Bohemians. Brain-weight and Skull Length. Females. 20—59 *years.*

Brain-weight (grams).

Skull length (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
150—154	—	1	—	—	—	1	2
155—159	—	2	1	—	—	—	3
160—164	1	1	18	4	3	—	27
165—169	—	4	16	9	3	1	33
170—174	1	2	18	33	14	3	71
175—179	—	—	6	4	7	—	17
180—184	—	—	—	4	1	1	6
Totals	2	10	59	54	28	6	159

TABLE 31.

Bohemians. Brain-weight and Skull Breadth. Males. 20—59 *years.*

Brain-weight (grams).

Skull breadth (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	1600—1699	1700—1799	1800—1899	Totals
110—114	—	—	—	—	—	1	—	—	—	1
115—119	—	—	—	—	—	—	—	—	—	—
120—124	—	—	—	—	—	—	—	—	—	—
125—129	—	—	—	—	—	—	—	—	—	—
130—134	—	—	—	—	2	1	—	—	—	3
135—139	—	—	4	2	1	1	—	—	—	8
140—144	1	—	8	19	21	7	1	1	—	58
145—149	—	—	8	21	32	17	—	—	—	78
150—154	—	—	1	22	35	26	11	—	—	95
155—159	—	—	—	2	9	14	11	2	2	40
160—164	—	—	—	—	1	6	2	2	—	11
165—169	—	—	—	—	—	3	—	1	—	4
170—174	—	—	—	—	—	—	—	—	—	—
175—179	—	—	—	—	—	1	—	—	—	1
Totals	1	—	21	66	101	77	25	6	2	299

TABLE 32.

Bohemians. Brain-weight and Skull Breadth. Females. 20—59 *years.*

Brain-weight (grams).

Skull breadth (millimetres).	1000—1099	1100—1199	1200—1299	1300—1399	1400—1499	1500—1599	Totals
130—134	—	—	4	1	1	—	6
135—139	2	4	9	5	—	—	20
140—144	—	6	28	19	7	—	60
145—149	—	—	14	20	6	4	44
150—154	—	—	4	6	11	1	22
155—159	—	—	—	2	3	1	6
160—164	—	—	—	1	—	—	1
Total	2	10	59	54	28	6	159

TABLE 33.

Bohemians. Skull Length and Skull Breadth. Males.

20—59 *years.*

Skull length (millimetres).

Skull breadth (millimetres).	*155—159*	*160—164*	*165—169*	*170—174*	*175—179*	*180—184*	*185—189*	*190—194*	*195—199*	Totals
110—114	—	—	—	1	—	—	—	—	—	1
115—119	—	—	—	—	—	—	—	—	—	—
120—124	—	—	—	—	—	—	—	—	—	—
125—129	—	—	—	—	—	—	—	—	—	—
130—134	—	—	2	—	—	1	—	—	—	3
135—139	—	1	3	3	1	—	—	—	—	8
140—144	—	6	12	14	8	17	1	—	—	58
145—149	2	5	14	17	24	10	5	1	—	78
150—154	—	2	10	25	30	20	6	2	—	95
155—159	—	—	1	5	11	11	5	4	3	40
160—164	—	—	—	3	5	1	—	2	—	11
165—169	—	—	—	—	—	1	2	1	—	4
170—174	—	—	—	—	—	—	—	—	—	—
175—179	—	—	—	—	—	—	—	—	1	1
Totals	2	14	42	68	79	61	19	10	4	299

TABLE 34.

Bohemians. Skull Length and Skull Breadth. Females.

20—59 *years.*

Skull length (millimetres).

Skull breadth (millimetres).	*150—154*	*155—159*	*160—164*	*165—169*	*170—174*	*175—179*	*180—184*	Totals
130—134	—	1	1	—	3	1	—	6
135—139	1	1	4	4	7	2	1	20
140—144	—	1	15	13	25	5	1	60
145—149	1	—	3	12	20	6	2	44
150—154	—	—	4	3	11	2	2	22
155—159	—	—	—	1	4	1	—	6
160—164	—	—	—	—	1	—	—	1
Totals	2	3	27	33	71	17	6	159

TABLE 35.

Bavarians. Brain-weight and Stature. "Total," Short Series. Males.

Stature (centimetres).

Brain-weight (grams).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—189	Totals
1000—1049	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	2
1050—1099	—	—	—	—	—	1	1	—	2	—	—	—	—	—	—	4
1100—1149	—	—	—	—	1	—	—	1	2	—	—	—	—	—	—	4
1150—1199	—	1	—	1	2	3	—	1	1	1	1	—	—	—	—	11
1200—1249	—	—	1	2	3	6	6	11	4	1	2	1	1	—	—	38
1250—1299	—	—	—	1	4	7	6	9	13	5	5	1	—	—	—	51
1300—1349	—	—	—	3	6	10	7	18	11	3	—	4	—	—	—	62
1350—1399	—	—	1	3	5	6	12	13	16	7	8	1	1	1	—	74
1400—1449	1	—	2	1	2	5	4	14	10	5	2	2	1	—	—	49
1450—1499	—	—	—	1	4	—	5	8	5	6	2	2	—	—	—	33
1500—1549	—	—	—	—	2	1	—	5	4	1	6	1	—	—	1	21
1550—1599	—	—	—	—	1	—	—	—	3	1	1	2	—	—	—	8
1600—1649	—	—	—	—	—	—	2	3	1	—	—	—	—	—	—	6
1650—1699	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
Totals	1	1	4	12	31	39	44	83	72	31	28	14	3	1	1	365

TABLE 36.

Bavarians. Brain-weight and Stature. "Total," Short Series. Females.

Stature (centimetres).

Brain-weight (grams).	132—134	135—137	138—140	141—143	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—177	Totals
800— 849	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
850— 899	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
900— 949	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
950— 999	—	—	—	—	—	1	—	1	2	—	—	—	—	—	—	4
1000—1049	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
1050—1099	—	—	—	—	1	3	2	2	4	2	—	—	—	—	1	15
1100—1149	—	—	—	1	3	6	7	6	7	3	2	—	—	—	—	35
1150—1199	—	—	—	1	2	4	9	4	15	4	2	—	—	—	1	42
1200—1249	—	1	1	—	4	5	7	9	10	8	1	1	—	—	—	47
1250—1299	1	—	—	—	6	6	13	5	9	7	3	—	1	—	—	51
1300—1349	—	—	—	—	1	4	1	6	1	3	1	1	1	—	—	19
1350—1399	—	—	—	—	—	1	1	1	4	6	1	—	—	—	—	14
1400—1449	—	—	—	—	—	—	—	1	2	1	—	—	—	—	—	4
1450—1499	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
1500—1549	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	3
1550—1599	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	1	1	2	17	31	41	36	54	37	11	3	2	—	3	241

TABLE 37.

Bavarians. Brain-weight and Age. "Total," Long Series. Males.

Brain-weight (grams).

Age (years).	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
20—24	—	—	—	1	3	3	6	10	5	1	3	—	—	—	32
25—29	—	—	—	—	6	7	8	15	8	5	—	3	2	—	54
30—34	—	1	—	2	7	12	10	18	9	7	2	6	3	1	78
35—39	—	2	1	4	10	14	22	14	16	7	8	3	1	2	104
40—44	—	1	—	—	7	5	4	11	13	8	2	—	1	—	52
45—49	—	—	1	—	5	6	9	10	3	6	3	1	1	—	45
50—54	—	—	—	1	5	10	7	14	9	4	3	2	3	—	58
55—59	—	—	1	1	3	4	8	8	6	1	3	—	—	2	37
60—64	—	—	—	2	6	2	2	6	4	3	1	—	1	—	27
65—69	1	—	—	2	1	3	6	4	1	1	—	—	—	—	19
70—74	—	—	1	2	—	5	2	2	2	1	—	—	—	—	15
75—79	1	—	—	1	1	3	—	1	—	1	—	—	—	—	8
Totals	2	4	4	16	54	74	84	113	76	45	25	15	12	5	529

TABLE 38.

Bavarians. Brain-weight and Age. "Total," Long Series. Females.

Brain-weight (grams).

Age (years).	800— 849	850— 899	900— 949	950— 999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	—	—	—	6	2	10	9	11	7	1	—	—	—	1	47
25—29	—	—	—	—	—	—	6	13	9	14	3	3	1	1	1	—	51
30—34	—	—	—	—	1	1	6	13	15	7	4	3	1	—	—	—	51
35—39	—	—	—	—	—	6	1	4	9	13	7	3	2	—	—	—	45
40—44	—	—	—	—	—	1	2	2	5	4	2	2	1	—	2	—	21
45—49	—	—	—	1	—	—	5	3	7	6	—	1	—	—	—	—	23
50—54	1	—	—	1	—	3	6	4	6	4	1	2	—	—	—	—	28
55—59	—	—	—	—	—	—	1	1	2	5	—	—	1	—	—	—	10
60—64	—	—	—	1	1	1	4	1	3	2	2	2	—	—	—	—	17
65—69	—	—	1	—	2	1	3	3	1	3	—	1	—	—	—	—	15
70—74	1	—	—	—	—	3	1	1	2	—	—	1	—	—	—	—	9
75—79	—	—	—	1	—	—	4	1	—	—	—	—	—	—	—	—	6
Totals	2	—	1	4	4	22	41	56	68	69	26	19	6	1	3	1	323

TABLE 39.

Bavarians. Brain-weight and Age. "Young," Long Series. Males.

Brain-weight (grams).

Age (years).	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	1600—1649	1650—1699	Totals
20—24	—	—	1	3	3	6	10	5	1	3	—	—	—	32
25—29	—	—	—	6	7	8	15	8	5	—	3	2	—	54
30—34	1	—	2	7	12	10	18	9	7	2	6	3	1	78
35—39	2	1	4	10	14	22	14	16	7	8	3	1	2	104
40—44	1	—	—	7	5	4	11	13	8	2	—	1	—	52
45—49	—	1	—	5	6	9	10	3	6	3	1	1	—	45
Totals	4	2	7	38	47	59	78	54	34	18	13	8	3	365

TABLE 40.

Bavarians. Brain-weight and Age. "Young," Long Series. Females.

Brain-weight (grams).

Age (years).	950— 999	1000—1049	1050—1099	1100—1149	1150—1199	1200—1249	1250—1299	1300—1349	1350—1399	1400—1449	1450—1499	1500—1549	1550—1599	Totals
20—24	—	—	6	2	10	9	11	7	1	—	—	—	1	47
25—29	—	—	—	6	13	9	14	3	3	1	1	1	—	51
30—34	—	1	1	6	13	15	7	4	3	1	—	—	—	51
35—39	—	—	6	1	4	9	13	7	3	2	—	—	—	45
40—44	—	—	1	2	2	5	4	2	2	1	—	2	—	21
45—49	1	—	—	5	3	7	6	—	1	—	—	—	—	23
Totals	1	1	14	22	45	54	55	23	13	5	1	3	1	238

TABLE 41.

Bavarians. Brain-weight and Body-weight. "Total," Short Series. Males.

Body-weight (kilograms).

Brain-weight (grams).	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	Totals
1000—1049	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
1050—1099	—	—	—	2	1	—	1	—	—	—	—	—	—	—	4
1100—1149	—	—	—	2	1	1	—	—	—	—	—	—	—	—	4
1150—1199	—	2	—	4	2	1	1	1	—	—	—	—	—	—	11
1200—1249	—	5	9	7	7	5	3	2	—	—	—	—	—	—	38
1250—1299	—	1	4	12	10	8	7	2	2	3	1	1	—	—	51
1300—1349	—	2	8	16	10	13	7	5	—	1	—	—	—	—	62
1350—1399	1	1	8	9	18	10	11	9	2	3	—	1	—	1	74
1400—1449	—	3	7	5	12	5	4	7	4	2	—	—	—	—	49
1450—1499	—	2	2	7	6	5	6	1	4	—	—	—	—	—	33
1500—1549	—	2	2	2	5	1	3	2	1	1	1	—	—	1	21
1550—1599	—	—	—	1	1	2	1	2	1	—	—	—	—	—	8
1600—1649	—	—	1	2	2	—	1	—	—	—	—	—	—	—	6
1650—1699	—	—	1	—	—	—	—	—	—	—	1	—	—	—	2
Totals	1	18	43	69	76	51	45	31	14	10	3	2	—	2	365

TABLE 42.

Bavarians. Brain-weight and Body-weight. "Total," Short Series. Females.

Body-weight (kilograms).

Brain-weight (grams).	15—19	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—90	Totals
800— 849	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
850— 899	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
900— 949	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
950— 999	—	—	1	—	1	—	—	1	—	—	—	1	—	—	—	4
1000—1049	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2
1050—1099	—	—	2	3	5	3	1	—	1	—	—	—	—	—	—	15
1100—1149	—	1	1	8	7	9	3	3	—	2	—	—	—	—	1	35
1150—1199	—	—	8	6	9	5	8	2	3	1	—	—	—	—	—	42
1200—1249	1	—	—	5	10	6	12	5	5	1	1	1	—	—	—	47
1250—1299	—	1	5	4	11	10	10	6	3	1	—	—	—	—	—	51
1300—1349	—	—	—	1	7	3	2	2	1	2	—	1	—	—	—	19
1350—1399	—	—	2	1	2	3	4	—	1	—	1	—	—	—	—	14
1400—1449	—	—	—	—	2	—	1	—	—	—	—	1	—	—	—	4
1450—1499	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
1500—1549	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	3
1550—1599	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	2	22	28	54	40	42	22	14	8	3	4	—	—	1	241

TABLE 43.

Bavarians. Body-weight and Stature. "Total," Short Series. Males.

Stature (centimetres).

Body-weight (kilograms).	144—146	147—149	150—152	153—155	156—158	159—161	162—164	165—167	168—170	171—173	174—176	177—179	180—182	183—185	186—188	Totals
25—29	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
30—34	—	—	—	2	5	4	1	4	2	—	—	—	—	—	—	18
35—39	1	—	3	—	4	7	7	16	3	2	—	—	—	—	—	43
40—44	—	1	—	7	8	9	5	19	12	4	1	2	1	—	—	69
45—49	—	—	1	1	5	5	17	17	15	3	6	6	—	—	—	76
50—54	—	—	—	—	4	9	4	13	10	7	3	1	—	—	—	51
55—59	—	—	—	1	2	4	3	7	16	6	2	1	2	—	1	45
60—64	—	—	—	1	1	—	4	4	8	4	5	3	—	1	—	31
65—69	—	—	—	—	—	—	1	1	3	3	6	—	—	—	—	14
70—74	—	—	—	—	1	1	1	2	3	—	1	1	—	—	—	10
75—79	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
80—84	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
85—89	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
90—94	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
Totals	1	1	4	12	31	39	44	83	72	31	28	14	3	1	1	365

TABLE 44.

Bavarians. Body-weight and Stature. "Total," Short Series. Females.

Body-weight (kilograms).

Stature (centimetres).	15—19	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—90	Totals
132—134	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
135—137	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
138—140	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
141—143	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
144—146	—	1	3	4	5	1	3	—	—	—	—	—	—	—	—	17
147—149	—	1	7	5	6	4	6	1	—	1	—	—	—	—	—	31
150—152	—	—	2	9	11	9	5	3	1	1	—	—	—	—	—	41
153—155	—	—	2	2	12	5	5	3	4	1	1	1	—	—	—	36
156—158	—	—	2	5	12	11	11	8	3	—	—	2	—	—	—	54
159—161	—	—	1	1	5	8	8	7	6	1	—	—	—	—	—	37
162—164	—	—	2	—	2	1	2	—	—	2	1	—	—	—	1	11
165—167	—	—	—	1	—	—	—	—	—	1	—	1	—	—	—	3
168—170	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
171—173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174—177	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	3
Totals	1	2	22	28	54	40	42	22	14	8	3	4	—	—	1	241

TABLE 45.

English. Weight of Cerebrum and Stature. Males.

Stature (inches).

Weight of Cerebrum (ounces).	*58*	*59*	*60*	*61*	*62*	*63*	*64*	*65*	*66*	*67*	*68*	*69*	*70*	*71*	*72*	Totals
28	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
29	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2
30	—	—	—	—	1	—	—	—	—	1	1	1	—	—	—	4
31	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
32	—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	3
33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
34	—	—	—	1	—	1	2	—	1	—	—	—	1	—	2	8
35	—	—	—	—	—	1	—	4	4	—	1	—	1	—	1	12
36	—	—	—	—	—	—	2	—	2	2	2	—	3	—	1	12
37	—	—	—	—	—	1	—	4	1	3	2	2	1	—	—	14
38	1	—	1	—	—	1	2	1	5	6	4	3	3	—	—	27
39	—	—	1	—	1	1	1	2	5	—	2	3	2	—	—	18
40	—	—	—	—	—	—	1	3	4	4	5	2	2	—	1	22
41	—	—	1	1	—	1	3	2	2	4	3	3	5	3	2	30
42	—	—	—	—	—	—	1	4	4	6	6	2	3	1	—	27
43	—	—	—	—	—	1	2	2	4	6	4	2	2	1	4	28
44	—	—	—	—	—	1	1	2	4	2	4	2	5	2	3	26
45	—	—	2	—	—	—	—	—	6	2	2	1	1	2	—	16
46	—	—	1	1	—	—	—	—	2	4	1	3	—	2	3	17
47	—	—	—	—	1	—	3	3	2	2	1	—	1	1	—	14
48	—	—	—	—	—	1	—	2	—	2	5	1	1	—	1	13
49	—	—	—	—	—	—	1	1	1	1	—	1	1	—	—	6
50	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	3
51	—	—	—	—	—	—	—	—	—	1	1	2	—	—	—	4
Totals	2	—	6	3	6	9	20	31	47	47	44	28	34	13	18	308

TABLE 46.

English. Weight of Cerebrum and Age. Males.

Weight of Cerebrum (ounces).

Age (years).	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	Totals
20—29	1	1	1	—	—	—	1	1	1	2	2	2	2	6	1	6	5	—	6	2	6	1	—	2	49
30—39	—	—	1	—	—	—	3	2	3	2	3	3	4	4	3	5	4	5	4	4	2	3	—	—	55
40—49	—	—	2	1	1	—	2	2	—	3	6	6	4	4	3	3	6	3	1	2	3	2	2	2	58
50—59	—	—	—	—	—	—	—	—	4	2	2	4	8	4	6	4	4	2	4	4	1	—	—	—	49
60—69	—	1	—	—	1	—	2	3	2	2	6	1	3	8	6	4	3	5	1	1	—	—	—	—	49
70—79	—	—	—	—	1	—	—	4	2	3	8	2	1	4	8	6	4	1	1	1	1	—	1	—	48
Totals	1	2	4	1	3	—	8	12	12	14	27	18	22	30	27	28	26	16	17	14	13	6	3	4	308

A STUDY OF THE RELATIONS OF THE BRAIN TO THE SIZE OF THE HEAD.

By REGINALD J. GLADSTONE, M.D.

One of the main objects of the investigation which forms the subject of this paper, has been to obtain a series of reconstruction formulæ, by which it will be possible, when in possession of certain chief measurements of the head, to predict within the limits of normal variation, the approximate weight of the brain.

The collection of data for this purpose, which has occupied a period of over three years, has been carried out with the permission of the Resident Medical Officer, in the 'post-mortem' room of the Middlesex Hospital, whereas the mathematical treatment, in the following paper, is chiefly the work of Mr J. Blakeman, M.Sc., of University College, London.

Briefly, the method which we have adopted has been to measure the heads of 'post-mortem' room subjects, and afterwards in each case to take out the brain and weigh it. In each case, therefore, a comparison can be made between the outside measurements of the head, and the weight of the brain.

The individuals measured have been a few presumably healthy subjects, who have died an accidental death; the ordinary subjects of post-mortem examination at the Middlesex Hospital, London; and a fairly large proportion of subjects who have died from malignant disease in the Cancer Department of the same Hospital. They may be regarded, therefore, as forming a fairly typical sample of what has been termed a 'hospital population'; containing, however, owing to the inclusion of 'cancer cases,' a rather larger proportion of individuals who have died from wasting diseases than is ordinary. A comparison, however, of the ratio that the brain-weight bears to the outside measurements of the head in a series of cases in which death took place from accident, or acute illness, with cases of death from wasting diseases, has shown that the diminution of brain-weight in the latter, although measurable, is small; and that their inclusion, therefore, with the acute cases does not seriously affect the general result.

Totals
49
55
58
49
49
48
308

No case has been included in which the brain showed a distinctly pathological condition which would have obviously affected its weight; nor have individuals with foreign or Jewish names been included.

For permission to make use of this valuable material, and for assistance in many ways, I am indebted to Dr R. A. Young, Pathologist, and Lecturer on Morbid Anatomy at the Middlesex Hospital, and to Mr W. T. Hillier, Pathological Assistant in the Cancer Department of the Middlesex Hospital.

In each case the following items have been recorded: (*a*) name, (*b*) age, (*c*) sex, (*d*) cause of death, (*e*) remarks: on general condition, degree of emaciation, &c., (*f*) stature, (*g*) measurements of head:

These were: *L*, the length of the head from the glabella to the occipital point.

B, the maximum transverse diameter of the head above the level of the zygomatic arches.

H, the height of the cranium as indicated by the vertical distance from the biauricular line * to the bregma.

U, the horizontal circumference, taken in a plane passing in front through a point just above the glabella, and behind through the occipital point.

S, the longitudinal or sagittal arc, measured from the glabella, over the vertex to the external occipital protuberance.

Q, the transverse or coronal arc, measured from the tragus of one side, over the vertex to the tragus of the opposite side.

These measurements of the head having been obtained, the scalp was reflected, and (𝔏) the length, (𝔅) the breadth, (𝔎) the height, and (𝔘) the circumference were measured on the bared skull. The vault of the skull was then removed, and the brain taken out and weighed in the usual manner, without removing the pia and arachnoid membranes†, the weight being recorded in grammes‡.

The longitudinal and transverse diameters of the head have been taken with a Flower's craniometer, made by Aston and Mander, of 25 Old Compton Street,

* Taken at the *centre* of the auricular orifices.

† The pia and arachnoid membranes vary considerably in their total weight and an increase in their weight accompanies an increase in age; thus according to Broca, the weight of the 'pia' at different ages in males, averages

20—30	years	...	45 grammes
31—40	,,	...	50 ,,
60	,,	...	60 ,,

The variations ranged between 38 and 130 grammes. The mean weight of the 'pia' in the case of 133 females was 48·7 grammes, and of 273 males 55·8 grammes. The term 'pia' as used by Broca obviously includes both the pia and arachnoid membranes.

‡ The weight of the brain in ounces was in a large number of cases also ascertained, and recorded in the Hospital Reports; and a reference to these reports proved to be a useful means of verifying the recorded weight in grammes, of certain cases in which there was a considerable deviation from the usual relation between the brain-weight, and the outside measurements of the head.

London, and the vertical diameter by an instrument, Fig. 1, similar to the one which I have previously described and figured in the *Report of the Proceedings of the Anatomical Society*, November, 1901; the instrument which we have used for the post-mortem measurements has, however, been improved by the adoption of a suggestion made by Mr J. Gray, namely, the substitution of a vertical screw to act upon the indicator, for the rack and pinion with which the first instrument was fitted. Any slipping of the measuring rod, from the instrument working loose, is thus avoided.

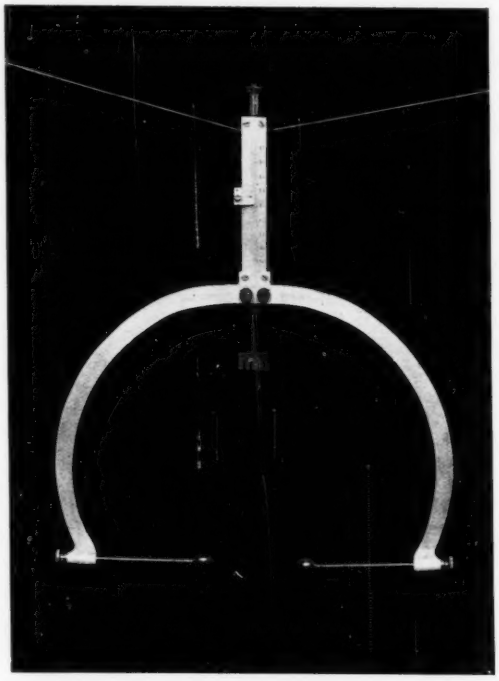


FIG. 1. Instrument for measuring the vertical height of the head, from the biauricular line to the vertex.

The circumference and the longitudinal and transverse arcs have been taken with a Chesterman's steel tape-measure.

Although the longitudinal and transverse arcs were measured, and are preserved in each case, we have not made use of them in constructing any of the tables or formulæ. The diameters being in our opinion the more trustworthy measurements, since they are less influenced by variations in the amount and thickness of the hair, and because the points between which the measurements are

taken are in the case of the diameters much more precise. In many individuals the external occipital protuberance, even when the muscles of the neck are thoroughly relaxed, is difficult to localize. This is the case in the living subject, and still more in the cadaver, so much so that in some subjects it is impossible to determine its exact position. Moreover, when prominent and easily felt, there may be as much as 1 cm. difference between the measurement taken by one individual and that by another, according to whether the top or bottom of the projection has been taken as the starting point. The small tubercle on the tragus from which the measurement is taken in recording the transverse arc is also relatively to the centre of the external auditory meatus, a somewhat variable point. I regard these measurements, therefore, as less reliable than the diameters. The measurement of the arcs and circumference has this important advantage, however—they can

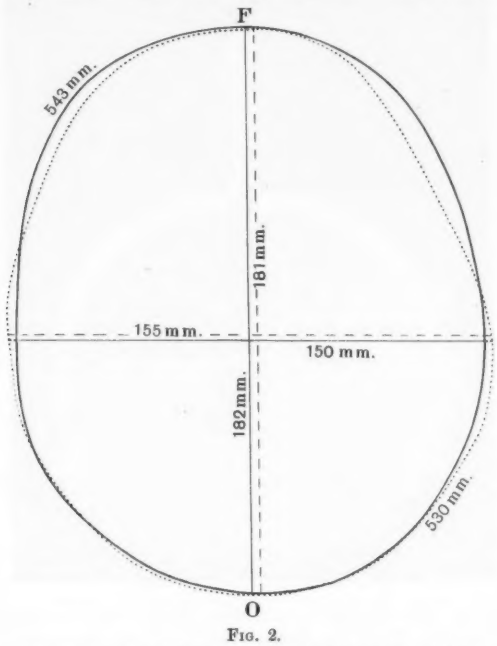


Fig. 2.

be taken by a tape measure, which can be carried in the waistcoat pocket, and they can thus be more readily ascertained by travellers who would be unable to carry a more bulky instrument. Moreover the measurement of the longitudinal and transverse arcs, and of the horizontal circumference, will in some cases give a truer estimate of size than the greatest diameters taken through the principal axes, as will be seen by the accompanying Fig. 2, which shows tracings of the horizontal circumference of two heads, one of which, represented by a continuous line, was narrow in the frontal region, but wide in the parietal, whereas the other, represented

by broken lines, although having a wider frontal region, had a less maximum transverse or interparietal diameter. The total area of the latter is greater than the former, as is indicated by the circumference, which is 543 mm. in the broken tracing as compared with 530 mm. in the continuous. The product of the two principal diameters of the continuous tracing ($L = 181 \times B = 155$) is 28,055, whereas that of the broken curve ($L = 182 \times B = 150$) is 27,300, or 755 less than the continuous. The product of the diameters thus gives in this case an erroneous indication of the area enclosed by the tracings*. We have accordingly made use of the horizontal circumference, although discarding the longitudinal and transverse arcs for the reasons stated above.

The correlation between brain-weight and the outside measurements of the *head* is naturally not nearly so close as that which exists between the outside measurements of the *skull* and the capacity of the *skull*. For in addition to variations in the thickness and shape of the skull, we have to deal with variations in the thickness of the scalp and hair, and also with the very considerable variations in the amount of space which exists between the surface of the brain and the internal surface of the skull. Moreover the weight of the brain is modified by variations in density†, the size of the cerebral ventricles‡, and by the amount of blood contained in its vessels.

The weight of the brain also, as is well known, varies considerably with age, a progressive decrease of weight and most probably also of size taking place after middle age. This decrease, however, appears according to both Boyd's§ and Vierordt's‖ statistics, to commence at a much earlier age, the highest average brain-weights for different ages occurring between 14 and 20 in both sexes.

The brain in children and in youth is both actually and relatively to the body weight and stature very large. According to Vierordt's statistics, the average brain-weight of 35 boys, between 12 and 16 years of age inclusive, was as much as 1423 grammes; and in a table published by the same author, showing the average brain-weight for each year from birth up to the age of 25, the highest average brain-weights, 1490 grm. ♂, and 1345 grm. ♀, occur at the ages of 15 and 14 respectively; whereas, according to Boyd's statistics, the average brain-weights of adult British subjects (mostly of the lower classes), are:

Males between 20 and 40 ..... 1360 grm. or 48 oz.
Females „ 20 and 40 ..... 1230 grm. or 43½ oz.

[* The problem really is which gives the better *average* result. It may be worth noting that the volume of an ellipsoid is proportional to the diametral product, but not to the product of the three circumferences. ED.]

† According to Professor Donaldson's statistics, "the average specific gravity of the entire encephalon should be for the adult male 1·0363, and for the adult female 1·0360." H. H. Donaldson: *Growth of the Brain*, p. 95.

‡ "The cast of the ventricles as made by Welcker displaces 26 cm.³ of water so that the fluid filling such a cavity would weigh a trifle over 26 grammes." H. H. Donaldson: *Growth of the Brain*, p. 87. The specific gravity of the cerebro-spinal fluid is about 1·009.

§ See table compiled from the observations of R. Boyd, *Phil. Trans.* 1860, in Quain's *Anatomy*, Vol. III. Part I. p. 178.

‖ Table 17, p. 104, H. H. Donaldson: *The Growth of the Brain.*

The maximum, average and minimum weights of the brain, in four groups, arranged according to age and sex, of the 'post-mortem' room subjects of the Middlesex Hospital, may be seen in the following table:

The weight of the brain is expressed in grammes, and each group contains 50 subjects.

Age	Males			Females		
	Max.	Aver.	Min.	Max.	Aver.	Min.
20—46 ...	1635	1370·5	1207	1520	1223·5	1027
46 upwards ...	1588	1316·1	1120	1408	1195·7	955

It will be noticed that in passing from the younger to the older groups there is a decrease in the average brain-weight of 54·4 grm. in the male, and of 27·8 grm. in the female, the mean decrease for both male and female being approximately 40 grammes.

This result corresponds very closely with the table compiled from the observations of R. Boyd, and published in Quain's *Anatomy*, Vol. III. Part I. p. 178, in which it is shown that "the brain is absolutely heavier between 14 and 20 years of age than at any other period of life, and that at the age of 80 it has lost about 90 grammes, or rather more than 3 ozs., i.e., $\frac{1}{15}$ of its total weight."

Not only is there a marked change in brain-weight corresponding with different periods of life, but there are also considerable variations in the thickness of the skull and its coverings.

The scalp is thin in infants and young children, and a sensible attenuation of the scalp occurs in old age, which is probably largely due to atrophy of the hair follicles following the loss of hair. The amount, although small, is measurable, and is sufficient to affect the general result of a statistical investigation. The amount of atrophy may be seen by a comparison of the figures in the following tables, which show the average differences between the diameters of the head and the diameters of the skull as measured before and after the scalp has been turned down in order to remove the 'skull cap.'

18 *Males between* 20 *and* 46 *years of age.*

					mm.
Height of head	minus	height of skull	=	4·05.
Length	„	„	length	„	=8·47.
Breadth	„	„	breadth	„	=8·27.

27 *Males from* 46 *years upward.*

					mm.
Height of head	minus	height of skull	=	3·79.
Length	„	„	length	„	=7·25.
Breadth	„	„	breadth	„	=7·40.

17 *Females from* 20—46 *years of age.*

				mm.
Height of head	minus	height of skull		=3·82.
Length	„	length	„	=7·52.
Breadth	„	breadth	„	=7·88.

27 *Females from* 46 *years upward.*

				mm.
Height of head	minus	height of skull		=3·50.
Length	„	length	„	=7·12.
Breadth	„	breadth	„	=6·98.

It will be observed that the scalp is thinner in female subjects than in males and that there is a diminution in the thickness of the scalp, which amounts to:

♂ : 0·26 mm. ♀ : 0·32 mm. at the vertex.
1·22 mm. 0·40 mm. at the glabella and occipital point together.
0·87 mm. 0·90 mm. on the two sides.

There is thus a diminution in the length of the principal diameters of the head, attributable to the atrophy attendant on old age, which amounts if the mean of both sexes be taken to:

0·29 mm. in the vertical diameter.
0·81 mm. „ longitudinal „
0·885 mm. „ transverse „

The thickness of the skull also varies with age, as will be noted on comparing the cut surface of the vault of a skull shown in Plate II. Fig. D, which is a photograph of the vault of the skull of a child about four years old, with that in Fig. C, which is the photograph of the vault of a normal adult skull.

The difference in thickness of the skull of a child, and that of an adult will also be seen on comparing Fig. A, Plate II. with Fig. B on the same plate. Fig. A represents the right half of the skull of a child about five or six years of age; Fig. B the right half of a thick, adult skull. Note also the absence of frontal sinuses in Figs. A and C.

The extent to which the thickness of the skull may be affected by disease, may be seen in Fig. 3, which represents part of the vault of a skull, from a case of 'osteitis deformans*,' the walls of which averaged about one inch in thickness. The increase in thickness of the skull in these cases takes place partly on the outer and partly on the inner surface of the skull, so that although the size of the head is enlarged the capacity of the skull is diminished, and compression exerted on its contents.

The reverse condition is met with in cases of chronic hydrocephalus in which the size of the head is increased by an augmentation in the volume of its contents.

* Specimen 1239. Museum of the Royal College of Surgeons, England.

An extreme case of this disease is seen in Figs. E and F in Plate III., which are photographs of specimen 3878 in the Hunterian Museum of the Royal College of Surgeons, England. It is described in the catalogue as "The skeleton of a man who died with hydrocephalus at the age of 25 years. Its greatest horizontal circumference is 914 cms.! Besides the enlargement of the frontal and parietal bones by which the greater part of the enlarged cranial cavity is formed rows of Wormian bones from 2·5 to 3·8 cms. in breadth are placed in the whole course of the lambdoid and sagittal sutures, and in great part of the length of the squamous sutures."

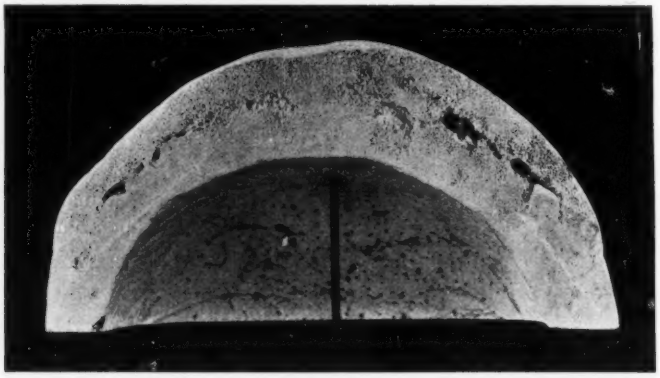


Fig. 3.

The enlargement of the head in hydrocephalus is associated with an increase in the amount of the cerebro-spinal fluid, which is found either in the space between the arachnoid and pia mater, outside the brain, within the cerebral ventricles, or in both these situations. In the case of James Cardinal, the skeleton of whom is preserved in the Museum of Guy's Hospital, seven pints of cerebro-spinal fluid were found between the membranes, whereas the ventricles contained one pint. In this case "it appeared that the fluid had been originally contained within the ventricles, but had burst through an opening in the corpus callosum *."

It is obvious that both in the case of *osteitis deformans*, and *hydrocephalus* the size of the brain cannot be gauged by the size of the head.

The enlargement of the head in hydrocephalus is also of interest, as it indicates one means by which the size of the cranium is increased in the course of normal growth, namely by an expanding force acting from within. In the early stages of development the fœtal brain has a smooth even surface, at a later stage from the 8th week of intrauterine life to the 4th month, what are called the 'transitory fissures' make their appearance; these are infoldings of the thin walls of the

* See description of case in *Tumours, Innocent and Malignant*, p. 448, J. Bland Sutton.

hemisphere vesicles, which appear to be occasioned by the growth of the brain being more rapid than the capsule which confines it. At a still later stage following a more rapid enlargement of the skull many of the early fissures disappear, but they are afterwards replaced by the permanent fissures or sulci of the adult brain. The appearance and disappearance of these fissures seem to indicate that there are two counteracting forces concerned in the growth of the brain, namely an expanding force due to increase in brain matter, and cerebro-spinal fluid, and a passive restraining force exerted by the skull.

In anencephalic monsters the brain is imperfectly developed; the hemisphere vesicles appear at an early stage of development to have ruptured and collapsed, and the cerebro-spinal fluid to have escaped through a triangular aperture, which may often be recognized at the time of birth in the position of the anterior fontanelle. The cerebro-spinal fluid thus passes into the cavity of the amnion, and the amount of fluid within this sac becomes thereby much increased.

In these cases the expanding force concerned in the growth of the skull is absent, and consequently the bones forming the vault of the skull, if they are developed at all, grow only to a sufficient size to cover over the base of the skull, and rudiments of the brain, whereas the bones of the face, and of the skeleton generally, grow to their normal size.

In rickets on the other hand the bones of the skull are abnormally soft and yielding, and apparently do not afford the usual resistance to the expanding pressure from within, and as a result the cranial portion of the skull enlarges, and the well-known square overhanging forehead of a typical rickety child is produced, while the face remains of the normal size and owing to the enlargement of the head above, appears unusually small.

In the course of normal growth the two forces continue to counteract one another until the brain has attained its maximum size. Later a progressive diminution in the weight and probably also of the size of the brain takes place, and the sutures of the skull afterwards, gradually consolidate.

Not only is there the difference mentioned above in the general thickness of the skull at different ages, but there is also a considerable difference in the frontal region, due to the development after puberty of the frontal air sinuses, which tend to increase in size with the advance of age, and which are usually considerably larger in the male than in the female. Their position and the size which they may attain are well seen in Fig. 4, D, and Plate II., Figs. B and C.

It is obvious, therefore, that in any attempt to calculate the weight of the brain age must be taken into account, since it affects not only the brain itself, but also the thickness of the scalp and of the skull. It may be noted, however, that after middle age the variations attributable to age are so far as the thickness of the scalp and the weight of the brain are concerned, in the same direction, namely diminution, so that the proportion between the outside measurements of the head and the brain-weight is not very materially altered. The diminution in brain-

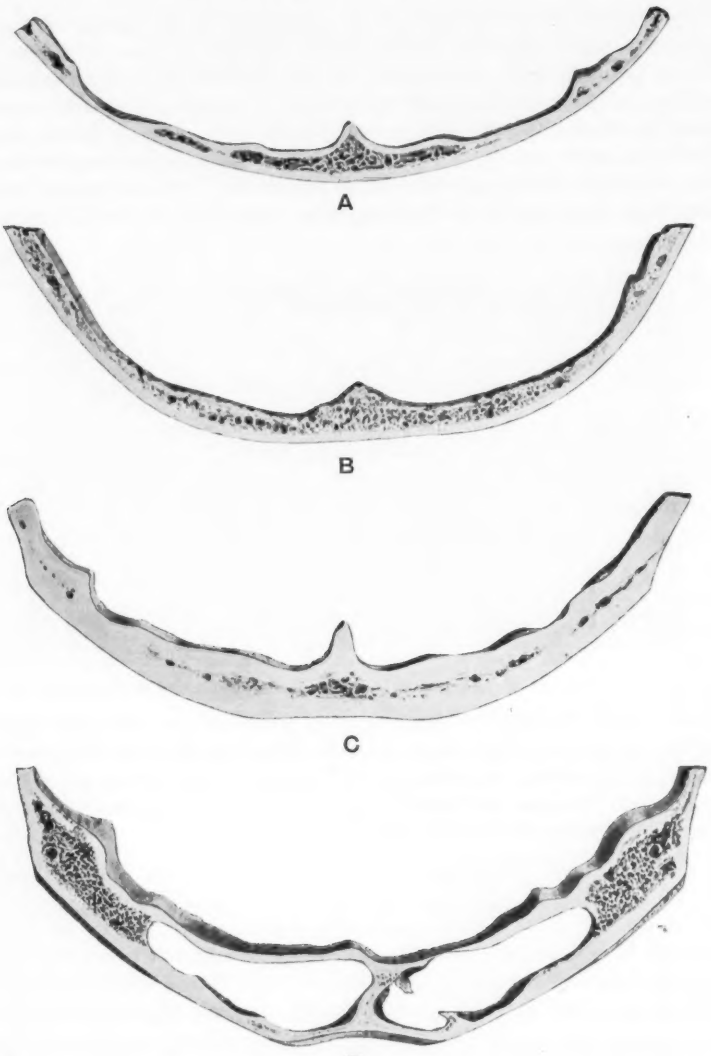


FIG. 4. Four horizontal sections through the frontal bone, showing the condition of the bone at various ages.

A. Male, aged 9. Bone thin, no frontal sinuses.
B. ,, ,, 14. Bone considerably thicker, no frontal sinuses.
C. ,, ,, 32. Illustrating the occasional absence of the frontal sinuses in an adult.
D. ,, ,, 54. Unusually large frontal sinuses.

From specimens in the Museum of the Royal College of Surgeons, London.

weight follows very closely the diminution in the size of the head, as is shown by the following table, which gives the proportion between these two quantities in 4 groups of 50, arranged according to age and sex. The size of the head is here indicated by a figure obtained from the product of the three principal diameters ($L \times B \times H$) expressed in millimetres, the first four figures of this number represent the number of cubic centimetres which would be contained in a rectangular block having the same diameters as the cranial portion of the head, and I shall refer to it subsequently as the 'index of size' of the head, thus:

$$(196) \times (156) \times (138) = 4219488.$$

4219, the 'index of size' of the head, represents cubic centimetres and may therefore be conveniently compared with the weight of the brain expressed in grammes.

Males	*H*	*L*	*B*	Index of size	Brain-weight	Index of Size / Brain-weight	Number in each group
Age, *20—46* ...	134·8	190·8	149·5	3876	1370·5	2·806	50
„ *46* and upwards	132·4	189·8	148·7	3736	1316·1	2·838	50
Females							
Age, *20—46* ...	129·8	183·4	144·9	3449	1223·5	2·818	50
„ *46* and upwards	128·2	182·1	143·5	3350	1195·7	2·801	50

By combining the male with the female groups, we obtain the following ratios between the size of the head and the brain-weight:

Age, 20–46 . . .	2·812
„ 46 and upwards .	2·819

In comparing the extreme ends of the series, however, it appears that the diminution in brain-weight, with the advance of age, is more rapid than the diminution in the size of the head; thus taking the mean of 4 ♂ and 3 ♀ cases over 70 years of age, the ratio between size of head and brain-weight is 2·821, whereas an equal number of cases between 20 and 25 years of age give 2·702.

The influence of stature upon the ratio between the outside measurements of the head and the brain-weight is apparently very small; but it has a marked influence on the actual size of the head as compared with the general mean, and a less but measurable influence on the proportion of the size of the head to the body in the individual. In other words, tall men in the aggregate have larger heads than short men, but proportionally to the size of their bodies their heads are considerably smaller than those of short men.

The formula which I have employed to express this relationship of the size of the head to the stature is the following:

$$100 \times \frac{\sqrt[3]{\text{Index of size}}}{\text{Stature in centimetres}} = \text{Capitulo-statural index.}$$

$$\text{Ex.} \quad \frac{\sqrt[3]{3921 \text{ cm.}^3} \times 100}{170 \text{ cm.}} = \frac{15{\cdot}77 \text{ cm.} \times 100}{170 \text{ cm.}} = \frac{1577}{170} = 9{\cdot}27.$$

It will be seen on referring to the table given below that a considerable diminution of the capitulo-statural index takes place with increase in stature; and also that the size of the head increases with the stature, but that the enlargement of the head does not keep pace with the increase in stature.

Table showing the proportion that the size of the head bears to the stature in groups of individuals arranged according to their height, the degrees of which are expressed in intervals of 3 inches.

Number of cases in each group	Stature		Index of size ($L \times B \times H$) cm.^{3}	$\sqrt[3]{\text{Index of size}}$ cm.	Capitulo-statural index $\frac{\sqrt[3]{\text{Index of size}} \times 100}{\text{Stature}}$
	Inches	cm.			
37	74—72	185·3	4135	16·05	8·66
120	71—69	177·7	4071	15·97	8·98
124	68—66	170·0	3993	15·87	9·00
67	65—63	162·4	3902	15·75	9·69
15	62—60	154·8	3778	15·56	10·05

This table is based upon the measurements of 363 male subjects of all classes in England above 20 years of age, the larger number of these belong to the professional class, so that both the mean stature and mean size of the head are slightly above the general average.

The increase in size of the head, accompanying increase in stature, is shown also in the following chart, which is planned from the same figures as the preceding table:

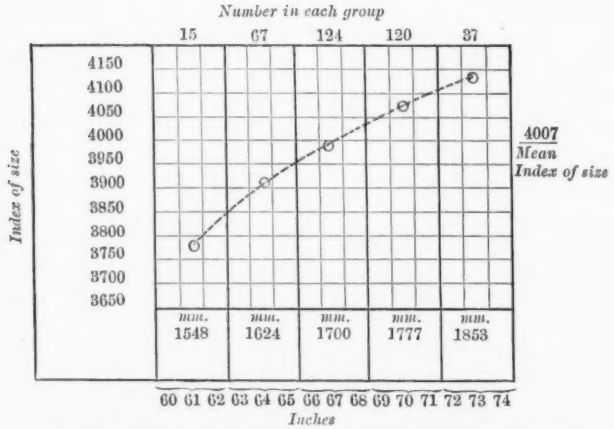
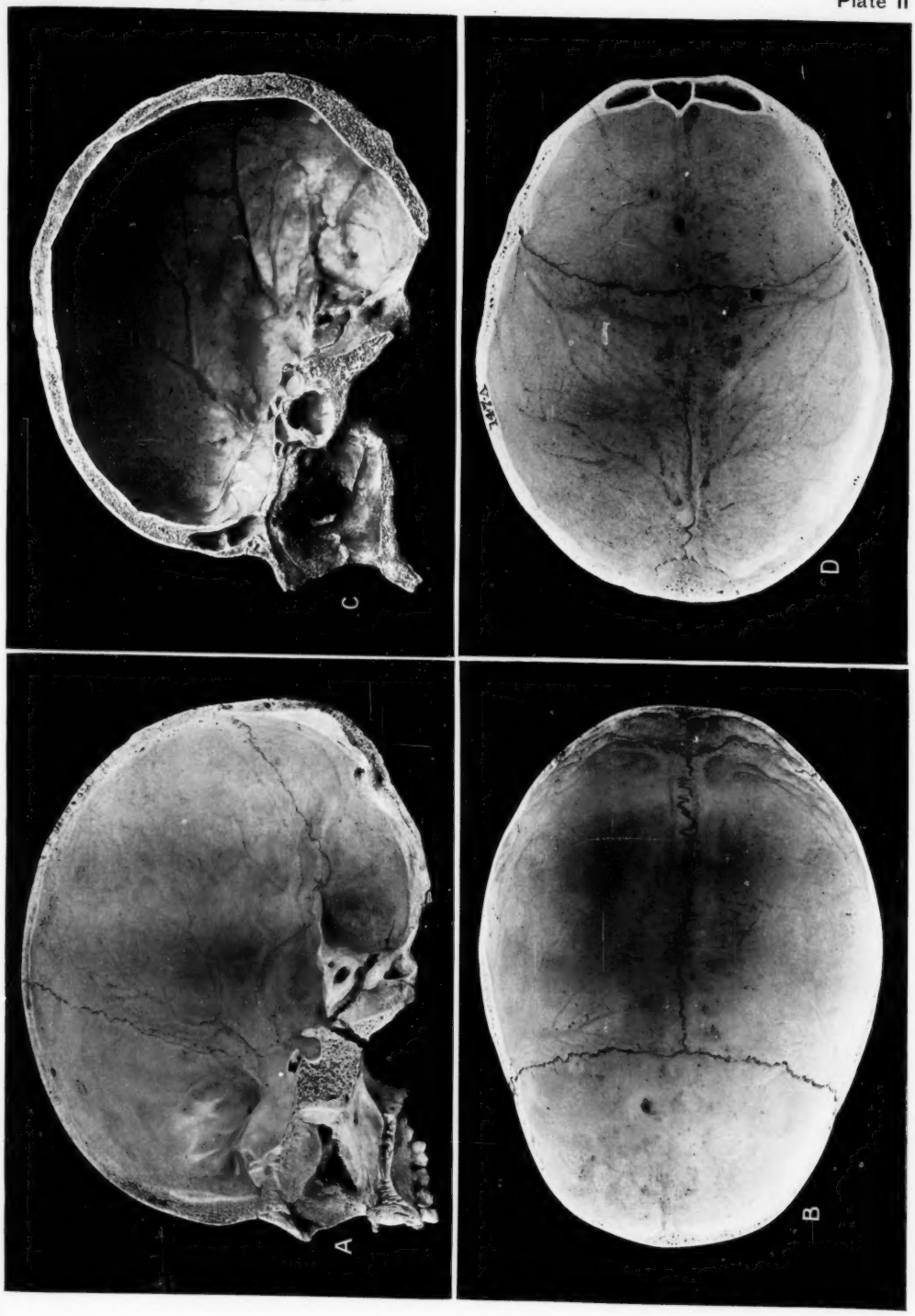
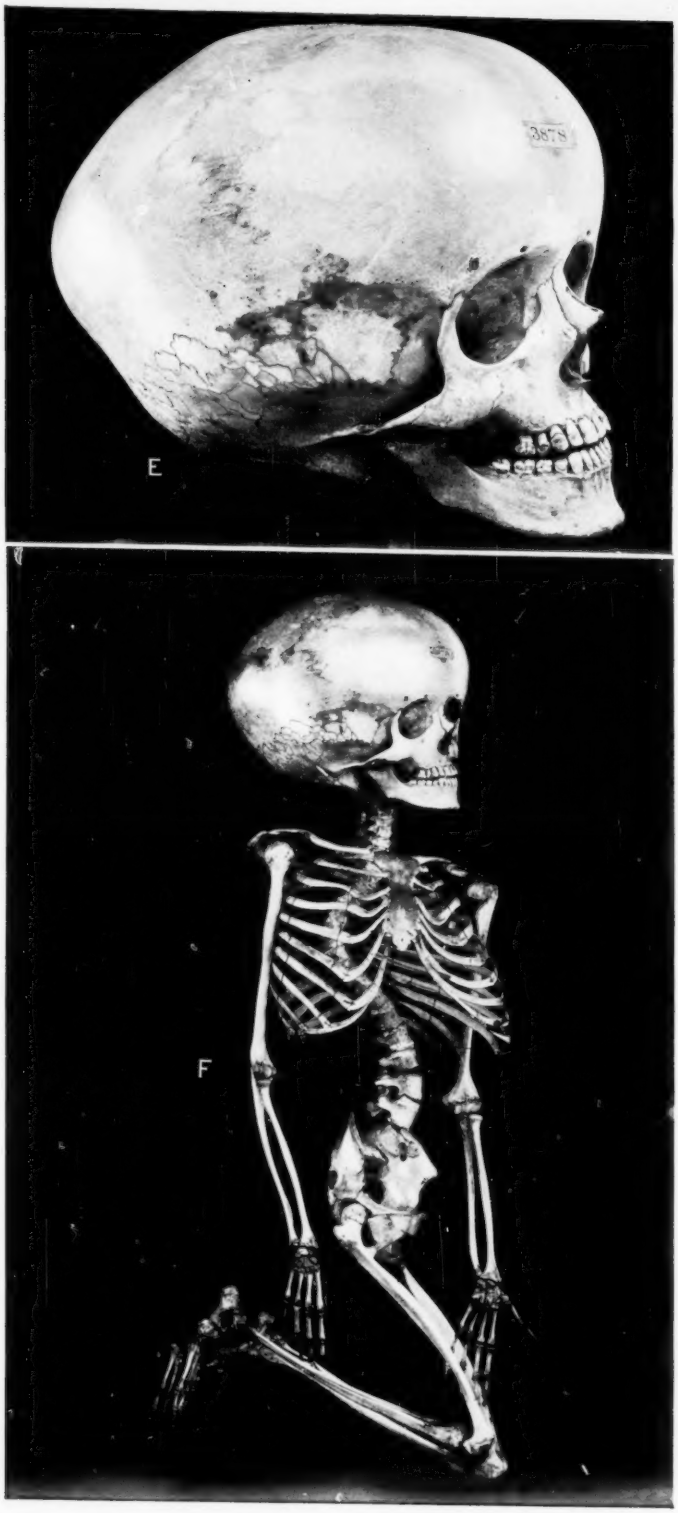


FIG. 5. Chart showing the increase in the size of the head which takes place with increase of stature. The chart is based on the measurements of 363 adult male subjects, above 20 years of age. The individuals have been grouped according to their stature, which is expressed in intervals of 3 inches. The figures indicating the size of the head are shown in the column on the left.


C
D
A
B


E
F

The following table is also of interest, as exemplifying the same law in certain extreme cases, in giants and dwarfs*.

	Stature			Index of size	$\sqrt[3]{\text{Index of size}}$	Capitulo-statural index $\frac{\sqrt[3]{\text{Index of size}} \times 100}{\text{Stature}}$
	ft.	in.	cm.			
Irish giant, O'Brian ...	7	7	231·1	4557	16·58	7·17
American giant, Freeman	6	9	205·7	4085	15·99	7·77
363 All classes, British, ♂	5	7	170·0	4007	15·88	9·33
Dwarf from Kiel, ♂ ...	4	0	121·9	2788	14·07	11·55
Dwarf from Holstein, ♂ ...	3	2½	97·8	2699	13·92	14·23

In conclusion I must express my thanks to Mr Freke Field, who has worked out most of the tables in this section of the paper, and to Mr J. W. Stockwell and Mr G. Fairclough, who have taken the majority of the measurements.

DESCRIPTION OF PLATES.

PLATE II. FIG. A. Longitudinal section through skull of a child between five and six years of age.

FIG. B. Longitudinal section through a normal adult skull. From the Anatomical Department, Middlesex Hospital.

FIG. C. Interior of the vault of the skull of an adult male. Note the thickness of the sawn edge of the bone, and the size of the frontal sinuses. From a specimen 417 A, in the Museum of the Royal College of Surgeons, London.

FIG. D. Interior of the vault of the skull of a child, aged about four. Note the thinness of the sawn edge of the bone, and the absence of the frontal sinuses. From a specimen in the Museum of the Royal College of Surgeons, London.

PLATE III. E skull, F skeleton of man of 25 years with hydrocephalus. From specimen 3878 in the Hunterian Museum of Royal College of Surgeons.

* For permission to measure the skeletons of the two giants, which are contained in the Hunterian Museum of the Royal College of Surgeons, England, I am indebted to Professor C. Stewart. In calculating the 'indices of size' of the heads, 4·5 mm. were added to the vertical diameter, and 9 mm. to the longitudinal and transverse diameters, as taken on the bare skull, thus allowing about 0·5 mm. on each diameter for shrinkage of the skull in drying. See table, p. 110, 'Males 20—46 years of age.'

APPENDIX OF MEASUREMENTS.

A. *Male Subjects between* 20 *and* 46. *Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cm.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
1	—	—	67	141	200	160	4512	1530	574	80·0	Chronic nephritis. Cardiac degeneration
2	*A*	39	68	137	182·5	149·5	3738	1297	550	81·9	Accident. Fractures of bones
3	*C*	35	71	139	201	152·5	4261	1335	569	75·9	Pulmonary tuberculosis
4	*C*	35	70	134·5	193	145·5	3777	1282	542	75·4	Malignant growth affecting mouth
5	*A*	37	68	140	195	153	4177	1590	576	78·5	Accident. Fracture of skull
6	—	34	65	132	186	146	3585	1300	547	78·5	Cerebral hæmorrhage
7	*C*	38	68	137·5	192·5	143	3785	1400	549	74·2	Pulmonary tuberculosis. Skull thin
8	*C*	45	69·5	134·5	189	140	3559	1255	535	74·1	Carcinoma of stomach
9	—	45	68	132·5	185·5	147	3613	1355	555	79·2	Pneumonia. Pleural effusion
10	—	40	64	135	196	150·5	3982	1375	565	76·7	Cerebral hæmorrhage
11	*C*	21	66	132	185	141	3443	1340	537	76·2	Meningitis. Mastoid disease
12	—	42	63	136·5	197	148·5	3993	1380	570	75·4	Cellulitis of upper lip. Septicæmia
13	—	25	69·75	136	188·5	142	3640	1355	558	75·3	Basal meningitis. Pulmonary tuberculosis
14	*A*	23	66·5	138	198	154	4208	1522	576	77·8	Fracture of skull. "Large pear-shaped head"
15	—	45	71	132	191	152	3832	1208	561	79·6	
16	*A*	41	60·5	135	194	148	3876	1405	569	76·3	Fracture of skull [hæmorrhage
17	*A*	28	63·5	130	183	147	3497	1358	551	80·3	"Run over." Rupture of liver. Cerebral
18	*C*	45	65	130	184·5	144·5	3466	1292	541	78·3	Malignant growth. R. ext. auditory meatus
19	*A*	20	71·5	126	178	138	3095	1340	517	77·5	Burns. Congestion of brain
20	*C*	43	68	144	192	160	4424	1400	580	83·3	Fatty liver and kidneys. Delirium tremens
21	*C*	43	68·5	137	190	149	3878	1357	568	78·4	Tubercular nephritis. Thin sku.
22	—	25	68·5	130	197	158	4046	1287	582	80·2	Fatty heart. Congestion of lungs
23	*C*	29	68·5	137·5	189·5	146	3804	1275	554	77·0	Morbus cordis
24	*C*	40	68·5	135	182	151	3710	1270	541	83·0	Carcinoma of œsophagus, and stomach
25	*A*	?	71	142	199	168	4747	1635	602	84·4	Impaction of food in pharynx
26	*A*	40	72·5	135	195	168	4423	1505	577	86·2	Inguinal hernia. Pulmonary embolism
27	—	29	69·25	135	198	151	4036	1490	564	76·3	Appendicitis. Thick skull. Foreign
28	—	35	70·5	138	193	151	4022	1485	570	78·2	
29	—	27	69·5	135	184	139	3454	1310	537	75·5	Appendicitis. General peritonitis
30	*A*	20	69	142	200	147	4175	1420	574	73·5	Purulent meningitis. Fracture of nasal bones
31	*C*	38	68·5	136	182	153	3787	1318	567	84·1	Malignant growth of kidney
32	—	24	69·75	135	190	148	3796	1432	564	77·9	Lymphatic leukæmia. Punctiform hæm. c'blm.
33	—	41	70·5	138	187	159	4103	1364	574	85·0	Lobar pneumonia. Enlgemt. of liver. Foreign
34	*C*	45	71	137	191	159	4161	1405	584	83·2	Chronic laryngitis. Thin skull. Scalp very thick
35	*C*	41	67	139	195·5	153	4158	1432	574	78·3	Malignant growth of lip. Metastasis
36	—	37	61	134	191	149	3814	1207	548	78·0	Fracture of tibia. Delirium tremens
37	—	29	65	135	184	142	3527	1375	530	77·2	Cerebral tumour
38	*A*	36	61·5	131	187	153	3748	1350	560	81·8	"Run over." Fractures of ribs and sternum
39	*C*	21	61	129	182	142	3334	1236	528	78·0	Pulmonary tuberculosis
40	*C*	38	62·5	129	192	141	3492	1250	540	73·4	Malignant growth in floor of mouth
41	*C*	42	61·5	134	194·5	152	3962	1350	558	78·1	Ulceration of larynx
42	*C*	43	62·5	129	190	143	3505	1320	539	75·3	Carcinoma of omentum. Emaciation
43	*C*	37	66	138·5	201	155	4315	1525	587	77·1	Cerebral abscess following empyema
44	*A*	31	67·5	130	190	154	3804	1570	550	81·1	Pulmonary embolism. Inguinal hernia
45	*C*	34	69	131·5	192	153	3863	1340	563	79·7	Diabetes
46	*C*	44	69	134	193	156	4034	1422	578	80·8	Cirrhosis of liver. Morbus cordis
47	*C*	24	69·5	142	192	158	4308	1506	569	82·3	Chronic nephritis. Œdema of lung. Foreign
48	*C*	28	68·5	124	181	141	3165	1215	510	77·9	Infective endocarditis. Skull very thin
49	*C*	44	67	134	190	143	3641	1311	545	75·3	Renal calculus
50	*A*	43	66·5	132·5	191	144	3644	1300	555	75·4	Pneumonia. Well nourished
51	*C*	44	68·5	130·5	193	154·5	3891	1224	570	80·1	Cirrhosis of liver. Well nourished
52	*A*	38	68·5	132	193·5	148·5	3793	1350	560	76·7	Fractures of sternum and ribs
53	*C*	35	70	136	195	161	4270	1335	558	82·6	Aortic regurgitation. Well nourished
54	*A*	39	69	132	190	162	4063	1390	557	85·3	Meningitis. Bullet in brain 14 years
55	*A*	36	69	135	193	154	4012	1400	569	79·8	Fall down lift. Well nourished
56	*C*	43	66	125	182	152	3458	1225	538	83·5	Fibroid degeneration of heart
57	*C*	42	60·5	134	191	152	3890	1310	549	79·6	Bronchitis and emphysema. Thick scalp

B. *Male Subjects from 46 and Upwards. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cm.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
58	—	50	68	140	192	155	4166	1560	560	80·7	Fatty degeneration of heart
59	*C*	47	69	139	190	149	3935	1330	544	78·4	Calculous nephritis. Pericarditis. Pleurisy
60	—	47	67	132	193	144	3669	1222	556	74·6	Pneumonia
61	*C*	49	64·5	133	190	153	3866	1415	553	80·5	Epithelioma of lower lip [ribs
62	*A*	50	69	130	180	145	3393	1175	530	80·6	Injuries to head. Fracture of clavicle and
63	*C*	48	69·5	139	201	159	4442	1330	585	79·1	Tubercular nephritis. Pulmonary tubercu-
64	—	47	69	136·5	201	155	4253	1485	585	77·1	losis. Foreign
65	*A*	50	72·25	129·5	195	148	3737	1470	568	75·9	Strangulated umbilical hernia
66	*A*	48	64·5	124·5	182·5	146·5	3329	1135	541	80·3	Cerebral hæmorrhage. L. hemisphere
67	—	47	67	132	183·5	141	3415	1310	548	76·8	
68	—	50	68	122	188	147	3372	1154	550	78·2	Fracture of base of skull
69	*C*	48	68	141	204	154	4430	1510	583	75·5	Carcinoma of stomach
70	—	49	68·5	139	197	160	4381	1415	589	81·2	Pleurisy, Pericarditis. Skull thick
71	*C*	48	69·5	138·5	191	151·5	4008	1468	564	79·3	Malignant growth of pharynx
72	—	49	69	135	188	152	3858	1390	563	80·9	Cerebral hæmorrhage into L. ventricle
73	—	48	66	137	188	160	4121	1380	576	85·1	Foreign
74	*A*	50	69·5	131	196	158	4057	1432	575	80·6	Subdural, and intrapontine hæmorrhage
75	*C*	69	69·5	136	190	148	3824	1240	560	77·9	Fatty degeneration of heart and kidneys
76	—	?	63	126	184·5	146	3394	1195	540	79·1	Cardiac failure
77	*A*	58	67	131	183·5	148	3558	1225	540	80·7	Tetanus
78	—	57	69	130·5	184	140	3362	1188	544	76·1	Lymphosarcoma of neck
79	*C*	67	68·5	137	187·5	153	3930	1252	569	81·6	Cerebral hæmorrhage. Fibroid kidneys
80	*A*	61	66	136	188	150	3835	1315	552	79·8	Fracture of spine, R. femur, and ribs
81	—	53	69	138	185	150	3830	1245	545	81·1	
82	*A*	60	68·5	139	190	146	3856	1430	564	76·8	Compound fracture of R. tibia and fibula
83	*C*	58	70	128	180	141	3249	1279	532	78·3	Malignant disease of R. lung
84	*C*	55	67·5	131	187	146	3577	1245	572	78·1	Carcinoma of soft palate
85	*A*	57	67	132	196	152	3933	1309	569	77·6	Fracture of skull
86	—	75	67·5	133	193	150	3850	1412	553	77·7	Burns. Receding forehead
87	—	63	61	125·5	187	141	3309	1120	534	75·4	Cerebral hæmorrhage. Heavy drinker
88	—	69	67·25	128·5	188	141	3406	1220	540	75	Cerebral hæmorrhage
89	*A*	?	65	130	186	145	3506	1280	538	78	Fracture of base of skull
90	*C*	52	63·25	132	196	151	3907	1440	552	77	Carcinoma of stomach. Skull thick
91	*C*	66	71·5	142	194	151	4160	1370	570	77·8	Carcinoma of œsophagus
92	*C*	67	69·5	128	180	144	3318	1192	531	80·0	Malignant growths in liver. Skull thick
93	—	67	67	138	183	145	3662	1230	524	79·2	Carcinoma of stomach
94	*A*	53	63	137	191	149	3899	1346	568	78·0	Compound fracture of tibia and fibula
95	*C*	55	63	132	196	143	3700	1290	551	73·0	Epithelioma of R. cheek
96	*A*	56	65	133	192	148	3779	1165	558	77·1	Fracture of base of skull
97	*C*	54	65	130	185·5	144	3473	1240	530	77·6	Epithelioma of tongue. Skull very thin
98	*C*	59	63	136	182	141	3490	1132	528	77·5	Coma. Alcoholic dementia. Skull thick
99	*A*	65	65·5	128	183	156	3654	1242	544	85·2	Fracture of skull and ribs. Skull and scalpthin
100	*C*	59	64	128	190	143	3478	1270	536	75·3	Carcinoma of stomach. Skull and scalp thin
101	*C*	60	65·5	128	184·5	148	3495	1218	525	80·2	Epithelioma of lips [drinker"
102	*A*	60	69	130·5	192	153	3834	1430	579	79·7	Fractured ribs. Syncope. "Heavy malt
103	*C*	60	68	135	194	148	3876	1588	555	76·3	Carcinoma of stomach. Skull and scalp thin
104	*C*	62	65	130·5	187	150	3661	1320	548	80·2	Chronic nephritis. Alcoholism. Skull thin
105	*A*	70	67	128	191	148	3618	1290	565	77·5	Rupture of heart. "Dropped dead in street"
106	*A*	59	67·5	126·5	191	151	3648	1260	558	79·0	Fracture of arm and ribs. Pneumonia
107	*A*	51	68	140·5	200	143·5	4032	1425	565	71·7	Acute colitis. Peritonitis
108	*C*	57	66	128·5	185	143	3399	1226	542	77·3	Malignant disease of liver. Emaciated
109	*C*	51	68·5	133·5	193	152	3916	1360	556	78·8	Malignant disease of glands in neck
110	*A*	65	70·5	139	203	157	4430	1620	585	77·3	"Brought in dead." Fatty heart. Syncope
111	*C*	56	66	130	189·5	150	3695	1310	549	79·2	Chronic interstitial nephritis. Cerebral hæm.
112	*C*	75	72·5	125·5	193	145·5	3524	1250	555	75·3	Chronic interstitial nephritis. Fatty heart
113	*C*	47	69·5	124	192	150	3571	1295	561	78·1	Carcinoma of tongue
114	*C*	57	67·25	129	187	149	3594	1290	555	79·7	Carcinoma of tongue. Emaciated

B. *Male Subjects from 46 and Upwards. Middlesex Hospital. (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cm.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
115	*C*	62	69·75	123	184	149·5	3383	1290	536	81·2	Carcinoma of liver
116	*C*	84	50·5	127	190	145	3499	1275	545	76·3	Epithelioma of lip. Scalp thin
117	*C*	48	61·5	128·5	190	147	3589	1250	549	77·4	Cardiac failure. Pulmonary tuberculosis
118	*C*	58	68·5	134	189	154	3900	1270	557	81·5	Carcinoma of tongue. Emaciated [& spleen
119	*A*	63	67·5	136	199	152	4114	1362	588	76·4	Fracture of skull and ribs. Rupture of liver
120	*C*	52	67	133	191	155	3937	1300	558	81·2	Carcinoma of rectum. Well nourished
121	*A*	57	66	126	183	147	3399	1173	544	80·3	Meningeal hæmorrhage. Cerebral compression
122	*A*	52	69	132	192	164	4200	1256	563	85·4	Fracture of scapula and ribs. Foreign. 18 stone
123	—	58	72	134	212	158	4488	1440	585	74·5	Cerebral hæmorrhage. Skull and scalp thick
124	*C*	63	65	129	188	149	3614	1180	551	79·3	Papilloma of stomach. Emaciated. Foreign
125	*A*	57	67·5	133	194	157	4051	1306	573	80·9	Cerebral hæmorrhage. Well nourished
126	*A*	46	69	134	196	144	3782	1350	556	73·5	Pneumonia. Fairly well nourished
127	*C*	61	64	128	184	144	3391	1125	531	78·3	Malignant ulceration of face. Emaciated
128	—	58	66	126	171	145	3124	1165	534	84·8	Foreign
129	—	55	65	141·5	186	154	4053	1312	558	82·8	Apoplexy. Skull thin. Scalp thick
130	*C*	50	66	125·5	189	151	3582	1300	547	79·9	Chronic interstitial nephritis. Scalp thin
131	*C*	52	66	129	196	145	3666	1270	556	74·0	Carcinoma of scrotum. Wasted
132	—	51	67	128	189	146	3532	1335	553	77·2	Tumour of stomach. Well nourished
133	—	63	69	131	198	156	4046	1450	583	78·8	Fracture of skull. Cerebral hæmorrhage
134	—	72	66·5	132	184	151	3667	1310	554	82·1	Fracture of skull. Subdural hæmorrhage

C. *Female Subjects from 20 to 46 Years of Age. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cms.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
135	*C*	36	—	114	179	140	2857	1027	570	78·2	Cirrhosis of liver. Excess of subarachnoid
136	—	20	63	133·5	184·5	139·5	3436	1235	523	75·6	Cerebro-spinal meningitis [fluid
137	—	35	63·5	132·5	187	153	3791	1260	558	81·8	Cellulitis and œdema of pharynx. Thick
138	—	40	60	126	182	144	3302	1165	532	79·1	Carcinoma uteri. Thin skull [skull
139	—	30?	51	126	178·5	138	3104	1080	498·5	77·3	Uræmia
140	—	33?	58	122·5	181	143	3171	1127	531	79·0	Valvular disease of heart. Emboli in brain
141	—	?	64	137	175	149	3572	1270	532	85·1	Pneumonia. Pericarditis
142	—	42	62	132	187	143	3530	1252	552	76·5	Fatty heart
143	—	35	66	126	180	140	3175	1200	521	77·8	Rheumatism. Bronchitis
144	—	31	66	131	181	145	3438	1290	539	80·1	Raynaud's disease
145	*C*	35	64	128	193	158	3903	1334	554	81·9	Carcinoma. Cervix uteri
146	*A*	23	69	142	176	156	3899	1380	555	88·6	Gun-shot wound in L. breast
147	*A*	38	65·5	129	185	142·5	3401	1140	542	77·0	Burns of neck and chest
148	—	23	67·5	128	181	141	3267	1243	540	77·9	Septicæmia
149	—	32	70	133	184	141	3451	1340	532	76·6	Pneumonia. Bronchitis
150	*C*	34	58	126	179	137	3090	1168	505	76·5	Carcinoma mammæ. Skull thin
151	*C*	37	60	132	186	139	3413	1322	532	74·7	Carcinoma mammæ. Skull and scalp thin
152	—	39	64	129·5	182	141	3323	1249	521	77·5	Broncho-pneumonia
153	—	34	63	134·5	190	144	3680	1321	552	75·8	Pneumonia. Skull very thick
154	—	41	66·5	127·5	183·5	147	3439	1192	533	80·1	Morbus cordis. Skull very thick, scalp thin
155	*C*	43	66·5	136	187	151·5	3853	1373	565	81·0	Carcinoma mammæ. Skull and scalp thick
156	*C*	33	59	128	180	137	3156	1170	530	76·1	Carcinoma of stomach. Skull thick behind
157	*C*	28	64	129	179	142	3279	1265	528	79·3	Intestinal ulceration probably syphilitic
158	*C*	33	67	132	186	151	3707	1235	560	81·2	Sarcoma of frontal bone
159	*A*	34	66	138	191	152	4006	1302	546	79·6	Pulmonary embolism. Fibromyoma of uterus
160	*C*	38	65·5	131	177	141	3269	1241	523	79·7	Carcinoma uteri. Scalp thin. Foreign

C. *Female Subjects from 20 to 46 Years of Age. Middlesex Hospital. (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cm.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
161	*C*	45	68	125	178	138	3071	1078	514	77·5	Carcinoma uteri. Thick scalp. Thick bushy hair
162	*A*	25	67	136	181	153·5	3779	1520	550	84·8	Unresolved lobar pneum. Cerebral congestion
163	—	39	62	133	184	145	3548	1460	535	78·8	Carcinoma uteri. Thick hair. Thin scalp
164	*C*	36	61	131	179·5	140	3292	1075	519	78·0	Carcinoma uteri. Skull thick, except at sides.
165	*C*	36	62·5	130	183	147	3497	1280	535	80·3	Carcinoma mammæ [Foreign
166	*C*	40	60	123·5	174·5	143	3082	1180	513	81·9	Malignant dis. R. side of face. Emaciated
167	—	22	58·5	126	179	144	3248	1250	513	80·4	Diabetic coma. Scalp rather thin
168	*C*	40	63·5	125	184	146	3358	1190	528	79·3	Carcinoma uteri. Scalp thin
169	*C*	40	61·5	130·5	193	151	3803	1374	548	78·2	Addison's disease. Skull rather thick [thick
170	*C*	40	65	128·5	185	150	3566	1306	535	81·1	Tubercular nephritis. Skull very thin. Scalp
171	*C*	30	63	126	177	141	3145	1202	502	79·7	Malignant disease of R. ear. Skull very thin
172	*C*	42	64·5	128·5	188	145	3503	1240	539	77·1	Carcinoma mammæ. Skull thick
173	*C*	34	63·5	131	188	145	3571	1316	540	77·1	Pernicious anæmia. Well nourished
174	*C*	42	62·5	130·5	189	151	3724	1280	543	79·9	Carcinoma uteri. Skull thin
175	*C*	39	63·5	126	190	151	3615	1350	555	79·5	Cirrhosis of liver. Ascites
176	*C*	39	62·5	128	175	143	3203	1180	515	81·7	Carcinoma uteri. Skull thin. Scalp thick
177	*C*	24	64	132	186	147	3609	1210	560	79·0	Pyonephrosis. Foreign
178	*C*	44	64·5	128·5	188·5	147	3561	1127	541	78·0	Softening and growth in brain
179	*C*	24	64·5	136	190	154	3979	1324	552	81·1	Bronchitis. Empyema. Dilated heart
180	*C*	30	63·5	134	187	141	3533	1210	540	75·4	Diabetic coma. Wasting. Skull thick
181	*C*	37	63·75	137	187	144	3689	1290	530	77·0	Carcinoma mammæ. Wasted
182	*C*	33	55	120	179	147	3158	1100	490	82·1	Carcinoma uteri. Emaciated. Alcoholic
183	*C*	38	67	132	197	154	4005	1280	578	78·2	Calcareous tumour of cerebellum
184	*A*	26	64·5	128	175	142	3181	1175	545	81·1	Chorea. Well nourished
185	*C*	25	61·5	129	186	145	3479	1160	542	78·0	Tuberculosis of lungs and intestine
186	*C*	35	63	129·5	190	148	3642	1205	547	77·9	Pulmonary tuberculosis. Emaciated
187	*C*	21	66·5	128·5	191	148	3632	1163	541	77·5	Sarcoma of maxilla. Emaciated. Scalp thin

D. *Female Subjects from 46 Years of Age and Upwards. Middlesex Hospital.*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) *H*	*L*	*B*	(Cm.^{3}) $H \times L \times B$	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
188	*C*	46	66	116	185	143	3069	1022	?	77·3	Carcinoma uteri
189	*C*	50	60	132	187	137·5	3394	1243	528	73·5	Carcinoma of breast, and pleura
190	*C*	46	66	139	185	144	3703	1350	532	77·8	Malignant growths in peritoneum
191	*C*	50	63	124	181	141	3165	1237	522	77·9	Epithelioma of scalp and face
192	*C*	46	67·5	130	183	141	3354	1204	522	77	Carcinoma of urethra
193	*C*	49	60·5	123	173	141	3000	1090	514	81·5	Carcinoma uteri. Scalp and hair thin
194	*C*	49	65·5	132	185	151	3687	1355	545	81·6	Cystic kidneys. Uræmia. Skull thin
195	*C*	48	64	129	185	149	3556	1250	530	80·5	Carcinoma uteri. Skull thin
196	*C*	47	66·5	120·5	177	130	2773	1076	516	73·4	Carcinoma mammæ. Skull very thick
197	*C*	53	64	130	168	140	3058	1120	515	83·3	Carcinoma uteri
198	*C*	56	60	131	187	136·5	3344	1220	531	73·0	Carcinoma mammæ
199	—	64	61·5	132	182·5	145	3493	1240	540	79·7	Diabetes. Gangrene of foot
200	*C*	69	62·5	125·5	185	142	3297	1220	525	76·8	Morbus cordis
201	*C*	60	61	130	182	142	3360	1095	515	78·0	Carcinoma uteri. Skull thick
202	*C*	55	61·5	129	180	139	3228	1235	518	77·2	Chronic bronchitis. Skull thin
203	*C*	55	59	129	174	146	3277	1105	506	83·9	Bronchitis. Alcoholism. Skull soft and thick
204	—	53	61	133	188	154	3851	1405	577	81·9	Cerebral hæmorrhage. Thick skull and scalp
205	*C*	60	60·5	126	179	136	3067	1150	518	76·0	Multiple sarcomata. Very thin skull & scalp

D. *Female Subjects from 46 Years of Age and Upwards. Middlesex Hospital. (Continued.)*

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
206	*C*	54	60·5	129·5	181	157·5	3692	1305	548	87·0	Carcinoma. Metastases in L. hip. Thin skull
207	*C*	60	64·5	132	179	144	3402	1220	523	80·4	Chronic interstitial nephritis. Scalp thin
208	—	62	62·5	136	192	153	3995	1296	561	79·7	Chr. nephritis. Fatty degen. of Heart. Very thick skull
209	*C*	53	61	128	180	144	3318	1175	523	80·0	Carcinoma mammæ. Skull thick. Scalp thin
210	*C*	68	61	120	173	131	2720	955	502	75·7	Carcinoma uteri. Scalp thin
211	*C*	63	62·5	126·5	172	135	2937	1070	520	78·5	Sarcoma of nose
212	—	51	66	134	183	146	3580	1320	532	79·8	Appendicitis. Hair and skull thin
213	*C*	78	58·5	121	176	138	2939	1060	509	78·4	Carcinoma recti. Skull very thin
214	*C*	57	61	123	180	135	2989	1130	525	75·0	Carcinoma mammæ. Emaciated
215	—	68	64	128	188	149	3586	1250	540	79·3	Carcinoma of pancreas. Well nourished
216	*C*	58	60·5	124	178	143	3156	1225	515	80·3	Carcinoma mammæ. Cerebral abscess
217	*C*	59	65	127	180	142	3246	1180	524	78·9	Carcinoma mammæ. Skull thin
218	*C*	49	57	126	181	139	3170	1178	512	76·8	Carcinoma uteri. Fairly nourished
219	*C*	56	60	122	186	144	3268	1142	541	77·4	Carcinoma. Sigmoid flexure of colon
220	*C*	58	59	133	182	140	3389	1130	520	76·9	Carcinoma uteri
221	*C*	62	62·5	128·5	184	143	3381	1185	528	77·7	Carcinoma uteri. Wasted
222	*C*	67	63	124	175	132	2864	1012	509	75·4	Carcinoma uteri. Well nourished
223	—	47	64	133	185	152	3740	1280	547	82·2	
224	*C*	64	59	131·5	180	147	3479	1103	532	81·7	Carcinoma mammæ
225	*C*	47	65	131	192	145	3647	1408	565	75·5	Carcinoma mammæ. Skull rather thick
226	*A*	74	63	128	191	152	3716	1300	564	79·6	Strang. femoral hernia. Well nourished
227	*C*	67	62	125	185	142	3284	1246	528	76·8	Cerebral hæm. Interstitial nephritis. Emac.
228	*A*	50	—	142	191	155	4204	1380	559	81·2	Fall. Rupture of aorta. Fracture of ribs. Fat
229	*C*	63	63	130	189	152	3735	1350	548	80·4	Malignant dis. of intestines. Very fat. Skull
230	*A*	74	62	121	186	143	3218	1060	542	76·9	Fractured ribs. Skull and scalp thick [thin
231	*C*	49	65·5	137	185·5	145	3685	1350	535	78·2	Carcinoma of omentum. Very fat. Skull thin
232	*C*	46	67	135	196	140	3704	1220	521	71·4	Carcinoma uteri. Emaciated. Skull thick
233	*C*	46	62·5	126·5	174	146	3214	1110	532	83·9	Pulmonary tuberculosis. Skull thick & spongy
234	*C*	50	64·5	125	181	150	3394	1215	542	82·9	Carcinoma uteri. Emaciated. Scalp thin
235	*C*	63	58·5	127	178	143	3233	1104	522	80·3	Carcinoma mammæ. Emaciated. Skull thin
236	*C*	56	64·5	126·5	184	144	3352	1170	533	78·3	Carcinoma of stomach. Slight wasting
237	*C*	63	62	128	179	148	3391	1120	537	82·7	Carcinoma uteri. Emaciated. Scalp thin

E. *Children* ♂.

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
238	*C*	2·5 ms.	22	89·5	125	95	1063	450	361	76·0	Broncho-pneumonia
239	*A*	9 ms.	28	114	155	117	2067	—	—	75·5	Peritonitis
240	—	1 y.–8 m.	37·5	119	168	125	2499	1052	481	74·4	Tubercular meningitis
241	—	2 y.–6 m.	31·75	111	160	131	2327	1015	474	81·9	Acute general tuberculosis
242	—	2 y.–6 m.	33	121	172	135·5	2820	1120	486	78·8	Broncho-pneumonia. Foreign
243	—	2 y.–8 m.	38	116	162	139	2612	1240	483	85·8	
244	*A*	3	37	124·5	181	137	3087	1310	514	75·7	Meningitis
245	*C*	3 y.–6 m.	35·5	121	170	129·5	2664	1095	476	76·2	Septic broncho-pneumonia. Emaciated
246	*C*	3 y.–10 m.	34	127	174	135	2983	1080	501	77·6	Otitis media. Emaciated
247	—	4	39	127	173	134	2944	1202	501	77·5	—. Skull very thin
248	—	5	37·5	125	170	132	2805	1200	490	77·7	Death by suffocation. Foreign
249	*C*	7	46	127	175	140	3112	1375	504	80·0	Tubercular cervical glands
250	—	8	43	131·5	192	140	3535	1450	530	72·9	Pericarditis. Head large and long

E. *Children* ♂. (*Continued.*)

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
251	—	10	52	137·5	187	149·5	3844	1417	539	79·9	Pericarditis. Foreign
252	*C*	11	55	130	181	145	3412	1412	532	80·1	Morbus cordis [spleen
253	*A*	12	65	136	183	141	3509	1379	540	77·1	"Run over." Fracture of ribs. Rupture of
254	*C*	12	48·5	122·5	183	142	3183	1322	528	77·6	Broncho-pneumonia. Pulm. tuberculosis
255	—	12–13	56·5	134	184	143	3526	1454	530	77·7	
256	*C*	15	62·5	141	190·5	151	4056	1618	547	79·2	Diabetes. Coma
257	*C*	17	67	129	189	150·5	3669	1095	536	79·6	Morbus cordis. Wasted. Foreign
258	*C*	18	71	131	172	142	3200	1230	520	82·6	Tubercular meningitis. Foreign
259	*C*	19	67	143	187	146	3904	1460	556	78·1	Psoas abscess. Tubercular meningitis
260	*A*	19	67·5	135	191	157	4048	1278	558	82·2	Bullet wound in head
261	*A*	19	65·5	133	188·5	147·5	3698	1412	540	78·2	Pneumonia
262	—	19	69·5	138	189	149	3886	1450	547	78·8	Cerebral abscess. Meningitis

F. *Children* ♀.

No.	Acute or Chronic	Age	(Inches) Stature	(Millimetres) H	L	B	(Cms.³) H×L×B	Grms. Brain weight	Mm. Circ.	Cephalic Index	Cause of Death, etc.
263	—	3 wks.	21·5	92·5	114	88·5	933	385	324	77·6	Bronchitis. "Brought in dead"
264	*C*	2·5 ms.	22	94·5	129·5	99	1212	490	369	76·4	Gastro-enteritis. Foreign
265	*C*	5 ms.	22·5	88·5	129	95	1085	512	368	73·6	Convulsions
266	*C*	6 ms.	24·5	100	132·5	104·5	1385	503	385	78·8	Malignant growth of orbit (? sarcoma)
267	*C*	8 ms.	25	118	144	126	2141	960	437	87·5	Rickets. Gastro-enteriti. Foreign
268	*C*	2 y.–6 ms.	33·5	124	172	132	2815	995	496	76·7	Tuberculosis. Emaciated
269	*A*	3 y.–6 ms.	38·5	114	164	126	2356	1100	475	76·8	Fracture of base of skull
270	—	7	47	121	169	131	2679	1120	493	77·5	Pulmonary tuberculosis. Meningitis
271	*C*	7	48	118	174	133	2731	1205	485	76·4	
272	*C*	15	61·5	128	185	139	3292	1205	518	75·1	Otitis media. Thrombosis of R. lateral sinus
273	*C*	17	61	125·5	178	143·5	3206	1180	524	80·6	Tubercular meningitis. Skull thick. Emac.
274	*C*	18	53·5	125	179	135	3021	1147	507	75·4	Mitral regurgitation. Gangrene of hands
275	*C*	18	66	139	186	151	3904	1419	554	81·2	Chronic otitis media. Cerebellar abscess
276	—	19	65	131	179	141	3306	1245	512	78·8	

A STUDY OF THE BIOMETRIC CONSTANTS OF ENGLISH BRAIN-WEIGHTS, AND THEIR RELATIONSHIPS TO EXTERNAL PHYSICAL MEASUREMENTS.

By J. BLAKEMAN, B.A., M.Sc., assisted by ALICE LEE, D.Sc., and KARL PEARSON, F.R.S.

(1) *Introductory.*

THE purpose of this paper is to present a biometric analysis of the measurements provided by Dr R. J. Gladstone and published in this volume. The conclusions reached are therefore of the same order of validity as the data upon which they are based. An attempt has been made to compare them with the fuller material reduced by Dr Raymond Pearl, and in many points where comparison was possible general confirmation of his conclusions has been obtained. Gladstone's statistical material differs from that used by Pearl in two essential points. It is in the first place more meagre, but in the second place it provides additional measurements which enable us to predict with a moderate degree of accuracy brain-weight from external measurements on the living subject. The characters recorded by Gladstone were the age (A), stature (S), cause of death, the maximum head-length (L), the maximum head-breadth (B), the auricular height (H), the horizontal circumference (U), and the brain-weight (w). The sagittal arc, nasion to inion, and the biauricular transverse arc (Q of our notation) were also measured, but are not considered, as the difficulty of determining the inion, and the impediment formed by the hair, especially in the case of the female subjects, was found to be so great as to make the measurements unreliable. The product $P = L \times B \times H$ was formed, and the present study is concerned with the biometric relations of w, A, S, L, B, H, P, and U. The distribution of the work has been as follows: The whole of the labour of obtaining the multiple regression formula, p. 146, for the males fell to J. Blakeman, the similar formula for the females to A. Lee. All the other statistical and calculating work throughout the paper as well as the preparation of Figures 3—6 is further due to J. Blakeman. K. Pearson is responsible only for suggesting the general lines the paper should take, and preparing the draft of its final form.

In dealing with Gladstone's material a reduction was first made in its amount by

(a) the exclusion of a small number of foreigners;

(b) the exclusion of those individuals for whom all the seven characters referred to above were not available;

(*c*) the exclusion of all males under 24 years of age and all females under 20 years of age. The reason for adopting this restriction was that from considerable experience of anthropometric measurements we have found that these ages roughly mark in man and woman a turning-point in the nature of the growth curves. Up to these ages there is a rather rapid increase, after these ages in most characters a slight but continuous decrease.

With the above exclusions we had 117 male and 94 female cases. These numbers are small, but the material is fairly homogeneous and the probable errors have in each case been given. The publication of the actual data (pp. 118—123) has been undertaken largely with the hope that other medical schools may collect similar material so that ultimately proper numbers will be available, large enough for due allowance to be made for the cause of death and the condition at death. No such separate classifications are possible at present.

(2) *On the Special Character of the Present Material.*

The present material is what has been described in this journal* as a "General Hospital Population,"—with perhaps in this case a rather large proportion of cancer cases. It cannot be too often insisted upon that such a population is not a fair sample of the "general population" of a given district. There is a larger amount of what it would be convenient to call "shrinkage" due to illness and defective nourishment. The well-nourished, physically well-developed middle classes are largely absent, and taking the brain-weights of these hospital returns as typical of the "general population"—especially applying them to problems in the relationship of intelligence to physical measurements—is liable to lead to very erroneous conclusions. An illustration of this may be given from Gladstone's classification into acute and chronic cases—the *A* and *C* of the second column of the tables. We find for average brain-weights:

TABLE I.

Ages	Cause of Death	No.	Male	No.	Female
20 to *46* ...	Acute	16	1430	5	1303
	Chronic	27	1331	32	1227
Over *46* ...	Acute	22	1318	3	1248
	Chronic	34	1354	40	1178

It will thus be seen that, with the exception of the males over 46, the cause of death makes a very substantial difference in the average brain-weight. The numbers are however far too few to allow us to make any separate classification, but the point is a vital one and must undoubtedly be properly dealt with when greater numbers are forthcoming. The reader will find as he proceeds further that we have other grounds for suspecting that the general hospital population differs in some essential features from the general population.

* Greenwood: *Biometrika*, Vol. III. p. 65.

(3) *On the Means and Variabilities of the Anthropometric Characters of the Present Material.*

Table II. contains the means, standard deviations, and coefficients of variation of all the characters dealt with.

We notice that the woman is *absolutely* more variable than man in four out of the nine characters considered, and *relatively* more variable in eight out of the nine. In the ninth, stature, the difference in variability between the two sexes is well within the probable error of the difference. This is a further illustration of the point that man is not essentially more variable than woman.

We can now make some comparisons between this general hospital population and other English groups. In the first place let us take the head-measurements:

TABLE II. *Means and Variabilities.*

Character	Mean		Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀	♂	♀
Head Length ... (L) in mm. ...	190·36 ± ·35	183·20 ± ·39	5·62 ± ·25	5·59 ± ·28	2·95 ± ·13	3·05 ± ·15
Head Breadth ... (B) in mm. ...	149·34 ± ·34	144·55 ± ·40	5·41 ± ·24	5·76 ± ·28	3·62 ± ·16	3·99 ± ·20
Auricular Height (H) in mm. ...	132·90 ± ·28	129·21 ± ·30	4·51 ± ·20	4·35 ± ·21	3·39 ± ·15	3·37 ± ·17
Cephalic Index ... (100 B/L) ...	78·48 ± ·17	78·93 ± ·20	2·76 ± ·12	2·87 ± ·14	3·52 ± ·16	3·64 ± ·18
Horizontal Circumference ... (U) in mm. ...	555·79 ± ·99	533·25 ± 1·17	15·84 ± ·70	16·83 ± ·83	2·85 ± ·13	3·16 ± ·16
Product ... ($P = L \times B \times H$) in cm.^{3} ...	3782·91 ± 18·11	3427·03 ± 19·68	290·47 ± 12·81	282·94 ± 13·92	7·68 ± ·34	8·26 ± ·41
Stature ... (S) in inches ...	67·16 ± ·19	63·05 ± ·19	3·05 ± ·13	2·79 ± ·14	4·55 ± ·20	4·42 ± ·22
Age ... (A) in years ...	49·26 ± ·76	45·90 ± ·95	12·11 ± ·53	13·69 ± ·67	24·60 ± 1·15	29·83 ± 1·59
Brain-Weight ... (w) in grs. ...	1327·69 ± 6·49	1224·90 ± 6·70	104·03 ± 4·59	100·16 ± 4·93	7·84 ± ·35	8·18 ± ·40

N.B. In working out all further biometric constants, regression formulae, etc., not the above values but the values as actually calculated to four decimal places were used.

TABLE III.

Results for Cephalic Index.

Group	Male	Female
General Hospital Population ...	78·48	78·93
3000 Criminals*	78·54	—
1000 Cambridge Graduates† ...	79·56	—
2000 School Children‡	78·92	78·29
100 to 200 British Association§	77·75	78·75

We see that as far as the cephalic index is concerned there is nothing to markedly differentiate our material from other groups of the general population that have been dealt with by other observers.

Turning next to the diameters we have

TABLE IV.

Group	Male			Female		
	L	*B*	*H*	*L*	*B*	*H*
General Hospital Population ...	190·4	149·3	132·9	183·2	144·6	129·2
3000 Criminals*	191·7	150·4	—	—	—	—
1000 Cambridge Graduates† ...	193·5	154·0	—	—	—	—
British Association‖	198·1	155·0	130·9	185·6	147·3	128·4
Congress of Anatomists‖ ...	198·4	157·2	133·1	—	—	—
University College Staff‖ ...	196·4	153·5	134·8	—	—	—
Bedford College Students‖ ...	—	—	—	189·7	146·8	132·7

These results show very clearly that the general hospital population has a smaller head than the middle class English population. The only exceptions are the male and female auricular heights for the British Association members. These were probably not measured with an ear-plug craniometer. The nearest approach is the criminal population. Before commenting on this let us consider the stature returns. We have

* Macdonell: *Biometrika*, Vol. I. p. 177 *et seq.*
† Pearson: *The Chances of Death*, Vol. I. p. 351.
‡ Pearson: *Biometrika*, Vol. III. p. 140.
§ Pearson: *The Chances of Death*, Vol. I. p. 351.
‖ Lee and Pearson: *Phil. Trans.* Vol. 196 A, p. 251 *et seq.*

TABLE V.

Group	Male Stature	Female Stature
General Hospital Population	67″·2	63″·1
Galton's South Kensington Returns* ...	67″·9	63″·3
3000 Criminals†	65″·5	—
Australian Criminals‡	66″·9	62″·2
Cambridge Students§	68″·9	63″·8
Pearson's Family Measurements‖, Offspring	68″·7	63″·9
" " " Parents	67″·7	62″·5

Thus we see that the general hospital population is shorter than the average of the upper middle classes, approaching closer to the lower middle classes, who would chiefly supply the South Kensington Exhibition returns. It is taller than the criminal classes either in England or New South Wales, and approaches closely to the average of Pearson's "Parent" Group, who must all have been over 40 years of age.

It would accordingly seem that our general hospital population belongs anthropometrically to a class intermediate between the classes from which criminals are drawn and the lower middle class. Its reduced head-measurements are probably due in part to this fact, in part to chronic illness, and in part to a concomitant of class differentiation, namely want of good nourishment and exercise in the period of growth. We might reasonably attribute a great deal to this want of nurture, and it seems as hopeless to settle the problem of correlation between size of head and intelligence by measuring different social classes, as to settle the correlation between stature and intelligence by the same process.

If we turn to age at death we shall get much confirmation of the view that we are dealing with a badly nourished, physically degenerate class. We have

TABLE VI.

Mean Age at Death in Years.

Group	Male	Female
General Hospital Population	49·3	45·9
Society of Friends ¶... ...	60·9	60·8
Peerage and Landed Gentry ¶	62·9	—

* Galton's *Natural Inheritance*, pp. 200—1; Pearson: *The Chances of Death*, Vol. I. p. 311.
† Macdonell: *Biometrika*, Vol. I. p. 191.
‡ Powys: *Biometrika*, Vol. I. p. 44.
§ *Biometrika*, Vol. I. p. 191.
‖ Pearson and Lee: *Biometrika*, Vol. II. p. 370.
¶ Pearson and Beeton: *Biometrika*, Vol. I. p. 64.

Here in the latter two groups adults, 21 years and over, have been taken, while in the hospital group among the males 24 was the minimum admitted age. Clearly we have to deal, even allowing for a larger percentage of accidents and an absence of some of the diseases of old age, for a population of lesser constitutional vitality as measured by expectation of life. It is to be noted that the female is shorter-lived than the male*.

For the horizontal circumference we have no reliable data for comparison. Dr Beddoe gives it† for 30 groups, containing various numbers from two to a hundred, and also for a hundred individuals, but in *not one single case* of the groups or of the individuals has he obtained a result as low as our *mean* of 558 mm. On the other hand it is almost exactly the value 560 Lewenz and Pearson‡ have found for Jeremy Bentham's head, which in other respects is close to the English mean type. There appears to be little doubt accordingly that Dr Beddoe has an individual method of measuring this circumference.

Lastly, turning to the brain-weights, we have, pooling the results of Reid and Peacock, Clendenning, and Sims, the following mean values.

TABLE VII.

English Brain-weights in grms.

Group	♂	♀
Gladstone's Middlesex Hospital Data ...	1328	1224
Reid and Peacock, Clendenning, Sims§ ...	1335	1235

Thus the average brain-weight of an English general hospital population appears to be fairly constant when we compare Gladstone's data with the means of other material. But this is of course no evidence that the general hospital population is at all representative of the general population.

On the contrary, the fact that for the head diameters, for stature, and for constitutional robustness, as measured by duration of life, the hospital population falls below the general population, would lead us to believe that it probably does so in brain-weight; and we should attribute the probable source of this differentiation to want of nourishment and "shrinkage" due to chronic disease.

(4) *Variation Constants.*

Turning first to the head measurements we may compare our results with those for other English groups. This is done in Table VIII.

* On this point see "Assortative Mating in Man," *Biometrika*, Vol. II. p. 488.

† *L'Anthropologie*, Vol. XIV. pp. 284, 291—4.

‡ *Biometrika*, Vol. III. p. 394.

§ Pearson: *The Chances of Death*, Vol. I. p. 321. Of course the pooling of the results of these different observers is open to considerable criticism.

TABLE VIII. *Variability of Head Measurements.*

Group	Length				Breadth				Auricular Height			
	S. D.		C. of V.		S. D.		C. of V.		S. D.		C. of V.	
	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
General Hospital Population	5·62	5·59	2·95	3·05	5·41	5·76	3·62	3·99	4·51	4·35	3·39	3·37
English Criminals* ...	6·05	—	3·15	—	5·01	—	3·33	—	—	—	—	—
Cambridge Graduates† ...	6·16	—	3·18	—	5·06	—	3·28	—	—	—	—	—
English Skulls	6·27	6·22	3·31	3·45	5·28	4·77	3·75	3·54	4·28	4·50	3·73	4·12

Very little of a definite kind can be deduced from these rather sparse results. In length the general hospital population seems less variable than either the English criminals or Cambridge graduates, but it is more variable in breadth. If we compare the variability of the diameters of the living head with those of the skull, we see that the head is clearly not definitely more variable than the skull, and accordingly the assumption often made that owing to the presence of the flesh it must be so, is erroneous. The paradox is explicable on the basis of a negative correlation between skull diameters and amount of covering flesh‡, i.e., individuals growing much bone must on the average have less flesh.

Taking next cephalic index we have for English groups:

TABLE VIII *bis.*

Group	Cephalic Index			
	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population ...	2·76	2·87	3·52	3·64
English Criminals§	2·74	—	3·55	—
Cambridge Graduates‖	2·90	—	3·70	—
School Children¶	3·31	3·99	4·20	5·09
British Association**	2·52	2·96	3·24	3·76
English Skulls††	3·26	2·98	4·38	3·99

* Macdonell: *Biometrika*, Vol. I. p. 202. † Fawcett: *R. S. Proc.*

‡ Let σ_l be S.D. of skull length, σ_L, of head length, σ_f of flesh thickness; then:

$$\sigma_L = \{\sigma_l^2 + \sigma_f^2 + 2\sigma_l\sigma_f r_{fl}\}^{\frac{1}{2}}$$

or σ_L must be greater than σ_l unless r_{fl}, the correlation between amount of flesh and bone size, be negative.

§ Macdonell: *Biometrika*, Vol. I. p. 191. ‖ *Idem*, p. 191.

¶ Pearson: *Biometrika*, Vol. III. p. 140. ** Pearson: *The Chances of Death*, Vol. I. p. 351.

†† Macdonell: *Biometrika*, Vol. III. p. 222.

We see at once that a general hospital population is about equally variable with other English groups, whether of lower grade, like the criminal, or of higher grade, like the middle class, chiefly drawn upon for Cambridge graduates or B.A. members. School children are more variable—probably a phenomenon of growth. But again we see that our skull series is more variable than the head series. For the circumference our comparable data are *nil*; we can only compare with the English skull. This provides:

TABLE IX.

Coefficients of Variation for English Horizontal Circumference.

Group	♂	♀
General Hospital Population ...	2·85	3·16
English Skull*	2·87	2·92

In the matter of the circumference we should therefore conclude that there is equal variability for head and skull measurements.

Stature provides us with more material. We find:

TABLE X.

Variation in Stature in Inches.

Group	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population	3·05	2·79	4·55	4·42
Galton's South Kensington Records† ...	2·55	2·40	3·95	3·79
English Criminals‡	2·59	—	3·88	—
Cambridge Students†	2·51	2·42	3·64	3·79
Pearson's Family Measurements, Offspring§	2·71	2·61	3·95	4·09
" " " Parents§	2·70	2·39	3·99	3·83

It will be seen that both absolutely and relatively the general hospital population is, in the case of stature, more variable than any of the other English groups. We believe this increased variability in stature to be due to the measurements being taken on the corpse, not on the living subject. As a rule we may say that the coefficient of variation of the living stature of the adults of any race is under 4. Pearl gives twelve cases of this coefficient for corpse measurements in his Table I, p. 23, and in ten of these cases the coefficient of variation is over 4.

* Macdonell: *Biometrika*, Vol. III. p. 222.
† Pearson: *The Chances of Death*, Vol. I. pp. 313 and 311.
‡ Macdonell: *Biometrika*, Vol. I. p. 181.
§ Pearson and Lee: *Biometrika*, Vol. II. p. 370.

In fact the mean values are ♂s 4·11, ♀s 4·39; and we may, we hold, safely conclude that the increased variability does not make a class difference, but is peculiar to measurements made under some difficulties*.

Lastly, we may turn to the variation in age at death:

TABLE XI.

Variation in Duration of Life.

Group	Standard Deviation		Coefficient of Variation	
	♂	♀	♂	♀
General Hospital Population ...	12·11	13·69	24·59	29·83
Society of Friends†	17·13	18·73	28·13	30·80
Peerage and Landed Gentry† ...	16·84	—	27·61	—
Yorkshire Tombstones‡ ...	12·97	16·40	18·97	24·53
Oxfordshire Tombstones‡ ...	12·70	14·67	18·55	21·68
London Tombstones‡	13·14	14·06	19·56	21·22

Thus the general hospital population, while more variable in duration of life than the groups taken from tombstone records, which were all of married adults, is less variable than the pedigree records of either the Society of Friends or the Peerage and Landed Gentry, which were simply for persons over 21. We think, therefore, that the general hospital population, while sensibly differentiated from the other classes in actual duration of life, is not sensibly differentiated in the variability of that duration.

We may sum up then the result of the last two sections as follows:

A general hospital population is sensibly differentiated in its average anthropometric characters from other groups of the same population. But there appears to be no markedly significant difference in variation. It would seem, therefore, that observations made on a hospital population may with due precautions be used for problems in variation, but they should not be used for drawing inferences as to the type of the surrounding general population without extreme care.

(5) *On the Relative Magnitude and Variability of English Brain-weights compared with those of other Races.*

Professor Raymond Pearl's reductions enable us to form a fairly extensive table of comparative brain-weights. We have added a few results for stature and skull capacity. In the bulk of cases these are dependent upon few individuals, and are far from reliable, so that they must be treated as merely suggestions until further data are available.

* See on difference between stature and corpse length Pearson: *Phil. Trans.* Vol. 192 A, p. 191.

† Pearson and Beeton: *Biometrika*, Vol. I. pp. 60 and 64.

‡ *Biometrika*, Vol. II. p. 488.

TABLE XII. *Comparative Brain-weights, etc.*

Group	Brain-weight in grs.				Skull Capacity in cms.				Corpse Stature in cms.				Ratio of Brain-weight to skull-capacity
	No.	Mean	S. D.	C. of V.	No.	Mean	S. D.	C. of V.	No.	Mean	S. D.	C. of V.	
Bohemians ♂ ...	372*	1455	114	7·8	10(!)**	1448	70(!)	—	266*	169	7·3	4·3	1·004 (!)
„ ♀ ...	197*	1311	97	7·4	—	—	—	—	133*	158	7·2	4·5	—
Swedish ♂ ...	416†	1400	106	7·6	—	—	—	—	416†	170	6·8	4·0	—
„ ♀ ...	233†	1253	101	8·0	—	—	—	—	262†	159	6·7	4·2	—
Hessian ♂ ...	475‖	1392	113	8·1	—	—	—	—	475‖	167	7·2	4·3	—
„ ♀ ...	281‖	1260	102	8·1	—	—	—	—	281‖	157	6·9	4·4	—
Bavarians ♂ ...	559§	1362	114	8·4	100¶	1504	116	7·7	390§	166	6·7	4·0	·906
„ ♀ ...	347§	1220	103	8·4	100¶	1336	109	8·2	266§	154	6·6	4·3	·913
English I ♂ ...	340††	1335	124	9·2									·904
„ ♀ ...	269††	1235	119	9·7	72§§	1477	122	8·3	177‖‖	171	7·7	4·6	·905
„ II ♂ ...	117‖‖	1328	104	7·8	80§§	1300	113	8·7	94‖‖	160	7·1	4·4	·899
„ ♀ ...	94‖‖	1225	100	8·2									·942
French I ♂ ...	292***	1325	121	9·2	56§§§	1473	107	7·3	50¶¶	166	—	—	·900
„ ♀ ...	140***	1144	105	9·1	7(!)‖‖‖	1296	101	7·8	50¶¶	154	—	—	·883

* Matiegka's data reduced by Pearl.

† G. Retzius' data reduced by Pearl.

‖ Marchand's data reduced by Pearl. For Pearl's reductions see *Biometrika*, Vol. IV. p. 23.

§ Bischoff's data reduced by Pearson: *The Chances of Death*, Vol. I. pp. 295 and 322.

¶ Ranke's data reduced by Pearson, *loc. cit.* p. 333.

** A few isolated crania taken from the German anthropological catalogues reduced by J. Blakeman.

†† Data of Reid, Peacock, Clendenning and Sims combined and reduced by Pearson, *loc. cit.* p. 321.

§§ Whitechapel data of Macdonell reduced by him, *Biometrika*, Vol. III. pp. 208 and 222.

‖‖ Gladstone's data reduced by Blakeman.

*** Data of *Société d'Anthropologie de Paris* reduced by Pearson, *loc. cit.* p. 322.

§§§ French crania in Munich Catalogue reduced by Pearson, *loc. cit.* p. 332.

‖‖‖ A few French crania in Barnard Davis' *Thesaurus Craniorum* collected and reduced by Blakeman.

¶¶ Rollet's data reduced by Lee and Pearson, *Phil. Trans.* Vol. 192 A, p. 180.

Now, although this table is very incomplete, and impresses upon one the need for the further collection of material, we can safely, we think, draw one or two conclusions:

(i) The order of racial average brain-weights is very far from the order of average racial intelligence. Nor is the order bettered if we allow in any manner for stature. We think that even allowing for differences in racial stature, it is quite impossible to compare the brain-weights of men of different races, and any consideration of intelligence and brain-weight which proceeds by "pooling" the brain-weights of distinguished men of a great variety of races is fundamentally fallacious*.

(ii) The variability, as measured by the coefficient of variation, of brain-weight is for the three races for which we possess fairly reliable data greater than that of skull capacity. Roughly, the average brain-weight variability is 9, the average skull capacity 8. On the whole the female shows very slightly greater variability in both cases.

(iii) The ratio of brain-weight to skull capacity has an average value of about ·9, and this is almost exactly the value of the ratio of the variation in brain-weight to that in skull capacity. It would appear that the cranial chamber is more closely packed with brain in the case of the female than in that of the male, but the evidence, which is based upon heterogeneous material, is not convincing†.

(iv) The standard deviation of brain-weight in grammes is fairly close to the standard deviation of capacity in cubic centimetres.

(v) Gladstone's results are, as far as the means are concerned, in close agreement with the results obtained by Pearson from mixing the data of a variety of observers. The increased variability, however, of the English Series I shows its real heterogeneity, and not too much stress ought accordingly to be laid on the sensible identity of the mean values. The means of the component groups, indeed, vary widely.

(6) *On the Correlations of the various Characters measured.*

Table XIII gives the values of the correlation coefficients and their probable errors. They are tabled to four decimal places, not because they are worth more than one or two, but because these values must be used if the regression coefficients are to be tested. We propose to consider the separate entries in this table, and test them against such scant data as are available at present from other sources. As there are nine observed characters, each sex would provide 36 coefficients of correlation, but only 17 have been calculated for each case, partly because of the

* Especially when the next stage is to compare this heterogeneous group with the fairly homogeneous results for a local general hospital population!

† Data are being collected on more homogeneous material, and it is further hoped to test more completely than has hitherto been done the relative densities of different parts of the male and female brain.

TABLE XIII. *Correlations of Brain-weight and other Characters.*

		L		*B*		*H*		*I*=100 *B*/*L*		*U*		*P*=*L*×*B*×*H*		*S*		*A*		*w*	
		♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
L	♂	1	—	—	—	—	—	—	—	—	—	—	—	—	—	−·0751 ±·0620	—	·5980 ±·0401	—
	♀	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	−·1526 ±·0679	—	·5450 ±·0489
B	♂	—	—	1	—	—	—	—	—	—	—	—	—	—	—	−·0910 ±·0618	—	·4885 ±·0475	—
	♀	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	−·2059 ±·0666	—	·6074 ±·0439
H	♂	—	—	—	—	1	—	—	—	—	—	—	—	—	—	−·2147 ±·0595	—	·5443 ±·0439	—
	♀	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	−·3334 ±·0618	—	·6734 ±·0380
I	♂	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	·0014 ±·0624	—
	♀	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	+·2110 ±·0665
U	♂	—	—	—	—	—	—	—	—	1	—	·8273 ±·0197	—	·3302 ±·0556	—	−·1095 ±·0616	—	·6227 ±·0382	—
	♀	—	—	—	—	—	—	—	—	—	1	—	·8067 ±·0249	—	·4085 ±·0580	—	−·1763 ±·0674	—	·6273 ±·0422
P	♂	—	—	—	—	—	—	—	—	·8273 ±·0197	—	1	—	·3116 ±·0563	—	−·1714 ±·0605	—	·7034 ±·0315	—
	♀	—	—	—	—	—	—	—	—	—	·8067 ±·0249	—	1	—	·3997 ±·0585	—	−·2878 ±·0638	—	·7493 ±·0305
S	♂	—	—	—	—	—	—	—	—	·3302 ±·0556	—	·3116 ±·0563	—	1	—	−·1499 ±·0610	—	·2885 ±·0572	—
	♀	—	—	—	—	—	—	—	—	—	·4085 ±·0580	—	·3997 ±·0585	—	1	—	−·3217 ±·0624	—	·3669 ±·0602
A	♂	−·0751 ±·0620	—	−·0910 ±·0618	—	−·2147 ±·0595	—	—	—	−·1095 ±·0616	—	−·1714 ±·0605	—	−·1499 ±·0610	—	1	—	−·2559 ±·0583	—
	♀	—	−·1526 ±·0679	—	−·2039 ±·0666	—	−·3334 ±·0618	—	—	—	−·1763 ±·0674	—	−·2878 ±·0638	—	−·3217 ±·0624	—	1	—	−·3724 ±·0599
w	♂	·5980 ±·0401	—	·4885 ±·0475	—	·5443 ±·0439	—	·0014 ±·0624	—	·6227 ±·0382	—	·7034 ±·0315	—	·2885 ±·0572	—	−·2559 ±·0583	—	1	—
	♀	—	·5450 ±·0489	—	·6074 ±·0439	—	·6734 ±·0380	—	·2110 ±·0665	—	·6273 ±·0422	—	·7493 ±·0305	—	·3669 ±·0602	—	−·3724 ±·0599	—	1

great labour involved (each coefficient being found without grouping), and partly because we realized at once that the product of the diameters and the horizontal circumference had the highest correlations with brain-weight, and that accordingly these, and not the individual diameters or cephalic index, were the appropriate characters to use in the multiple regression equations, if the latter were not to be too unmanageable. The relationship of the head diameters to each other and to stature has already been determined for more copious and better material, and was accordingly hardly worth repeating here*.

(*a*) We will consider first the influence of sex on the observed characters. We note that with two exceptions, the correlations of length of head with brain-weight and the product of the diameters with the horizontal circumference, the female has higher correlations than the male. Even in these cases the difference is within the probable error of the difference. This confirms the result already reached by Pearson and Lee that in civilised races the organs of the women are more highly correlated than those of the men†. It will be remembered that this was attributed to more intense selection in the case of the male.

(*b*) We turn next to the consideration of the influence of age on the other characters. We note at once that all twelve coefficients of correlation are negative, or the average value of each physical character dealt with decreases with age. We conclude that at any rate for a general hospital population, the *physical characters all shrink with age.* While the brain-weight shrinkage is most marked the auricular height is also very significant, then follows the product of the diameters, while the shrinkage of stature only comes fourth on the list.

If age be measured in years the following are the regression lines for Gladstone's data for stature, together with those for Swedes deduced from the reduction of Retzius' data by Pearl‡.

Shrinkage of Stature with Age.

Males:	English:	$S = -\cdot0378\,A + 69\cdot02$ (unit of stature = inch)
		$S = -\cdot0960\,A + 175\cdot31$ (unit of stature = cm.)
	Swedes:	$S = -\cdot0807\,A + 173\cdot42$ („ „ „)
Females:	English:	$S = -\cdot0655\,A + 66\cdot05$ (unit of stature = inch)
		$S = -\cdot1662\,A + 167\cdot78$ (unit of stature = cm.)
	Swedes:	$S = -\cdot1008\,A + 163\cdot51$ („ „ „).

From these we conclude that the English shrink in stature more than the Swedes with age, and that women of the General Hospital Population in both races shrink more than the men. The lines are represented in Fig. 1 below, which gives a fair idea of the effect of age on stature. We now inquire whether

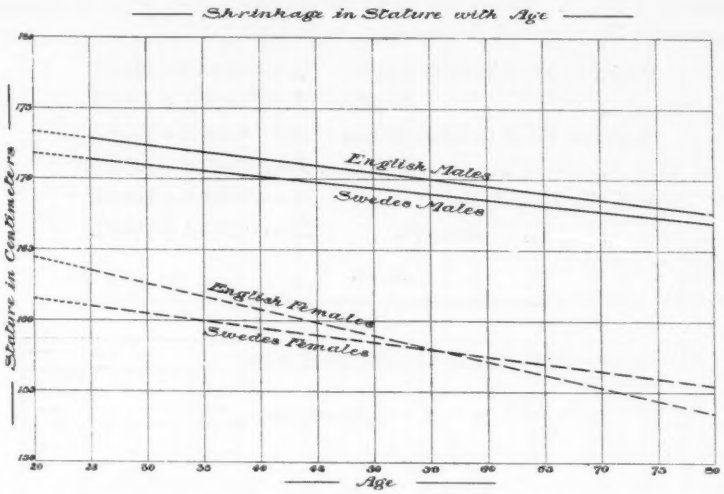
* See for example Macdonell's results for 3000 criminals, *Biometrika*, Vol. I. p. 202, and Fawcett and Pearson's results for 1000 Cambridge graduates, *R. S. Proc.*, Vol. 66, p. 23.

† *R. S. Proc.*, Vol. 61, pp. 365.

‡ *Biometrika*, Vol. IV. pp. 53, 54.

this shrinkage is peculiar to the General Hospital Population, or extends to the normal population of a district. The only comparative results that we know

FIG. 1.



of are the regression lines calculated for A. O. Powys' experience of New South Wales criminals by K. Pearson*. We have:

English Males, Hospital Population:	$S = -\cdot0378A + 69\cdot02$
New South Wales Males, Criminal Population:	$S = -\cdot0337A + 68\cdot34$
English Females, Hospital Population:	$S = -\cdot0655A + 66\cdot05$
New South Wales Females, Criminal Population:	$S = -\cdot0361A + 63\cdot61$

In all cases the unit of stature is an inch.

Hence it would appear that the hospital population of males shrinks slightly more rapidly than the criminal population, and the hospital population of females very sensibly more. This is probably due to the larger proportion of male acute cases and of female chronic cases. We do not think the present results do more than confirm the conclusion of Powys' paper that there is for normally healthy individuals a shrinkage of a little more than $\frac{1}{3}''$ per ten years.

(c) If we take in the next place head diameters we are not able to compare our General Hospital Population with Matiegka's data, for his paper† gives no clue to the ages of the individuals whose head measurements he has taken, and further those measurements are scarcely comparable with external measurements

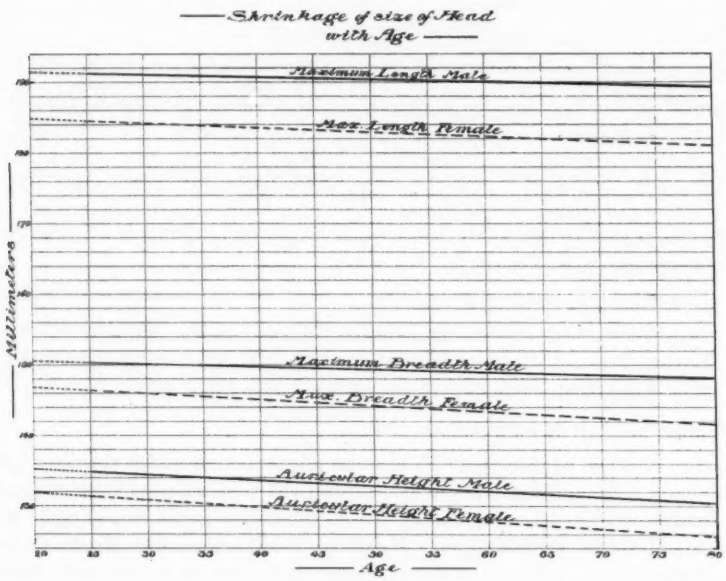
* *Biometrika*, Vol. I. p. 49.

† *Sitzberichte des k. böhmischen Gesellschaft d. Wissenschaften, Math. Naturw. Classe*, Jahrg. 1902, pp. 1—75.

on the head. Our results therefore must at present stand unsupported. They show a sensible shrinkage of the head diameters, greater in the woman and greatest in the auricular height for both sexes. The amount of it is represented graphically in Fig. 2. The equations to the regression lines are as follows, age being measured in years and the diameters in centimetres:

Maximum Head Length, Males: $L = -\cdot0348A + 192\cdot07$,
" " Females: $L = -\cdot0623A + 186\cdot06$,
Maximum Head Breadth, Males: $B = -\cdot0406A + 151\cdot34$,
" " Females: $B = -\cdot0867A + 148\cdot53$,
Auricular Height, Males: $H = -\cdot0799A + 136\cdot84$,
" " Females: $H = -\cdot1059A + 134\cdot07$.

FIG. 2.



It will thus be seen that the woman shrinks with age about twice as fast as the man in head length and breadth, but only about $\frac{5}{4}$ as fast in auricular height. The very sensible shrinkage for both sexes in head height is peculiarly noteworthy, as it of course contributes to the shrinkage in stature. It seemed worth while investigating whether these results are peculiar to a hospital population. Accordingly 1306 criminals were taken, whose age, head length and head breadth but (unfortunately) not head height were recorded, and the correlations worked out for age and these two diameters. The correlation tables are given as

Tables XIV. and XV. below. The actual values obtained are, ages running from 21 to 70:

Mean Age: 33·32 ± ·19. Standard Deviation: 10·2219 yrs. ± ·1349.
Maximum Length: 191·44 ± ·11. Standard Deviation: 6·0025 cms. ± ·0792.
Maximum Breadth: 150·30 ± ·10. Standard Deviation: 5·1329 cms. ± ·0677.
Correlation of Age and Head Length + ·1138 ± ·0184.
Correlation of Age and Head Breadth: + ·0349 ± ·0186.

TABLE XIV.

1306 *Criminals. Correlation between Head Length and Age.*

Age.

Head Length.	21—25	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	Totals
171	—	1	—	—	—	—	—	—	—	—	1
172	—	—	—	1	—	—	—	—	—	—	1
173	1	—	1	—	—	—	—	—	—	—	2
174	—	—	—	—	—	—	—	—	—	—	—
175	1	—	—	1	—	1	—	—	—	—	3
176	—	2	—	1	—	—	—	—	—	—	3
177	1	4	—	—	—	—	—	—	—	—	5
178	3	3	—	1	—	—	—	—	—	—	7
179	5	2	3	—	—	1	—	1	—	—	12
180	4	3	2	3	—	—	—	1	—	—	13
181	7	5	3	1	—	1	—	—	—	—	17
182	7	7	6	3	3	1	—	—	1	—	28
183	6	6	2	6	2	2	—	—	—	—	24
184	15	15	2	4	1	1	4	—	—	1	43
185	18	17	6	5	4	1	1	—	1	—	53
186	19	15	7	5	7	3	1	—	—	—	57
187	17	15	3	7	5	3	4	—	1	—	55
188	26	10	14	7	6	2	2	1	—	—	68
189	25	16	13	15	4	6	—	1	3	—	83
190	24	19	14	13	5	4	3	2	—	1	85
191	34	27	12	3	9	3	2	1	2	3	96
192	30	14	21	15	7	7	3	3	1	1	102
193	22	20	15	12	3	3	2	—	2	—	79
194	28	12	14	10	9	3	2	1	4	—	83
195	18	13	10	5	7	8	2	—	3	—	66
196	14	16	11	10	4	6	3	1	—	1	66
197	18	6	10	5	6	8	2	—	1	—	56
198	15	6	10	6	2	1	2	—	—	1	43
199	6	10	4	6	5	1	1	1	1	—	35
200	5	9	5	4	2	1	1	2	—	1	30
201	6	4	1	2	—	4	1	—	2	—	20
202	6	9	4	—	1	—	1	1	2	—	24
203	3	4	3	2	1	—	1	—	—	—	14
204	1	7	2	2	—	—	1	—	—	—	13
205	2	2	—	2	—	1	—	1	—	—	8
206	—	1	1	1	—	—	—	—	—	—	3
207	—	1	—	1	—	2	1	—	1	—	6
208	—	—	—	—	—	—	—	—	—	—	—
209	—	—	—	—	1	—	—	—	—	—	1
210	—	—	—	—	1	—	—	—	—	—	1
Totals	387	301	199	159	95	74	40	17	25	9	1306

TABLE XV.

1306 *Criminals. Correlation between Head Breadth and Age.*

Head Breadth.	Age. 21—25	26—30	31—35	36—40	41—45	46—50	51—55	56—60	61—65	66—70	Totals
134	—	1	—	—	—	—	—	—	—	—	1
135	—	—	—	—	—	—	1	—	—	—	1
136	2	—	1	1	—	—	—	—	—	—	4
137	1	—	2	—	—	—	—	—	—	—	3
138	2	3	1	—	—	—	—	—	—	—	6
139	1	6	—	—	—	—	—	—	—	—	7
140	2	3	—	2	1	1	—	—	—	—	9
141	11	4	3	—	—	—	—	—	—	—	18
142	9	5	2	2	1	1	—	1	—	—	21
143	12	6	5	4	2	5	2	1	2	—	39
144	14	7	11	8	5	1	—	—	—	—	46
145	21	15	7	11	4	4	1	1	1	—	65
146	26	9	18	16	8	5	4	—	1	3	90
147	25	18	18	8	8	7	3	2	2	1	92
148	18	25	11	13	8	4	4	—	3	—	86
149	32	31	10	7	7	4	3	2	2	—	98
150	27	24	14	13	6	5	—	1	1	1	92
151	26	22	19	12	6	9	5	1	3	1	104
152	31	23	14	9	9	4	2	2	2	1	97
153	26	21	16	11	5	6	2	1	3	—	91
154	28	18	8	7	7	4	1	—	—	—	73
155	15	13	10	8	3	2	1	1	2	1	56
156	13	15	11	9	4	—	4	—	—	1	57
157	13	9	5	7	2	4	2	1	3	—	46
158	8	7	4	5	1	2	—	—	—	—	27
159	6	5	4	4	2	—	1	—	—	—	22
160	2	6	2	2	2	3	1	—	—	—	18
161	5	2	2	—	—	—	—	1	—	—	10
162	5	2	—	—	2	1	2	—	—	—	12
163	1	—	1	—	2	—	—	1	—	—	5
164	3	1	—	—	—	1	—	—	—	—	5
165	—	—	—	—	—	1	1	1	—	—	3
166	1	—	—	—	—	—	—	—	—	—	1
167	—	—	—	—	—	—	—	—	—	—	—
168	—	—	—	—	—	—	—	—	—	—	—
169	1	—	—	—	—	—	—	—	—	—	1
Totals	387	301	199	159	95	74	40	17	25	9	1306

A comparison of Table II. shows that the criminal is younger and less variable in age than the General Hospital Population. To test the effect of the more youthful population, we omitted the criminals less than 26 years of age. The results were as follows for head length of 916 cms.:

Mean Age: 37·46 ± ·21. Standard Deviation: 9·5254 ± ·1498.

Maximum Length: 191·73 ± ·14. Standard Deviation: 6·1534 ± ·0968.

Correlation of Age and Head Length: + ·1062 ± ·0220.

Clearly the correlation is not sensibly affected, while the age is sensibly and the mean head length slightly raised. We think we must conclude from this result

that in the criminal population there is no shrinkage of head breadth, perhaps a slight increase, with age, and a sensible increase, not a shrinkage, of head length with age. Unfortunately we could not test the auricular height change. It follows from this that the shrinkage of head size with age appears to differentiate the General Hospital from the criminal and very possibly from the normally healthy general population. Hence in determining the probable brain-weight from size of the head in a normal population probably little stress ought to be laid on any change of the diameters and therefore their product with age. It is possibly in our case a result of the increasing amount of chronic illness in the older members of the General Hospital Population. The possible shrinkage of the head with age in the general population would form an interesting subject for investigation.

Of course when the three diameters have negative correlation with age, we should expect to find, as we do find, that the product would shrink with age. We have no data to compare with the observed shrinkage of the horizontal circumference with age.

(*d*) The correlation of the horizontal circumference with stature, ·33, is we believe new, but is very much what we might have anticipated. So also is the correlation, ·31, of stature with the diametral product. Thus Macdonell found for 3000 criminals the correlation of stature and maximum head length, ·34, of stature and maximum head breadth, ·18, and of stature and face breadth, ·35*.

With regard to the correlation between the horizontal circumference U and the product P of the three diameters L, B, H, we note that if V be a coefficient of variation we must have:

$$r_{PU} = (V_L r_{UL} + V_B r_{UB} + V_H r_{UH})/V_P$$
$$= (2{\cdot}95 r_{UL} + 3{\cdot}62 r_{UB} + 3{\cdot}39 r_{UH})/7{\cdot}68$$

for English males from Table II. Now we are unacquainted at present with any determinations of r_{UL}, r_{UB} and r_{UH}, but in round numbers r_{UL} and r_{UB} cannot differ much from ·7 and r_{UH} from ·5. Substituting we find $r_{PU} = {\cdot}82$; so that the observed value ·83 for males appears quite probable. A similar investigation gives ·79 as against the observed ·80 for females.

(*e*) We turn lastly to the correlations between brain-weight and the other characters.

We see in the first place that cephalic index is the least important of these correlated characters, and further that its correlation with brain-weight varies markedly from male to female. It is within the limits of random sampling zero for men, and has the quite sensible value ·2 for women. We have no similar data to compare our results with. All we can do is to consider the correlation of skull capacity with cephalic index, which has been determined in a number of cases. The irregular and divergent values of this correlation were first pointed out by

* *Biometrika*, Vol. I. p. 202.

Dr A. Lee*. Thus she found it sensibly zero for ♀ Theban mummies and −·15 for male Thebans. A list of values is given by Lewenz and Pearson† in which it is shown that ♂ English crania give a practically zero value for the correlation between cephalic index and capacity. That value was, however, obtained indirectly, i.e. from the L and B correlations with each other and the capacity, and accordingly it seemed worth while to investigate the matter directly on the Whitechapel crania. Accordingly we extracted from Macdonell's tables‡ the capacities and cephalic indices of 70 ♂ and 80 ♀ crania and worked out directly the correlation between the two characters. We found:

Correlation of Cranial Capacity and Cephalic Index for Whitechapel Crania:

Males: $+\cdot093 \pm \cdot080$. Females: $-\cdot099 \pm \cdot075$.

Neither value is really significant having regard to the probable error, and we might therefore conclude that this correlation is sensibly zero for English crania. Such a result would be in good accordance with the sensibly zero correlation between brain-weight and cephalic index for males, but not in accordance with the result for English females. Assuming that there might be an error somewhere in the arithmetic the correlation between brain-weight (w) and cephalic index I was tested indirectly by the formula

$$r_{Iw} = (V_B r_{Bw} - V_L r_{Lw})/V_I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2),$$

with the following results:

TABLE XVI.

Cephalic Index and Brain-weight.

Sex	Direct Calculation	By Formula (2)
Males ...	+·0014	+·0373
Females ...	+·2110	+·2084

The degree of closeness is as great as we might expect from the two methods, the square of $V/100$ being neglected in obtaining formula (ϵ). There is, therefore, we think, no doubt that, while the correlation between cephalic index and capacity is sensibly zero for English crania of both sexes, the females of a General Hospital Population have a sensible correlation between brain-weight and cephalic index. It is conceivable that this is due to some exaggeration of breadth in the measurement of the heads of the younger females, for whom the brain-weight would be larger and as a rule the hair interfere more than would be the case for older women or for the males§. On the whole we think we have very adequate grounds for omitting the cephalic index entirely from our prediction equations for the brain-weight.

* *Phil. Trans.* Vol. 196 A, p. 232. † *Biometrika*, Vol. III. p. 374.

‡ *Biometrika*, Vol. III. Tables after p. 244.

§ The hair difficulty occurs *twice* in the case of measuring B and only *once* in measuring L. Hence B would tend to be more exaggerated than L.

Looking generally at the brain-weight correlations we see that predictions based on the diametral product and the circumference will give us the best results. Age and stature have formed such a large part of the anatomical discussions on the subject that we feel compelled to retain them, but clearly a knowledge of any diameter of the head is of more importance in predicting brain-weight than either stature or age. After careful consideration we determined to make our most complex prediction formula depend upon a knowledge of the diametral product, the horizontal circumference, the stature and the age of the subject.

Before considering prediction formulae, however, it is well to compare our correlations with any available material.

In the first place we take the English skull.

TABLE XVII.

Comparison of Skull and Head Correlations.

Character		General Hospital Population			Whitechapel Crania *	
		♂	♀		♂	♀
L with ...	w	·598 ± ·040	·545 ± ·049	C	·597 ± ·051	·691 ± ·040
B with ...	w	·489 ± ·048	·607 ± ·044	C	·631 ± ·048	·646 ± ·044
H with ...	w	·544 ± ·044	·673 ± ·038	C	[·501 ± ·059	·600 ± ·049]
U with ...	w	·623 ± ·032	·627 ± ·042	C	·820 ± ·026	·848 ± ·021
P with ...	w	·703 ± ·032	·749 ± ·031	C	[·750 ± ·035	·823 ± ·025]

We notice that the correlations of capacity and external measurements are all greater than the corresponding correlations for the brain-weight. The apparent exception (C and L, ♂ is within the limits of probable error), namely C and H for both sexes, arises, we think, from the fact that H is here the basio-bregmatic and not the auricular height of the skull, and no stress can thus be laid upon this exception†. The conclusion, which we might have anticipated, is therefore forced upon us that it is easier to predict capacity from external measurements on the skull than brain-weight from external measurements on the living head.

The values of the correlation between C and P, although obtained indirectly, are probably very close: thus A. Lee‡ gives for the corresponding values in Bavarian crania: ♂, ·700 ± ·034 and ♀, ·814 ± ·023, the differences for the two races being well within the limits of random sampling.

* Cited from Macdonell, *Biometrika*, Vol. III. p. 232. Except those for P and C which are deduced from the formula: $r_{CP}=(V_L r_{CL}+V_B r_{CB}+V_H r_{CH})/V_P$. All these constants are known for the Whitechapel crania except r_{CH} the correlation of capacity and auricular height, we have used for this the known correlation of capacity and basio-bregmatic height, cited in brackets under C and H.

† If the auricular height be more highly correlated with capacity than the basio-bregmatic height, the result is very satisfactory, because the former is all that is obtainable on the living head. The point should be investigated as ample data exist.

‡ *Phil. Trans.* Vol. 196 A, p. 237.

If we pass from intra-racial to inter-racial comparisons, Pearl's memoir offers valuable material. Thus he has calculated the correlation of w with L and B for Bohemians, based, it is true, on Matiegka's rather doubtful skull measurements. Again, he provides us with the brain-weight and stature and brain-weight and age correlations for Swedes and Hessians. These are reproduced in the accompanying table. Considering how widely correlation results differ from race to race the general agreement is not without significance. It increases our confidence in the possibility of good results being obtained from careful post-mortem room measurements.

TABLE XVIII.

Comparison of English and other European Brain-weight Correlations.

Correlation	Middlesex Hospital Data		Other European Data		
	♂	♀	Group	♂	♀
w and L	·598 ± ·040	·545 ± ·049	Bohemian	·548 ± ·027	·360 ± ·047
w and B	·489 ± ·048	·607 ± ·044	Bohemian	·466 ± ·031	·504 ± ·040
w and S	·289 ± ·057	·367 ± ·060	Swedes	·183 ± ·032	·349 ± ·039
" "	" "	" "	Hessians	·182 ± ·030	·183 ± ·039
w and A	− ·256 ± ·058	− ·372 ± ·060	Swedes	− ·249 ± ·031	− ·234 ± ·042
" "	" "	" "	Hessians	− ·167 ± ·030	− ·360 ± ·035

The general agreement is, we have said, not bad considering the differences in method of measuring and probably in heterogeneity of material. The Swedish are quite fairly concordant with the English. So are the Bohemians (considering the poor method used by Matiegka for determining the breadth and length of the skull), except for the correlation of weight and length for women, where there is a bad and inexplicable fall, although a slight fall might have been expected from the English results. The Hessian values appear to be irregular, and one is inclined to doubt the accuracy with which the corpse length has been determined, or the relative homogeneity of the series; individuals under 20 ought not to have been included in the age and stature correlations.

(7) *On the Regression Equations of Brain-weight on other Characters.*

(*a*) The following table will allow the reader to see at a glance the average changes in brain-weight w and product P involved in unit change from the mean in each of the observed characters.

As illustration, take the following case, which is closely that of Jeremy Bentham:

Stature: Mean (?), $U = 560$, $L = 192$, $B = 153$, $H^* = 127$, and Age = 85 yrs.

* Measured from *top* of auricular orifice H is 121 (see *Biometrika*, Vol. III. p. 394), we must add, say, 6 mm. to compare with Gladstone's data from centre of orifice.

TABLE XIX.

Increase of Brain-weight in grs. and Diametral Product in cm.³ above their means for each unit increase in the other observed Characters.

Regression Coefficient of Character with Probable Error.

			One mm. in *L*	One mm. in *B*	One mm. in *H*	One mm. in *U*	One Year in *A*	One inch in *S*
Mean of Character		♂	190·36	149·34	132·20	555·79	49·26	67·16
		♀	183·20	144·55	129·21	533·25	45·90	63·05
Brain-weight Increase in grs.		*Mean*						
	♂	1327·69 grs.	11·0728 ±·9254	9·3936 ±1·0463	12·5628 ±1·2074	4·0895 ±·3204	−2·1974 ±·5245	9·8277 ±2·0338
	♀	1224·90 grs.	9·7599 ±1·0446	10·5593 ±·9607	15·5037 ±1·1841	3·7344 ±·3224	−2·7245 ±·4721	13·1934 ±2·3271
Diametral Product Increase in cms.	♂	3782·91 cm.³	—	—	—	15·1711 ±·6424	−4·1097 ±1·4730	29·6394 ±5·6361
	♀	3427·03 cm.³	—	—	—	13·5656 ±·6913	−5·9478 ±1·3769	40·5999 ±6·4775

A man of his age would have lost 78·5 grs. of his original mean age brain-weight, or have on this account only 1249 grs. instead of 1328 grs., the average at age 49·26. He has an excess of 1·64 mm. in head length, an excess of 3·66 mm. in head breadth, and a defect of 5·20 mm. in auricular height. Also an excess of 4·21 mm. in horizontal circumference. Thus on the first count his brain-weight would be 18·15 grs. above the average, on the second 34·38 grs. above the average, on the third 65·33 below the average, and on the fourth 17·22 grs. above the average. But it would not be proper to club all these together and say that on account of his head measurements we should expect Bentham to be 4·42 grs. above the average when at the mean age. For all these characters are correlated, and we have seen that they, especially the auricular height, are influenced by age. Hence it is absolutely needful to use multiple regression formulae. These will be applied to Bentham's measurements later, but the above will suffice to show how a single character may have its influence appreciated.

(*b*) We now pass to the multiple regression equations. These were found in the usual way, namely, by the calculation in this case of 25 constituent determinants and the minors of their first row of constituents. The work is straightforward, but laborious. For purposes of comparison the prediction formulae based on diametral

product and horizontal circumference together (without age and stature), and on each of these alone, are also added.

Prediction Formulae.

(i)[a] ♂: $w = \cdot1987P + \cdot8644U - 1\cdot1910A + 1\cdot7508S + 36\cdot8559$.
Probable error of prediction: 48·5 grs. Mean error: 57·4 grs.

(i)[b] ♀: $w = \cdot2195P + \cdot5067U - 1\cdot2395A + 1\cdot0721S + 191\cdot8298$.
Probable error of prediction: 43·2 grs. Mean error: 51·0 grs.

(ii)[a] ♂: $w = \cdot2136P + \cdot8486U + 47\cdot9443$.
Probable error of prediction: 49·6 grs. Mean error: 58·7 grs.

(ii)[b] ♀: $w = \cdot2466P + \cdot3893U + 172\cdot2420$.
Probable error of prediction: 44·7 grs. Mean error: 52·8 grs.

(iii)[a] ♂: $w = \cdot2519P + 374\cdot7628$.
Probable error of prediction: 49·9 grs. Mean error: 59·0 grs.

(iii)[b] ♀: $w = \cdot2653P + 315\cdot8281$.
Probable error of prediction: 44·7 grs. Mean error: 52·9 grs.

(iv)[a] ♂: $w = 4\cdot0895U - 945\cdot1785$.
Probable error of prediction: 54·9 grs. Mean error: 64·9 grs.

(iv)[b] ♀: $w = 3\cdot7344U - 766\cdot4563$.
Probable error of prediction: 53·0 grs. Mean error: 62·8 grs.

We see from these results: first, that owing to the higher correlations it is possible to predict female brain-weights somewhat more accurately than male brain-weights from external measurements of the head. Secondly, that the fundamental character is the product of the three diameters. A comparison of (iii) and (iv) shows that it is worth a good deal more than the circumference, and further comparing (iii) with (i) we see that if we predict from the diametral product *alone*, we shall only make 2 grs. worse mean error than if we predict from all four characters. From this standpoint it is clear that age and stature allowances are not nearly as important matters as head diameters if we wish to predict brain-weights from physical measurements.

The formulae were then applied to 24 ♂ and 24 ♀ cases selected at random, being every fourth or fifth case in the observation book. It does not seem needful to reproduce the individual results, but the general conclusions are given in Table XX.

Our actual results are therefore somewhat in defect of the mean errors to be expected by theory, especially in the case of (i), (ii), and (iii) for the males. In the case of the females the defect is very little, and the theoretical and observed mean errors are close. We think it quite safe to say that the brain-weight of a living individual can be predicted with a mean error of not more than 50 grs. It is

noteworthy that Dr Lee's diametral formula* gave a mean error in predicting skull capacity also of about 50 cm.³ It seems to us that no linear formula based upon external measurements is likely to give any better result. For all such measurements are more or less closely correlated, and, as has been shown elsewhere, increasing the number of correlated variables does not much narrow the range of variation of the predicate†.

TABLE XX.

Prediction of the Brain-weights of 48 *Individuals from External Measurements.*

Mean Error, in each instance of 24 cases.

Formula	♂ sex	♀ sex
(i)	47·50 grs.	50·65 grs.
(ii)	48·83 grs.	50·65 grs.
(iii)	52·63 grs.	49·10 grs.
(iv)	60·21 grs.	60·15 grs.

(*c*) It may be of interest to try these formulae on one or two actual heads.

In the *Société d'Anthropologie de Paris, Bulletins* 1866, II. Série, T. I. p. 209 are given the measurements of a skull reputed to be that of Dante. Unfortunately all the measurements we require are not given. We find for cranial lengths: $L = 178$ mm., $B = 140$ mm., $U = 525$ mm. The bi-auricular transverse arc, doubtless measured from centre to centre of auricular orifice, was 310 mm., and the total height of the skull, i.e. the basio-bregmatic height, 140 mm. By examining crania with a similar transverse arc and basio-bregmatic height, we consider that 118 mm. would be a close value of the auricular height of Dante's skull. The height of the corresponding skeleton was 155 cms. The mean stature of the Tuscans is 165 cms., and of the more northern Piedmontese 162 cms. We shall almost certainly exaggerate Dante's stature if we put it at 164 cms. We have next to convert skull measurements into head measurements, and in doing this we have rather magnified the values in the light of recent investigations on the thickness of the flesh. Dante died at 56 years of age. Alongside Dante we can take the measurements of Jeremy Bentham's head‡, and for further illustration the head measurements of one of ourselves. We have then:

* *Phil. Trans.* Vol. 196 A, p. 238 *et seq.* *Biometrika*, Vol. III. p. 371.
† *Phil. Trans.* Vol. 190 A, p. 466.
‡ *Biometrika*, Vol. III. p. 394.

TABLE XXI.

Special Individual Heads.

Characters	Dante	Bentham		P.
L	188	192		191
B	150	153		150
H	124	126*		135
U	525	560		555
Age	56	85		47
Stature ...	164 cms.	Mean (?)		71″
	At 56 years	At 85 years	At 49 years	At 47 years
Probable Brain-weight (i)	1244	1272	1351	1353
,, ,, (ii)	1240	1314		1328
,, ,, (iii)	1256	1307		1329
,, ,, (iv)	1202	1345		1324

The average English brain-weight for a "General Hospital Population" of the mean age of 49 years is 1328 grs. It is therefore clear that P.'s brain-weight is essentially mediocre. The great Bentham could have had a brain-weight which at the mean age was at most only a few grammes above the normal. This corresponds exactly to the general mediocrity which has been already noticed in Bentham's physical measurements†. As for Dante his probable brain-weight was 80 to 90 grs. below that of the mean of the English General Hospital Population, and this corresponds well with the observed skull capacity. If the reputed skull were really Dante's we are forced to conclude either (i) that there are most striking individual exceptions to any rule that extreme ability is associated with large brain-weight, or (ii) that it is impossible to apply the formulae deduced from one race to determine the brain-weight of a member of a second. As the present paper and Pearl's memoir both show that at least (ii) is true we may at once reject (*a*) any intra-racial reconstruction formula used inter-racially, and (*b*) any demonstration of the association of great intellectual power with large brain-weight, which is based on lists of the brain-weights of distinguished men of all nations, clubbed together without any regard to age, stature, or race, and compared with the mean brain-weight of a General Hospital Population.

We think, therefore, that the above formulae should be confined to English persons. That if they are to be applied to any other race the mean values of w, P, U, A, and S, say $\overline{w}$, $\overline{P}$, $\overline{U}$, $\overline{A}$, and $\overline{S}$, should be adopted for that race. The fundamental formulae (i) then become:

* Formulae are calculated for *H* measured from centre of auricular orifices, and *OH* in *loc. cit.* was from the top.

† *Biometrika*, Vol. III. p. 394.

(i)[a] ♂: $w-\overline{w}=\cdot1987\,(P-\overline{P})+\cdot8644\,(U-\overline{U})+\cdot6893\,(S-\overline{S})-1\cdot1910\,(A-\overline{A})$,

(i)[b] ♀: $w-\overline{w}=\cdot2195\,(P-\overline{P})+\cdot5067\,(U-\overline{U})+\cdot4221\,(S-\overline{S})-1\cdot2395\,(A-\overline{A})$,

where w and $\overline{w}$ are in grs., P and $\overline{P}$ in cm.^{3}, U and $\overline{U}$ in mm., S and $\overline{S}$ in centimetres, no longer inches, and A and $\overline{A}$ in years. These formulae may enable the reader to obtain approximations for other groups, for which some of the mean characters are determinable. For example: a knowledge of $\overline{L}$, $\overline{B}$, and $\overline{H}$ for the Swedes would enable us to reach an approximate formula for Swedes from Retzius' data for $\overline{w}$, $\overline{S}$ and $\overline{A}$. The assumption made is that the multiple regression coefficients change far less from race to race than the "type*."

(8) *On the Graphic Representation of "Scatter" of Brain-weight about the Probable Value.*

In the course of the present investigation we have given the probable errors and mean errors of brain-weight prediction as deduced from the arithmetical deduction of our data. It seems desirable, with a view of impressing on anatomists the real character of variation in physical measurements, and how hopeless is the hunting for anything like an exact formula for brain-weight, to put the relationship between brain-weights and diametral products and brain-weights and horizontal circumferences in a graphical form for the present English data. In these diagrams each dot represents an actual observed case, and the best fitting straight line to the system of dots, calculated by the usual correlation method, is drawn slanting across the diagram. Figs. 3 and 4 give the results for males and females deducible from the diametral products. Figs. 5 and 6 give similar results for the horizontal circumferences. We see that the scatter is less for the diametral product than for the horizontal circumference, but we see further how hopeless it would be to attempt from these intra-racial data to achieve more by any curve than can be done by a straight line. When we pass, as in formula (ii), to double regression results we have merely such a system of dots in space of three dimensions, and multiple regression data, as in formula (i), only represent a more complicated system of dots in many dimensioned space. So soon as this conception of the actual conditions of variation is realized, we believe that anatomists will once for all abandon any attempt to represent by mathematical formulae, other than correlation results, this "scatter" which marks all the inter-relationships of anthropometric characters. We see in a general way also that increasing the number of dimensions of our space does not in the least necessitate any sensible reduction in the extent of the scatter—for if it did it would involve the principle that a plane must fit a system of points in space more closely than a line a system of points in a plane. A few variables highly correlated with the predicate and lowly correlated with each other is the ideal system to be hunted for, but it is not always to be found, even if it exists†.

* *Phil. Trans.* Vol. 200 A, pp. 21 *et seq.*

† For example, two parents give better results for predicting the character of an individual than two of his brethren, because although the latter may be more highly correlated with him, these are more highly correlated together than the parents are.

FIG. 3. Male Brain-weights and Diametral Products.

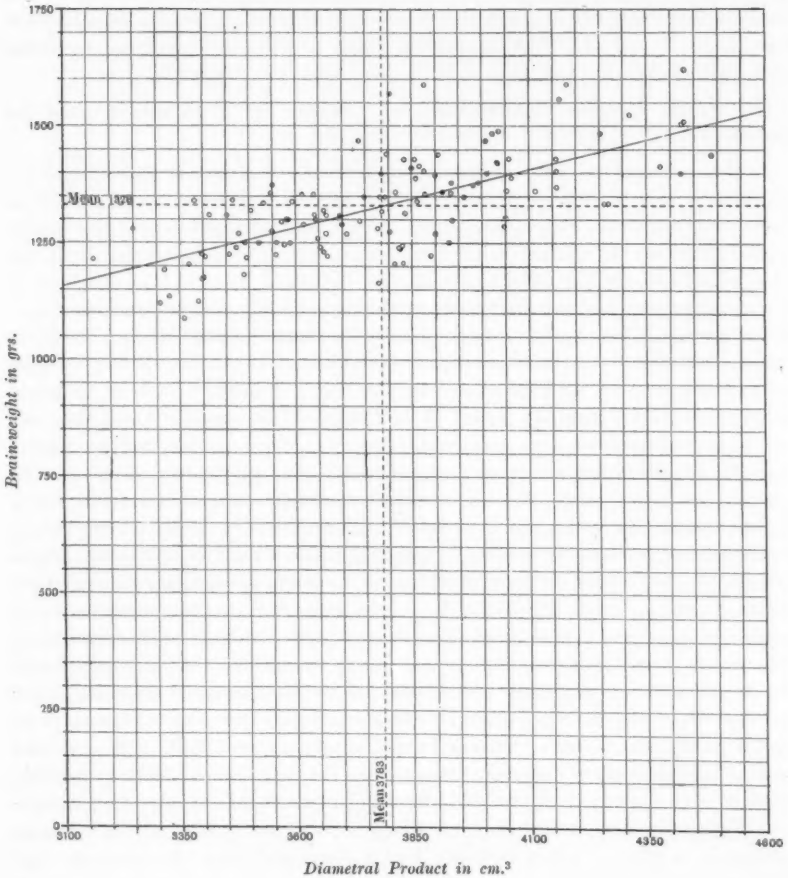


FIG. 4. Female Brain-weights and Diametral Products.

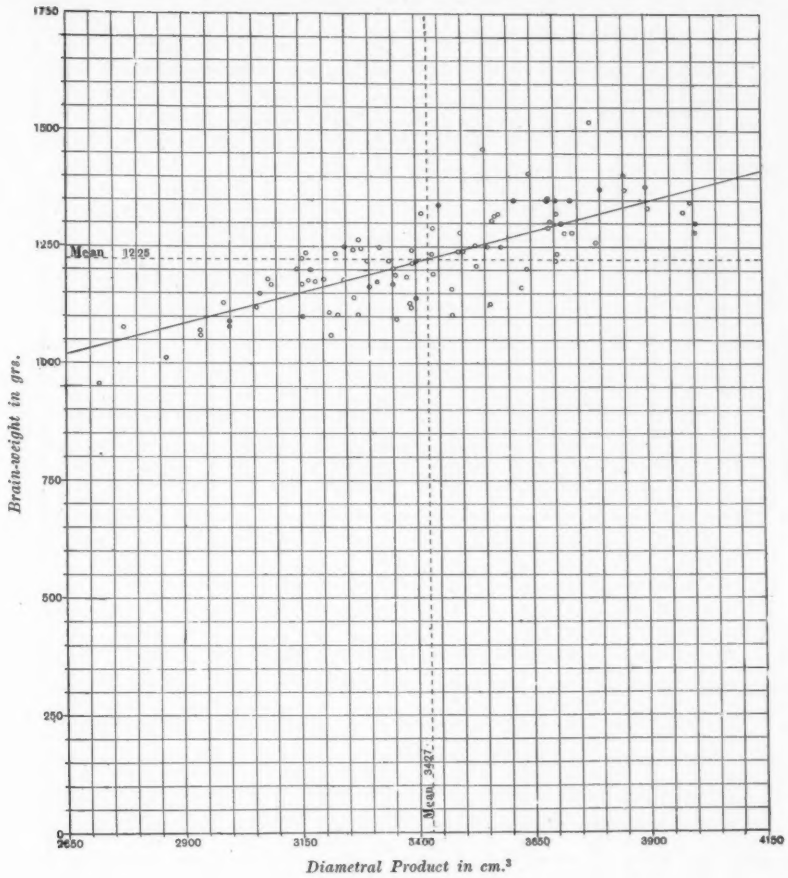


FIG. 5. Male Brain-weights and Circumferences.

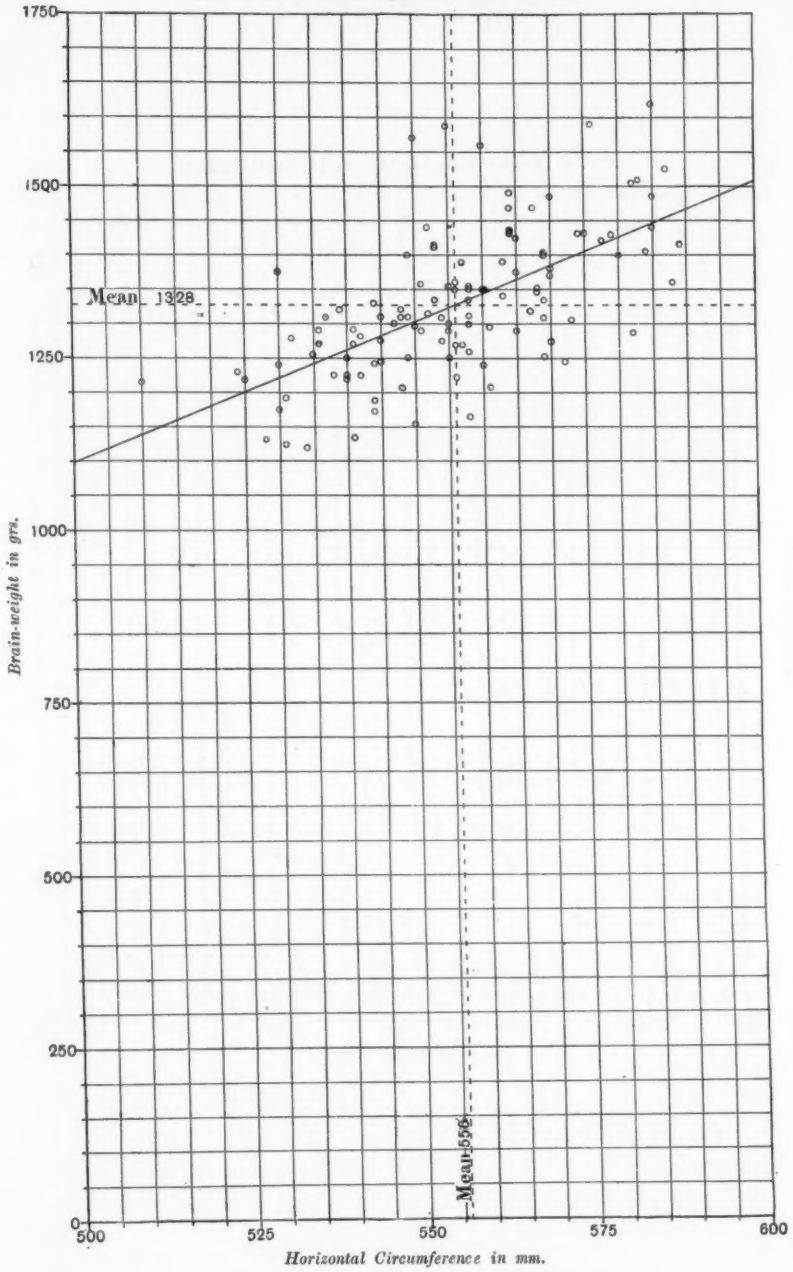
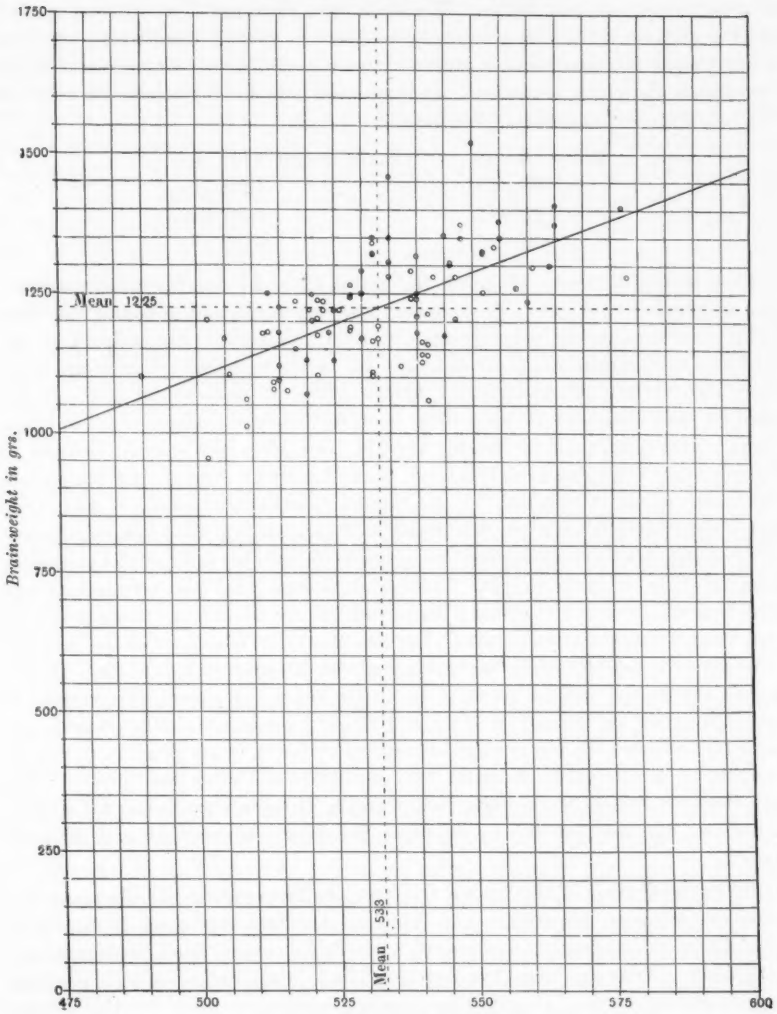


FIG. 6. Female Brain-weights and Circumferences.



(9) *On the Human Prime in the Case of Brain-weight.*

Pearl cites* Marchand as stating that the human brain-weight reaches its maximum between 15 and 20 years, and remains practically constant until 50, when senile degeneration begins. Gladstone cites Boyd and Vierordt to the effect that the maximum brain-weights are to be found in youth. Gladstone's own data certainly show this maximum in almost extreme youth, but the degeneration begins immediately the prime is reached. It is true the data are very scanty, but such as they are, they are suggestive. We have:

Males	8—19,	Brain-weight	1407 grs.
„	20—25,	„	1394 grs.
„	25—46,	„	1380 grs.
„	over 46,	„	1320 grs.

Even among the 8 to 19 group the group 12 to 15 has the largest average brain-weight, i.e. 1443 grs. The numbers are very small, and of course the cases of chronic illness in the older groups are more numerous. But Table I, p. 125 shows that even among the acute cases the fall in brain-weight owing to age alone is very substantial. Accordingly the human prime in brain-weight seems to fall before 20 years, and after 20 years there is, on the whole, a continuous fall in brain-weight. This result must be steadily kept in view when any attempt is made to correlate brain-weight and intelligence. It can hardly be argued that the human intellectual prime corresponds to the human brain-weight prime. And while brain-weight steadily changes, first rising and then falling with age, intelligence will be shown in a forthcoming memoir by one of our number to be almost stationary during the rise of brain-weight. Further, it is clear that the correlations and regressions will be much modified if we include in our results the rising period of brain-weight with the falling period. For this reason we have limited the material of this paper to cases in which the individual was 20 years and upwards. In this way our regressions will be much more nearly linear. A glance, for example, at Powys's diagram for change of stature with age† would, we think, convince the reader that without the use of skew correlation curves it is not possible to mingle observations on a physical character for ages before and after the "prime." We think that the source of the irregularity in the Hessian correlations for weight, stature, and age may be due to including individuals between 15 and 20 years of age‡.

The determination of the "prime" in every anthropometric measurement is a matter of very great importance and interest. Our own experience is that up to the prime we get a curved line of growth not widely divergent from the logarithmic in character. After the prime the degeneration is very gradual, and for practical purposes almost linear. It would be simple, therefore, to allow for this by age

* *Biometrika*, Vol. IV. p. 20.

† *Biometrika*, Vol. I. p. 47.

‡ Stature has a later prime than brain-weight, and this might well account for the irregular Hessian results in Pearl's Table IX. See p. 44 of this number of the Journal.

correlations, were it not that the prime appears to vary sensibly from one character to another. In other words, the "human prime" is purely relative to the special organ considered. There appears to be also no *period* of prime, but rather on the average an *instant* of prime, the time-graph of the character not being horizontal for any very sensible period. In the determination of the prime we ought to proceed by repeated measurements of the *same* series of individuals, and then reducing the individual time-graphs to a mean time-graph. This, of course, is absolutely impossible in the case of brain-weight and other internal organs. We are in these cases thrown back on the measurements of populations at different ages, the population at one age not consisting of the same individuals in whole or even in part as the population of another age. This method is admittedly defective, for it tacitly dismisses the whole question of natural selection*, and may lead us into strange pitfalls†; but it is often the only possible, and occasionally —since anthropometers will hardly work entirely for the future—the only practical method of approximating to the prime.

(10) We are able to a certain extent now to answer a problem which has much exercised anatomists. Has the woman *relatively* to the man less brain-weight, or is the absolute difference in brain-weight merely the result of the different sizes of the two sexes? There is, we think, only one proper method of answering this question. We must ask: What would be the brain-weight of a man having all his physical characters identical with that of the mean woman? Would he have more or less average brain-weight? Conversely, given a woman with all her physical characters identical with those of the mean man, would she have more or less brain-weight? Now unfortunately we do not know *all* the physical characters in either case, but we can judge from age, stature and the diametral product.

Using the multiple regression equation (i)^a we find that:

The Englishman of the same age, stature and diametral product as the mean woman has 1235 grs. brain-weight, or only 10 grs. more than the average woman.

Further using (1)^b we find that:

The Englishwoman of the same age, stature and diametral product as the mean man has 1315 grs. brain-weight, or only 13 grs. less than the average man.

But we are not certain that this even is the limit of difference because the woman reaches her prime rather sooner than the man and declines more rapidly. It would be really more reasonable to compare the man physically like the woman, when at his prime, and the woman physically like the man when at her prime. Brain-weight primes are so little determined at present that it seems impossible to do this. But we think that there is probably as in other physical

* The selective death-rate in man is probably 60 to 80 p. c. See *Biometrika*, Vol. I. p. 74.

† Pfitzner's conclusions drawn from the dead are probably to be largely discounted for this reason: see *Biometrika*, Vol. III. p. 465.

characters several years' interval between the two primes. If we change the problem and ask, what is the brain-weight of a man of *male mean age* equal to the mean woman in physical characters, we find it is 1230 grs. or only 5 grs. more than the brain-weight of the mean woman. On the other hand the brain-weight of a woman of female mean age equal to the mean man in physical characters is 1319 grs. or only 9 grs. less than the brain-weight of the mean man.

These differences are extremely slight, and would probably disappear entirely if we took another important cranial character like frontal breadth into consideration and gave the woman equal frontal breadth with the mean man and *vice versâ*. On the whole as far as present evidence goes, we can safely conclude that there is no sensible *relative* difference in the brain-weights of man and woman, the absolute differences observed are quite compatible with the differences which result from the relative sizes of the two sexes. In other words any argument as to the relative intellectual capacity of men and women based upon their absolute brain-weights applies equally to the thesis that the average big man has more intellectual capacity than the average small man, because he has on the average greater absolute brain-weight.

While our results thus apparently contradict those of Pearl on p. 51 of this Journal, the contradiction is only on the surface, for we have been able to use a far more complete system of physical measurements; and it is clear that the more measurements taken the closer is the approach to relative equality in the brain-weights of man and woman.

(11) *On the Capitulo-Statural Index.*

Gladstone has introduced a quantity equal to $100 \times \sqrt[3]{\text{Diametral Product}}/\text{Stature}$ or to $100P^{\frac{1}{3}}/S$ in our notation and termed it the capitulo-statural index*. He shows that it decreases with stature—an almost necessary result of its algebraic form. As he has not discussed it with special relation to the Middlesex Hospital measurements, it appears worth while to discuss some of its properties and correlations. We will represent it by the letter χ and take

$$\chi = P^{\frac{1}{3}}/S.$$

Clearly, since it involves no new measurement, all its properties can be deduced from the biometric constants already tabled.

We have at once $\delta\chi = \frac{1}{3}\overline{P}^{-\frac{2}{3}}\delta P/\overline{S} - \overline{P}^{\frac{1}{3}}\delta S/\overline{S}^2$,

whence squaring, summing and dividing by the number of individuals, we find:

$$\sigma_\chi^2 = \{\tfrac{1}{9}\overline{S}^2\sigma_P^2 + \overline{P}^2\sigma_S^2 - \tfrac{2}{3}\overline{P}\overline{S}\sigma_P\sigma_S r_{PS}\}/\overline{S}^4\overline{P}^{\frac{4}{3}},$$

or
$$V_\chi^2 = (100\sigma_\chi)^2/\chi^2 = \tfrac{1}{9}V_P^2 + V_S^2 - \tfrac{2}{3}r_{PS}V_S V_P \quad \text{.....(i).}$$

Again, multiply the value of $\delta\chi$ above by δS, sum and divide by number of individuals, and we have:

$$r_{\chi S} = (\tfrac{1}{3}r_{PS}V_P - V_S)/V_\chi \text{.....(ii).}$$

* *Biometrika*, Vol. IV. p. 115.

Here V_J denotes the coefficient of variation of any character J, or $V_J = 100\sigma_J/\bar{J}$, and $\bar{J}$ is a mean value.

For the practical purposes of the present inquiry we may take $\bar{\chi} = \bar{P}^{\frac{1}{3}}/\bar{S}$ although this is not absolutely exact*. As $\bar{P}$ is in cm.^{3} we must reduce $\bar{S}$ to cm. We then find:

TABLE XXII.

Capitulo-Statural Index.

Sex	Mean $= 100\,\chi$	Standard Deviation $= 100\sigma_\chi$	Coefficient of Variation $= \frac{100\sigma_\chi}{\bar{\chi}}$	Correlation with Stature	Regression with Stature
♂	9·1340 ± ·0255	·4095 ± ·0180	4·4828 ± ·1977	− ·8366 ± ·0187	− ·0442 ± ·0018
♀	9·4136 ± ·0273	·3924 ± ·0193	4·1686 ± ·2051	− ·7957 ± ·0255	− ·0441 ± ·0024

From the regression coefficients we at once find:

For ♂s:

$$\text{Capitulo-Statural Index} = 16{\cdot}6674 - {\cdot}04416S,$$

For ♀s:

$$\text{Capitulo-Statural Index} = 16{\cdot}4812 - {\cdot}04413S,$$

S being measured in centimetres.

Comparing this with Gladstone's Table on p. 116 of this volume of the Journal we have:

TABLE XXIII.

Capitulo-Statural Index for Different Statures.

Stature	Gladstone's Observed Values on Males	Values deduced from Post-mortem data
185·3 cms. ...	8·66	8·48
177·7 cms. ...	8·98	8·82
170·0 cms. ...	9·00	9·16
162·4 cms. ...	9·69	9·50
154·8 cms. ...	10·05	9·83

Probably the fair homogeneity of the post-mortem material gives sensibly better results, especially as it is smoothed by the correlation process, than the

* See *R. S. Proc.* Vol. 60, pp. 491, 492.

averages of Gladstone's groups, which were composed of post-mortem, work-house, and professional class measurements *.

Generally we see that the woman has a rather higher capitulo-statural index than the man; absolutely it is about equally variable, but its coefficient of variation is less. These coefficients, however, have very much the values which have been found for other bone measurements on man. The drop of this index per centimetre of stature is almost identical in male and female.

For anthropometric purposes the index appears to be of little value, for the change in the diametral product with stature is given at once by the regression coefficient tabled in Table XIX., where the reader will also find the influence of age and horizontal circumference on this product. In fact, the theory of linear correlation shows that we should expect $P^{\frac{1}{3}}$ for a given S to be closely of the form $P^{\frac{1}{3}} = C_1 S + C_2$, and accordingly if C_2 is positive we might not unnaturally anticipate that $P^{\frac{1}{3}}/S$ would decrease with S. Such an index would only be of value if C_2 were zero, and up to the present a wide experience shows that even a sensible approach to zero in C_2, if it ever does occur, is an occurrence of the most marked variety †.

(12) *On the Index = Ratio of Diametral Product to Brain-weight.*

What we have said at the conclusion of the previous article applies with a good deal of force to the index: Diametral Product/Brain-weight, which we will represent by the letter z. All the properties of it are involved in the correlations already tabled between the diametral product, the brain-weight, and the other characters. Still there is a physical side to this index which may be of some interest. It measures in a rough sort of way the ratio of volume to weight of enclosed brain-matter. We will investigate how far this index changes with age. All the requisite constants are given in Table XXIV.

The results were deduced, as in the case of the capitulo-statural index, from the formulae:

$$V_z^2 = (100\sigma_z/\bar{z})^2 = V_P^2 + V_w^2 - 2V_P V_w r_{Pw} \quad \text{.....(iii),}$$

$$r_{zA} = (V_P r_{PA} - V_w r_{wA})\, V_z \quad \text{.....(iv),}$$

* If we apply the above formula to male giants and dwarfs (see Gladstone, p. 117), we find:

Individual and Stature	Gladstone's Value	Formula
Irish Giant, O'Brian, 231·1 cms. ...	7·12	6·46
American Giant, Freeman, 205·7 cms.	7·77	7·59
Dwarf from Kiel, 121·9 cms. ...	11·15	11·29
Dwarf from Holstein, 97·8 cms. ...	13·78	12·35

but quite different values have been given by other authorities for these statures and accordingly no great stress can be laid on such divergences as occur between the formula and "observed" values.

† Cf. Lewenz and Pearson, *Biometrika*, Vol. III. pp. 373 and 374.

by aid of the known values of the coefficients of variation V_P, V_w (see Table II.), and the known correlations r_{Pw}, r_{PA}, and r_{wA} (see Table XIII.)*.

TABLE XXIV.

Index: Diametral Product/Brain-weight.

Sex	Mean	Standard Deviation	Coefficient of Variation	Correlation with Age	Regression Coefficient on Age
♂	2·8492 ± ·0106	·1703 ± ·0075	5·9771 ± ·2636	+ ·1153 ± ·0615	+ ·0016 ± ·0008
♀	2·7978 ± ·0113	·1628 ± ·0080	5·8189 ± ·2863	+ ·1150 ± ·0696	+ ·0014 ± ·0008

The mean value of the index is closely the same, 2·8 for both sexes. The difference of the variations is also within the difference due to random sampling. The influence of age is, within the limits of the probable errors, the same for both sexes, and for the sixty years covered by the data would not alter the index by as much as ·1 or by sensibly less than four per cent. We have thus the interesting result that while age tends to shrink diametral product and brain-weight its effect on their ratio is comparatively insignificant. It tends slightly to increase it, the shrinkage of the brain-weight being somewhat more than the shrinkage of the diameters. Turning to the regression coefficients in Table XIX., we see that the shrinkage of brain-weight in 60 years is 132 grs. on 1328 grs. average, or roughly 10 per cent., and the shrinkage in diametral product 247 cm.^{3} on 3783 cm.^{3} average, or roughly 6·5 per cent. We have thus the source of the 4 per cent. change in the index. It is, therefore, not surprising that the index increases and does not shrink like the simple characters with increasing age.

Supposing the index were absolutely constant, then equations (iii) p. 158 show us that it would only be a very inexact method of determining brain-weight from diametral product; for the constant terms are here 25 per cent. of the total probable brain-weights.

(13) *General Conclusions.*

While the writers of the present paper would admit that many of its conclusions are only tentative as the data are too sparse, they yet consider that they will be useful as suggesting lines for further research and as confirming and supplementing some of the results of Pearl's more elaborate study. They hold that:

(i) The biometric constants determined fit in reasonably well with the biometric results hitherto obtained for other brain-weight material.

* See footnote, however, to Table II.

(ii) They are, allowing for the differentiation of a general hospital population, consonant with what we know of other English anthropometric and craniometric material.

(iii) They show that a general hospital population differs in mean from other groups of the general population, but that the difference in variability and correlation is not very marked.

(iv) Important results would be reached if the change in physical characters with age were more completely studied; especially if the human prime for various physical measurements were more adequately determined.

(v) Allowance can be made for the effect of age and stature on brain-weight, but neither of these factors is nearly as important for brain-weight prediction as the knowledge of diametral product or horizontal circumference.

(vi) The probable brain-weight of an individual can be determined from stature, age, diametral product, and horizontal circumference with a mean error of about 50 grs. The mean deviation in brain-weight from the mean of the group is about 80 grs. In other words, we improve our prediction in the ratio of 8 to 5 by taking into account these characters. Nothing better can probably be achieved (see Figures 3 to 6) by introducing further external characters, or by considering regression as curved instead of plane.

(vii) Generally Pearl's conclusion that brain-weights for homogeneous material, uniformly measured, can be dealt with as satisfactorily as any other anthropometric characters is confirmed. We would add that the material ought to be restricted so that the highest prime of any character dealt with is the minimum age included in the data.

(viii) The apparent shrinkage of the diameters of the head with age is probably peculiar to general hospital material. Its absence in criminal data is significant. The subject, however, is at present being directly investigated by one of the present writers.

(ix) There is no sensible *relative* difference between the brain-weights of man and woman, when proper allowance is made for the relative difference in size of man and woman.

A QUANTITATIVE STUDY OF THE RESEMBLANCE BETWEEN MAN AND WOMAN.

By E. TSCHEPOURKOWSKY, Moscow.

In my studies on the cephalic index, which are partly published in Russian and partly in German journals, I affirm that sexual selection plays a very important *rôle* in the development of its varieties. Since then I have collected from different sources a certain amount of data for the study of interracial and sexual correlation, which can possibly give an answer to the following question: "If different characters of the human frame are inherited and if we admit that some of them are the result of natural selection, then the environment—in a wider sense—not being the same for both sexes we may expect that some characters are characteristic for one sex only. How far are they inherited in comparison with others by the opposite sex?" We may suppose *à priori* that the cephalic or nasal index has not the same importance in the struggle for life as for instance the arm length. If all kinds of characters are inherited equally by both sexes, the coefficients of correlation of their male and female values in different races must be equally high for them all.

In order to investigate this point I have used the data of various Russian authorities for living subjects; firstly, because they are collected in a very uniform manner, and secondly, because the diversity of races within the large territory of Russia is itself sufficient. The work of collecting the data was very much facilitated by the new compilation of Iwanowsky: "Anthropological Constituents of the Population of Russia." The raw material is represented in the tables given below in which the number of cases and the authors' names are also indicated. The numerical values for crania which I have used are partly extracted from different authors but mostly deduced from my own measurements made in the various anthropological museums of Europe. The cross-correlations can also be studied from these tables by anyone who wishes to do so*.

* See the remark on the next page.

The number of races is not very high, but the persistently high value of the coefficients of correlation and the fact that the measurement of men and women of the same race was made by the same investigator make the results very valuable.

TABLE I.

Constants for Interracial Variation and Correlation between Man and Woman.*

Source	No.	Character	Standard Deviation		Coefficient of Variation		Mean		Correlation	Sexual Ratio
			♂	♀	♂	♀	♂	♀		♀ to ♂ %
Iwanowsky	31	Stature	3·59	3·11	2·20	2·04	163·16	152·42	+·842	93·4
,,	33	Head Length ...	3·61	3·17	1·92	1·76	187·09	179·76	+·844	96·0
,,	45	Cephalic Index ...	2·59	2·54	3·16	3·10	81·71	81·93	+·875	100·2
,,	22	Face Index†	1·81	1·89	2·39	2·43	77·91	77·73	+·726	99·7
,,	25	Nasal Index	4·43	4·60	6·51	6·75	68·00	68·08	+·836	100·1
,,	22	Relative Arm Length‡	1·30	1·50	2·86	3·32	45·41	45·18	+·731	99·4
Crania ...	49	Cephalic Index ...	4·53	4·00	5·92	5·18	76·51	77·16	+·929	100·8
Means ...	—	—	2·73	2·60	3·12	3·07	—	—	+·826	—

TABLE I A.§

Character	Mean	Mean	Standard Deviation		Correlation	No.	Coefficient of Variation	
	♂	♀	♂	♀			♂	♀
Stature	164·00 ± ·42	152·28 ± ·39	4·10 ± ·21	3·76 ± ·19	·836 ± ·023	49	2·50	2·47
Head Length ...	187·11 ± ·37	178·36 ± ·33	3·76 ± ·26	3·36 ± ·23	·909 ± ·009	49	2·01	1·88
Relative Arm Length	45·82 ± ·29	45·41 ± ·27	2·00 ± ·20	1·85 ± ·19	·832 ± ·033	22	4·36	4·09
Cephalic Index ...	81·94 ± ·24	82·37 ± ·24	2·53 ± ·12	2·50 ± ·12	·869 ± ·017	43	3·08	3·03
Facial Index ...	77·35 ± ·29	77·94 ± ·29	2·04 ± ·29	2·06 ± ·29	·687 ± ·057	23	2·67	2·61
Nasal Index ...	68·00 ± ·64	68·01 ± ·60	4·96 ± ·36	4·66 ± ·40	·688 ± ·054	27	7·28	6·84

* In forming the correlation tables of this memoir as well as those of a second to be shortly published in *Biometrika*, I have sometimes used not all the given data but have chosen from the means of the same nationality only a few for groups which are geographically isolated.

† The relation of Bizygomatic Breadth to the total Face Length (from the top of the forehead to the chin).

‡ Ratio of the absolute arm length to stature.

§ I owe this table to the kindness of my friend M. Çekanowsky (Zürich-Warshawa), who formed (using my material quite independently from me) the correlation tables and calculated the coefficients with their probable errors. The number of cases is different, because he had not seen my tables—(and I excluded the means deduced from a small number), but the constants calculated by him are not very different from mine. This table can be used as control table. The mean resemblance (interracial) between man and woman is close to ·8.

We see firstly, that the coefficients lie between ·726 and ·929, and in the mean they are equal to ·826 (Table I.). Secondly, that in sexual resemblance there does not exist a large difference between characters which are apparently indifferent and those which can be presumed to play an important *rôle* in the struggle for life. This is shown also by the means of indices and relative lengths, which are nearly the same for both sexes. Two explanations of this fact are I think possible: 1. The racial proportions of the human frame have no special value in the struggle with environment, or they have the same value for both sexes. 2. The characters acquired in this struggle under the influence of natural selection by one sex are inherited by another.

If we admit that the racial characters of the human frame are evolved under the influence of an environment which is different for each sex, then why are they not characteristic for one sex only, i.e. why in the species *Homo* is the male not different from the female, as in many other species of animals and plants? It shows I think that in the formation of the different races of man many characters are probably the result of homogamic and endogamic instincts. From this point of view a race is very clearly defined by Karl Pearson as a population which by geographical, religious, or other causes has been isolated during a time sufficient for thorough intercrossing and for the work of selection. If that be so, the degree of resemblance between male and female of the same race is the product of this selection. Within the limits of one race the degree of resemblance between the closest relatives of the opposite sex is nearly ·4. If a part of the members of this race with a special deviation of any one character become isolated and form a new race, then the resemblance between their male and female elements increases to ·8. The difference ·4 is perhaps the measure of the work of selection, and the sexual correlation coefficient is the criterion of the formation of a new race.

From the Table I. it follows, that judged by coefficients of interracial variation woman is in the mean less variable than man; but the difference is very small. In three characters out of seven she is more variable than man. But in at least five of the seven cases the difference is not sensible, having regard to the probable error of the difference.

APPENDIX I.

TABLE II.

Cephalic Index-Skulls.

Nationality	♂	♀	Authority	Nationality	♂	♀	Authority
Whitechapel ...	74·7	75·0	Pearson *	Fijians	69·6	68·9	Flower, etc.
French Paris, City	79·2	78·0	„	Duke of York Islanders	70·9	71·8	Godefroi Collection
Italian	80·8	80·0	„	Australians	71·0	72·7	Different authors
Ancient Romans ...	78·1	78·7	„	Micronesians	74·4	75·0	„ „
Dutch	80·0	79·4	„	Lenftemberg	84·2	85·1	Niederle
Bavarians	83·4	83·1	„	Australians	68·8	71·4	Krause
Waichenfeld Graves	83·9	84·6	„	Czechs	83·2	83·6	Matiegka
Badeners	83·8	83·4	„	Australians	69·0	72·0	Challenger Expedition
Swiss	78·9	80·5	„	Wurtembergers ...	82·6	82·7	German Catalogue
Row-graves ...	73·7	74·1	„	Kalmuek Astrackan ...	81·6	81·0	Iwanowsky
Ancient Gaulish ...	78·4	75·4	„	Minousinsk	79·6	75·9	Goroshenko
Bretons	77·3	76·8	„	Thebans	75·1	76·5	Pearson
Romans	77·3	79·0	„	Bashkirs	81·0	81·0	Nikolsky
Friesians	77·7	79·0	„	Hottentots	71·4	74·9	Broca (registers)
Anglo-Saxons ...	75·0	75·0	„	Eskimo	71·4	71·8	„ „
Swedes	77·9	78·1	„	New Caledonians ...	71·3	74·4	Crania Ethnica
Etruscans	78·5	78·2	„	Hawaiians	75·5	78·4	„ „
Ancient Egyptians	75·1	76·5	„	Loyalty I.	69·8	71·8	„ „
Naqadas	73·2	74·6	„	Dzungarians	78·2	80·7	„ „
Ainos	76·5	77·7	„	Taithi	75·1	77·4	„ „
Negroes	74·3	74·8	„	Tasmanians	76·1	74·8	Broca
Panjabis	70·7	72·3	„	Chinese	77·9	76·5	„
Kanakas	79·3	80·3	„	Javanese	81·5	80·9	„
Andamanese ...	80·6	82·7	„	Marquesans	75·0	78·2	„
Torres Straits ...	68·3	70·1	Thomas				

* Extracted from "Variation in Man and Woman." *Chances of Death*, Vol. I. pp. 349—372.

APPENDIX II. TABLE III. *Living Races of Russia (Mean Values).*

Nationality	Stature		Head Index		Nasal Index		Head Length		Facial Index		Relative Arm Length		Authority
	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	
Aino	1471 (69)	1567 (91)	784 (71)	773 (95)	679 (54)	687 (87)	184 (71)	194 (85)	780 (55)	781 (73)	451 (66)	460 (90)	Koganei
Armenians	1529 (19)	1685 (231)	841 (49)	858 (292)	649 (48)	646 (291)	177 (49)	183 (292)	—	—	—	—	Chantre
Bielorussians*, Smolensk	1544 (275)	1652 (100)	825 (275)	802 (445)	711 (275)	694 (445)	179 (275)	190 (445)	789 (275)	763 (445)	—	—	Shadrowizky
„ Minsk	1539 (17)	1648 (57)	823 (17)	815 (57)	633 (17)	680 (57)	—	—	754 (17)	759 (57)	444 (10)	447 (57)	Rosdestwensky
Bouriats	—	—	807 (40)	824 (100)	686 (40)	725 (100)	182 (40)	188 (99)	—	—	442 (40)	445 (100)	Porotow
Great Russians, Moskwa	1528 (411)	1644 (647)	—	—	—	—	—	—	—	—	—	—	Rosdestwensky
„ „ Kazan	1530 (100)	1651 (325)	840 (100)	815 (325)	—	—	177 (100)	188 (325)	775 (100)	771 (323)	—	—	Worobiew
„ „ Twer	1528 (100)	1660 (200)	833 (100)	830 (200)	674 (100)	697 (200)	172 (100)	179 (200)	750 (100)	746 (200)	464 (100)	464 (200)	Galay
„ „ Kursk	1525 (50)	1640 (235)	810 (50)	817 (235)	—	—	178 (50)	185 (235)	784 (50)	775 (235)	454 (50)	462 (235)	Lokine
„ „ Semiriechensk (Siberia)	1569 (37)	1671 (51)	822 (36)	813 (51)	—	—	179 (36)	187 (51)	—	—	—	—	Zealand
Jews, Warchau	1506 (125)	1610 (200)	829 (125)	819 (200)	607 (100)	620 (147)	176 (25)	184 (200)	757 (100)	737 (147)	—	—	Elkina
„ Mohilew	1507 (100)	1617 (139)	807 (100)	817 (86)	683 (86)	600 (70)	178 (80)	183 (139)	748 (48)	762 (148)	449 (43)	448 (27)	Jakowenko
„ Kiew	—	—	830 (206)	829 (438)	—	—	—	—	—	—	—	—	Talko-Grinzewitch
„ South of Russia	—	—	824 (50)	824 (100)	—	—	176 (50)	183 (100)	—	—	—	—	Weissenberg
„ Different Goubernians	—	—	825 (23)	808 (51)	—	—	—	—	—	—	—	—	Jkow
Zirians	1537 (21)	1628 (97)	836 (16)	822 (20)	648 (13)	647 (98)	—	—	756 (13)	764 (98)	442 (93)	431 (98)	Sommier
Kalmueks, Astrackan	1504 (19)	1635 (30)	826 (30)	811 (96)	746 (30)	734 (47)	181 (30)	189 (96)	800 (30)	793 (85)	456 (9)	460 (93)	Korolew
„ „	—	—	833 (13)	821 (36)	—	—	—	—	—	—	—	—	Sommier
Karagass	—	—	—	—	—	—	—	—	—	—	430 (10)	471 (20)	Korolew
Karakirgiz	—	—	897 (10)	873 (40)	775 (10)	762 (40)	—	—	—	—	446 (10)	453 (39)	Zealand
Karaims	—	—	856 (10)	848 (20)	590 (10)	622 (20)	—	—	—	—	—	—	Weissenberg
Kirgiz Sredni Orda	1505 (30)	1651 (99)	863 (30)	894 (99)	722 (30)	718 (99)	176 (30)	181 (99)	830 (30)	804 (99)	451 (30)	445 (99)	Iwanowsky
	—	—	—	—	—	—	—	—	—	—	423 (10)	419 (31)	Tronow
Courds from Transkaukasian	—	—	770 (60)	776 (300)	626 (60)	623 (302)	182 (60)	189 (30)	760 (60)	752 (300)	451 (60)	453 (300)	Iwanowsky
„ „ „	—	—	764 (15)	762 (67)	648 (15)	658 (97)	—	—	—	—	—	—	Chantre
„ Turkmens	—	—	776 (34)	785 (140)	702 (34)	677 (140)	183 (34)	193 (140)	—	—	—	—	Chantre
„ from Syria	—	—	825 (11)	825 (35)	718 (11)	728 (35)	—	—	—	—	—	—	Chantre
Courtins (Transkaukasia)	1524 (60)	1686 (300)	—	—	—	—	—	—	—	—	—	—	Iwanowsky
Latish	1563 (40)	1705 (60)	796 (40)	805 (60)	636 (40)	638 (60)	185 (40)	190 (60)	—	—	444 (40)	449 (60)	Waeber
Lituanians	1548 (70)	1659 (130)	835 (40)	818 (60)	711 (40)	729 (60)	184 (40)	191 (60)	—	—	467 (40)	469 (60)	Brennsohn
„	—	—	825 (30)	827 (70)	—	—	178 (30)	184 (70)	777 (30)	752 (70)	468 (30)	472 (70)	Baronas
Little Russians, Chernigor	—	—	836 (40)	856 (120)	716 (40)	716 (120)	175 (40)	182 (120)	781 (40)	778 (120)	—	—	Bielodied
„ Woronief	1529 (48)	1632 (102)	805 (48)	811 (102)	—	—	179 (48)	187 (102)	786 (48)	772 (102)	—	—	Koshuchow
Morawa	1546 (75)	1643 (162)	—	—	686 (75)	667 (162)	181 (75)	187 (162)	—	—	489 (75)	472 (162)	Maïnow
Ossetins	1573 (60)	1690 (30)	819 (60)	815 (300)	—	—	182 (60)	190 (300)	777 (60)	813 (120)	443 (60)	449 (120)	Riskine
Ostiaks	1441 (27)	1563 (95)	790 (30)	792 (95)	—	—	185 (30)	193 (95)	—	—	—	—	Sommier
Poles, Warchau	1533 (144)	1640 (191)	813 (149)	808 (226)	609 (149)	633 (226)	177 (49)	185 (223)	783 (145)	768 (207)	—	—	Elkind
„ Lublin	1537 (83)	1655 (116)	836 (85)	827 (116)	734 (85)	729 (100)	178 (85)	186 (116)	772 (85)	753 (100)	458 (50)	459 (100)	Tezinsky
„ Radom	1531 (59)	1617 (131)	827 (59)	830 (131)	—	—	179 (59)	186 (131)	—	—	—	—	Olechnowiz
„ Lublin *bis*	—	—	845 (36)	837 (60)	—	—	181 (36)	190 (60)	—	—	—	—	Olechnowiz
Samoieds	1430 (46)	1550 (84)	839 (38)	844 (66)	—	—	179 (39)	189 (68)	—	—	449 (10)	453 (21)	Sommier
Soïots	—	—	826 (20)	830 (72)	—	—	—	—	—	—	—	—	Goroshanko
Tatars, Kazan	1521 (37)	1645 (206)	818 (37)	821 (206)	—	—	179 (37)	186 (207)	774 (36)	786 (205)	—	—	Warushkine
„ Azerbeisan	—	—	767 (18)	769 (114)	628 (18)	642 (114)	—	—	—	—	—	—	Chantre
Finns, Wiborg	1545 (100)	1666 (150)	819 (100)	804 (150)	667 (100)	812 (150)	181 (100)	191 (150)	771 (100)	768 (150)	—	—	Kolmogorow
„ Tawastgouse	1552 (31)	1679 (22)	833 (31)	823 (24)	—	—	181 (31)	186 (24)	793 (30)	794 (24)	—	—	Retzius
Cheremiss	1517 (45)	1615 (48)	767 (29)	790 (48)	—	—	182 (29)	184 (48)	—	—	—	—	Malieff
„	—	—	802 (16)	794 (38)	—	—	—	—	—	—	—	—	Sommier
Jakuts	1509 (105)	1617 (324)	809 (62)	827 (207)	—	—	186 (62)	191 (207)	789 (62)	792 (207)	457 (50)	461 (100)	Maïnow
Tasiks	1541 (20)	1693 (392)	—	—	—	—	—	—	—	—	—	—	Maslowsky
Jakuts *bis*	—	—	829 (15)	823 (46)	—	—	—	—	—	—	471 (16)	475 (45)	Witashewsky

* White-Russians.

APPENDIX III.

CORRELATION TABLES.

I. *Stature.*

Female.

Male.	*143*	*144*	—	*147*	*148*	*149*	*150*	*151*	*152*	*153*	*154*	*155*	*156*	*157*	*158*	Totals
155	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
156	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
157	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
162	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
163	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	3
164	—	—	—	—	—	—	—	—	2	2	—	1	—	—	—	5
165	—	—	—	—	—	—	1	—	—	1	3	—	—	—	—	5
166	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	2
167	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	2
168	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	2
169	—	—	—	—	—	—	—	—	1	—	1	—	—	1	—	3
170	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	1	1	—	1	—	—	2	3	4	7	6	3	1	2	—	31

II. *Cephalic Index-Skulls.*

Male.

Female.	*68*	*69*	*70*	*71*	*72*	*73*	*74*	*75*	*76*	*77*	*78*	*79*	*80*	*81*	*82*	*83*	*84*	*85*	*86*	Totals
69	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
70	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
71	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
72	—	1	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
73	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
75	—	—	—	1	—	1	2	2	1	—	1	—	—	—	—	—	—	—	—	8
76	—	—	—	—	—	—	—	2	—	1	1	—	1	—	—	—	—	—	—	5
77	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
78	—	—	—	—	—	—	—	2	1	—	1	1	1	—	—	—	—	—	—	6
79	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	3
80	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
81	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	—	—	4
82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
83	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	3
84	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
85	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	2
Totals	1	3	1	6	—	1	3	7	2	2	6	3	3	4	1	3	3	—	—	49

III. *Head Index, Living Subjects.*

Female.

Male.	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	Totals
75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
77	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
78	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
79	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	3
80	—	—	—	—	—	1	—	2	—	—	—	—	—	—	—	—	3
81	—	—	—	—	—	1	1	4	1	1	—	—	—	—	—	—	8
82	—	—	—	—	—	—	3	3	5	1	—	—	—	—	—	—	12
83	—	—	—	—	—	—	1	1	4	1	—	—	—	—	—	—	7
84	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
85	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
86	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
87	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
88	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
89	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
90	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	—	1	3	2	1	3	5	10	10	7	—	2	—	—	—	1	45

IV. *Nasal Index.*

Female.

Male.	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	Totals
60	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
63	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
64	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
65	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
66	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
67	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
68	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
69	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
70	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
72	—	—	—	—	—	—	—	—	—	—	1	—	—	2	—	—	—	—	—	3
73	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	3
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	2	—	3	1	2	—	1	2	2	2	2	3	1	1	1	—	1	25

V. *Head Length.*

Female.

Male.	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	Totals
179	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
180	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
181	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
182	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
183	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	3
184	—	—	—	—	1	—	1	—	—	—	1	—	—	—	—	—	3
185	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2
186	—	—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	4
187	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	3
188	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	2
189	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	3
190	—	—	—	—	—	—	—	1	—	1	1	—	—	1	—	—	4
191	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	3
192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
193	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	2
194	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
195	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
196	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	—	1	3	3	4	6	—	5	4	1	2	2	1	—	33

VI. *Facial Index.*

Female.

Male.	74	75	76	77	78	79	80	—	83	Totals
74	—	—	1	—	—	—	—	—	—	1
75	—	1	1	1	1	—	—	—	—	4
76	—	2	1	—	—	1	—	—	—	4
77	—	—	—	2	2	1	—	—	—	5
78	—	—	—	—	1	—	1	—	—	2
79	—	—	—	1	—	2	1	—	—	4
80	—	—	—	—	—	—	—	—	1	1
81	—	—	—	—	1	—	—	—	—	1
82	—	—	—	—	—	—	—	—	—	—
83	—	—	—	—	—	—	—	—	—	—
Totals	—	3	3	4	5	4	2	—	1	22

VII. *Relative Arm Length.*

Female.

Male.	42	43	44	45	46	47	48	49	Totals
42	1	—	—	—	—	—	—	—	1
43	—	—	1	—	—	—	—	—	1
44	—	—	1	1	—	—	—	—	2
45	—	—	3	4	—	—	—	—	7
46	—	—	—	2	4	—	—	—	6
47	—	1	—	—	—	3	—	1	5
48	—	—	—	—	—	—	—	—	—
49	—	—	—	—	—	—	—	—	—
Totals	1	1	5	7	4	3	—	1	22

"DAS FEHLERGESETZ UND SEINE VERALLGEMEINERUNGEN DURCH FECHNER UND PEARSON*." A REJOINDER.

By KARL PEARSON, F.R.S.

THERE is not much profit as a rule in complaining of the treatment one receives at the hands of critics, but still I think the needlessly hostile tone of Dr K. E. Ranke and Dr Greiner's review of my memoir on skew variation requires some protest on my part. As an illustration of the want of courtesy of which I complain I would cite for example the following statement (p. 323):

> Er (Pearson) weist aber einmal darauf hin, dass zwar seine eingeschriebenen Kurven teilweise unbegrenzt werden können, das Gesetz selbst, die hypergeometrische Reihe aber nicht, und vertröstet auf eine spätere Arbeit, in der an Stelle der eingeschriebenen Kurven die Anpassung der Reihen selbst gegeben werden solle. Diese spätere Arbeit ist nie geschrieben worden.

This is not an isolated instance of the manner in which the authors criticise my work. It was quite open to them to have examined the customary sources of bibliographical information, or even to have written to me and asked if the memoir in question had been published. But here as elsewhere they assumed, without making proper investigation, that I could say nothing further and therefore had said nothing further. The memoir in question appeared so long ago as 1899 in a well known British scientific journal† from which the authors actually cite another paper of mine. Although my memoir is *nie geschrieben*

* The criticism of my work appeared in the *Archiv für Anthropologie*, Bd. II. pp. 295—331, 1904. The proper place to reply to an attack of the kind would be in the *Archiv* itself. Professor J. Ranke accepted a rejoinder and asked that it should be in German and not exceed 40 pp. I have heartily to thank Miss M. Lewenz for the labour of a translation, which I much regret I cannot make use of, because the Editors of the *Archiv* do not now see fit to publish this reply to K. E. Ranke's attack. As the reply was an endeavour to give an historico-critical account of the theory of skew variation it may interest readers of *Biometrika*, and will possibly reach in the course of time some readers of the *Archiv für Anthropologie*.

† "On certain Properties of the Hypergeometrical Series, and on the fitting of such series to Observation Polygons in the Theory of Chance," *Phil. Mag.* 1899.

worden according to my critics, it was actually written in 1895 and its substance given in academic lectures of the same or the following year; it was not published till some years afterwards owing to that want of leisure for preparing matter for press which every teacher who has to lecture four hours a day will appreciate. Dr Ranke says that had the paper been written, it would not have influenced his judgment. That is quite possible, and I only cite the matter here to indicate the tone adopted by my critics.

A very similar instance occurs in Dr K. E. Ranke's treatment of my fitting of Professor J. Ranke's data for 900 Altbaierisch crania. I used this example purposely because it had already been used by Stieda. I should not myself have mixed, even for the cephalic index, ♂ and ♀ data as Stieda did, but I wished to compare the results reached by the generalised curve with those reached by the Gaussian curve. I actually spoke of the resulting curves in my memoir, whether the generalised curve or the Gaussian, as being "quite good for this type of statistics*." My object of course was to show that the generalised method did not fail where the Gaussian succeeded, but surpassed it. Now how does Dr Ranke treat this instance? He cites an example (actually inserted† by me!) in a memoir by Palin Elderton giving the Ranke'sche Messungen as an illustration of my method of testing goodness of fit in the case of the normal curve. Undoubtedly as I said in 1894 the Gaussian curve is quite good for J. Ranke's data, but it does not follow that the Type IV. frequency curve does not give a *better fit*, and is not significant for constants which the Gaussian process cannot deal with. Now it was open to Dr Ranke to test the values given for the distance from mode to mean, the skewness and the other constants in the case of the Altbaierisch crania. I have given the probable errors of these constants in my memoir: *On the Mathematical Theory of Errors of Judgment*, etc., *Philosophical Transactions*, Vol. 198 A, see p. 278. Had Dr Ranke fairly tested my results he would have found that the asymmetry was not significant, and that the mode sensibly coincided with the mean, but that the constant β_2, which should equal 3 for the normal curve, has a value 3·65 with a probable error of only about ·11. Now this constant and its probable error have no relation at all to any particular theory of variation. They follow quite easily from the general Gaussian theory. It is accordingly extremely improbable that Ranke's measurements are truly given by a Gaussian distribution in all their features. The fact that β_2 is >3 points to an emphasis

* p. 389.

† The same remark applies to the illustrations of goodness of fit given by Fawcett, cited in footnote Ranke u. Greiner, p. 326. The reference to Powys is inexact; his paper shows that in at least three cases the Gaussian curve is quite impossible. Dr Macdonell's work on the English skull shows that at least in 4 out of 13 cases the asymmetry is significant. "Die englische Schule," by which Dr Ranke refers to workers in my Biometric Laboratory has not discovered a truth which had escaped me; they have shown that the Gaussian curve is of wide applicability, but not of universal truth in anthropometric measurements. This result was reached with a view to testing whether the theory of inheritance, so far as it is based on the Gaussian theory, might be safely applied to human characters. In testing this validity of the Gaussian theory, it was of course needful to have a more general theory from which to determine the chief physical constants involved in non-Gaussian distributions.

of the modal frequency and to a reduction of the extreme frequencies which are inconsistent with the Gaussian curve. This is actually shown in my Plate 11 and referred to in my text, and corresponds to a sensible deviation from the Gaussian law. It was open to Dr Ranke to attribute this exaggeration of the modal at the expense of the extreme variates to heterogeneity in the material. But he had no right when the material for a judgment was before him in my memoirs to conclude that, because the *general* distribution of frequency was not on the average incompatible with the Gaussian law, the deviation of a particular constant of the distribution from its Gaussian value might not be most significant. This case of the deviation of the Ranke's measurements from the Gaussian form is of special interest, for it is not one of asymmetry, but of a non-Gaussian type of symmetry. Dr Ranke suggests that because my test of goodness of fit shows that the Gaussian curve is "quite a good fit," my generalised method of dealing with frequency is idle. Assisted by a mathematician he ought to have recognised that the expression $\beta_2 - 3$ (which measures whether the frequency towards the mean is emphasised more or less than that required by the Gaussian law) had a sensible value, and that my method not only led to the discovery of this deviation but provided a method of allowing for it in the description of the frequency. Had Dr Ranke read my memoir on errors of observation (*Phil. Trans.*, Vol. 198 A, pp. 274—286), he would have recognised that the two tests (*a*) whether special physical constants of the distribution satisfy the Gaussian law, and (*b*) whether the general distribution of frequency satisfies within reasonable limits the Gaussian law, are not necessarily identical. Finally had he concluded that (*b*) for the Altbaierisch crania was satisfied, but not (*a*), and that there was thus no necessary discrepancy between my memoir of 1894 and the statement in Palin Elderton's paper of 1900, he might indeed have fallen back on his customary assumption that when frequency is not Gaussian it is heterogeneous. But at any rate had he adopted this course he would have avoided the appearance of criticising his author without endeavouring to understand what the meaning of his investigations was, or striving to elucidate them by a study of his other memoirs on the same subject.

I do not wish to say anything further on this point. I want merely to indicate by these two out of several cases that the reader must not look for a really impartial statement of my position from Drs Ranke and Greiner. I am quite unable to account for the peculiar tone they have at times given to their criticism.

(2) With this preamble I should like to divide my reply under these headings:

(A) The need for generalised frequency curves, even in anthropological science.

(B) The nature of the assumptions made in the Gaussian theory and their insufficiency.

(C) The hypotheses made to generalise the Gaussian law.

(i) Poisson, Laplace and the early writers.

(ii) Francis Galton and D. McAlister.

(iii) Fechner.

(iv) Edgeworth and Kapteyn.

(v) General Results for Asymmetry*.

(D) The criticisms of my theory by Dr Ranke and Dr Greiner and the reply to be made to them.

A. *The need for Generalised Frequency Curves, even in Anthropological Science.*

I have already pointed out that even Prof. Ranke's measurements are not fully in accordance with the Gaussian theory—for the odds are great against a quantity exceeding its probable error more than five times. It is perfectly true that the English School have found that many characters, especially craniological characters, are for *practical* purposes sufficiently described by the Gaussian curve. But it is equally true that they have found other cases in which the deviation from the Gaussian curve is significant, and that they have only been able to measure this significance because they had a wider theory to base their researches upon. Dr Ranke entirely disregards the statements of Miss Fawcett and Dr Macdonell on this point. Both find a definite number of cases, the one in Egyptian skulls, the other in English skulls, in which the deviation from the Gaussian law is definitely significant†. Both conclude as I have done that in the case of many characters for a variety of practical purposes the Gaussian curve is sufficient; this is, however, not a *theoretical* justification of the Gaussian curve, but an argument in favour of its empirical use in a certain definite number of cases. Dr Ranke may of course say that the exceptions that we have found are due to heterogeneity of our material. If so he must face the difficulty that the *same* set of crania can be homogeneous and give the Gaussian curve for their length and be heterogeneous for their breadth, deviating therein largely from the Gaussian curve. If he asserts that this is quite possible then he must meet the further difficulty that they can be homogeneous for their cephalic indices, which are based upon the ratio of the supposed heterogeneous to the homogeneous material! The fact is that no unprejudiced observer can examine the constants by which we have defined the deviations from the Gaussian law without seeing that they present every variety of value, starting from the values to be expected on the Gaussian theory and rising to values which are absolutely incompatible with any Gaussian theory at all. In fact he must come to the conclusion that some theory is absolutely needful, which will provide a curve or series of curves capable of representing the

* I have left out of consideration the general method of Thiele, followed in Germany by Lipps, because I have dealt with these authors in a recent memoir.

† *Biometrika*, Vol. I. p. 443, and *Biometrika*, Vol. III. p. 227.

fundamental deviations of any distribution from the Gaussian curve and determining whether these deviations are significant or not. Looked at solely from this standpoint—which I am very far from accepting—my curves provide an empirical series which accurately measures the deviations from the Gaussian law and enables the enquirer to determine how far that law is applicable. Each one of them passes into the Gaussian curve if that curve is the better fit to the observations. This is not true of many of the other remedies which have been proposed to supplement what I venture to call the universally recognised inadequacy of the Gaussian law. They cannot as we shall see in the sequel effectively describe the chief deviations from the Gaussian distribution.

The chief physical differences between actual frequency distributions and the Gaussian theoretical distribution are:

(i) The significant separation between the mode or position of maximum frequency and the average or mean character.

(ii) The ratio of this separation between mean and mode to the variability of the character—a quantity I have termed the *skewness*.

(iii) A degree of flat-toppedness which is greater or less than that of the normal curve. Given two frequency distributions which have the same variability as measured by the standard deviation, they may be relatively more or less flat-topped than the normal curve. If more flat-topped I term them *platykurtic*, if less flat-topped *leptokurtic*, and if equally flat-topped *mesokurtic*. A frequency distribution may be symmetrical, satisfying both the first two conditions for normality, but it may fail to be *mesokurtic*, and thus the Gaussian curve cannot describe it.

The Gaussian curve is usually fitted from the mean square deviation, but it may also be fitted from the probable error, or the mean error, or again from the mean fourth power of the deviations—μ_4 in my notation. Whichever method is adopted we ought to get the same result within the errors of random sampling. When I first began to describe frequency data by the normal curve, I was startled to find the very large number of cases in which these different processes led to Gaussian curves, differing widely from one another, i.e. beyond all the limits of probable error. I was soon led to see that in actual statistics two distributions might have equal total frequency, be sensibly symmetrical, and have the same standard deviation and yet differ largely in their flat-toppedness. The mesokurtosis of the Gaussian curve is not a universal characteristic of frequency distributions.

When we test a theoretical distribution of frequency against observation, we may find an excellent fit for the *total* distribution and yet the distinction between mode and mean, the skewness, and the deviation from mesokurtosis may be most significant. The reason for this is that the test for goodness leaves a margin of variation which may be due to random sampling, or to the non-normal character of an important constant of the distribution. For example, 10 coins are tossed a hundred times, and the proportion of cases with five and more heads is somewhat

in excess of the theoretical distribution $100\,(\frac{1}{2}+\frac{1}{2})^{10}$, but within the limits of a random sample. It is quite conceivable that if the returns for each individual coin were analysed it would be found that those of one exceeded in proportion of heads the limits of random sampling, and that the coin proved to be loaded when delicately tested. Thus as I have shown in my memoir on errors of observation, we have not only to test for general goodness of fit, but also to consider the probable errors of the fundamental constants of the distribution. Because the general distribution of frequency is given within the limits of random sampling by a normal curve it does not follow that the system will be mesokurtic.

Consider for example the two curves:

$$y = y_1 \left\{1 - \frac{x^2}{2\sigma^2 (m_1 + \frac{1}{2})}\right\}^{m_1 - 1}, \qquad \text{where:} \quad y_1 = \frac{N}{\sqrt{2\pi}\sigma} \frac{\Gamma(m_1 + \frac{1}{2})}{\sqrt{(m_1 + \frac{1}{2})}\,\Gamma(m_1)},$$

and

$$y = y_2 \left\{1 + \frac{x^2}{2\sigma^2 (m_2 - \frac{1}{2})}\right\}^{-(m_2+1)}, \qquad \text{where:} \quad y_2 = \frac{N}{\sqrt{2\pi}\sigma} \frac{\Gamma(m_2 + 1)}{\sqrt{(m_2 - \frac{1}{2})}\,\Gamma(m_2 + \frac{1}{2})}.$$

They are both symmetrical, they both for any value of m_1 or m_2 which is moderately large are indistinguishable in appearance from the Gaussian curve. If they represented actual observations, we should try to fit them (i) by finding the area, (ii) by finding the standard deviation. The former for both curves is N and the latter for both curves is σ. Hence we should fit them with

$$y = \frac{N}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \qquad \text{.....(i).}$$

But this in both cases would be incorrect. Both cases would only pass into the Gaussian curve when m_1 and m_2 are theoretically infinite, practically large. No Gaussian fitting could distinguish one of these curves from the other. Why?—Because it does not proceed further than the standard deviation. To measure the difference between either of the above distributions and the Gaussian curve we must proceed to higher moments. Let $N\mu_n$ be the nth moment about the mean, i.e. if $\bar{x}$ be the mean value of x,

$$N\mu_n = \int y\,(x - \bar{x})^n\,dx,$$

where the limits of the integral are those of the range. Then if $\beta_2 = \mu_4/\mu_2^2$, we easily find:

$$m_1 = \frac{3\,(\beta_2 - 1)}{2\,(\beta_2 - 3)} \quad \text{and} \quad m_2 = \frac{3\,(\beta_2 - 1)}{3 - \beta_2},$$

or

$$\beta_2 - 3 = \frac{6}{2m_1 - 3}, \qquad 3 - \beta_2 = \frac{6}{2m_2 + 3}.$$

Thus we reach one of the conditions for the Gaussian curve, i.e. $\beta_2 = 3$, in either case when m_1 and m_2 are considerable, but if β_2 be > 3, m_1 will be positive and if $\beta_2 < 3$, m_2 will be positive. Now since $\dfrac{m_1 - 1}{m_1 + \frac{1}{2}}$ is always less than $\dfrac{m_2 + 1}{m_2 - \frac{1}{2}}$, it is easy

to show that in the neighbourhood of the origin y/y_1 is always greater than y/y_2 for the same value of x. In other words, the first curve is flatter topped than the second, and both lie on different sides of the corresponding Gaussian curve. The first curve type is platykurtic and the second leptokurtic.

Now there is nothing to prevent us fitting curves of the above types to any series of frequency observations. Supposing those observations are truly normal, then m_1 or m_2 will be so large that $\beta_2 = 3$ within the error of random sampling. Now the probable error of β_2 for a Gaussian distribution of total frequency N*:

$$= \cdot 67449 \sqrt{\frac{24}{N}},$$

and if β_2 differs from 3 by several times this probable error, it is absolutely impossible to treat the system as mesokurtic. In any such case one or other of the above curves *must* give a truer representation than the Gaussian curve. It is easy to show that for leptokurtic distributions the maximum frequency is greater than that given by the normal curve and for platykurtic distributions it is less. The Gaussian curve compels us to assert that the product of the maximum frequency into the standard deviation is a constant (i.e. $y_0\sigma = N/\sqrt{2\pi}$). This condition of mesokurtosis is unfulfilled—within the limits of random sampling—for a great variety of frequency distributions.

Further it is absolutely certain that divergencies from the Gaussian or normal curve are not exclusively in the direction of either platykurtic or leptokurtic distributions. Thus the *symmetrical* binomial is essentially leptokurtic, i.e. $\beta_2 < 3$, and therefore cannot be used for a great variety of distributions. In general all skew binomials with $p > \cdot 2113$ and $< \cdot 7887$ are leptokurtic; outside these limits they are platykurtic.

The test whether a curve satisfies the mesokurtic condition has nothing to do with my particular views on frequency, it is merely deduced from the general principles of probability and is a test of normal distribution. Of course there are many other conditions to be satisfied, e.g. μ_{2n} should equal $(2n-1)\mu_{2n-2}$. But as I have shown elsewhere the probable errors of the high moments increase so rapidly, that it becomes easier and easier to satisfy such conditions within the errors of random sampling, and without very large numbers they are of little practical value.

The following are significantly platykurtic distributions:

The Maximum Breadth in English ♂ skulls,
The Nasal Breadth in English ♂ skulls,
The Cephalic Index of Altbaierisch skulls,
The Auricular Height in ♀ Naqada skulls.

* Pearson: *Phil. Trans.* Vol. 198 A, p. 278.

As a rule the data available in craniological investigations are too sparse to give any real test of mesokurtosis, and this is the true reason why we must content ourselves with the Gaussian curve.

Again Mr Powys found out of twelve frequency distributions for the stature of men and women that 11 were leptokurtic and the twelfth essentially mesokurtic. This tendency to leptokurtic distributions—which can hardly be due to chance—is actually given by Ranke and Greiner as a case in favour of the Gaussian curve! (*Anmerkung* S. 326). They further cite Fawcett and Lee in the following manner:

> In der an letzter Stelle zitierten Arbeit ist der Nachweis einer bestimmt gerichteten Asymmetrie für die Mehrzahl der Masse und zwar in der nach Fechner zu erwartenden Richtung besonders beachtenswerth.

They do not say that of the 24 curves given by Fawcett and Lee 14 are leptokurtic and that Fechner's curve can only represent platykurtic distributions. They do not draw attention to the fact that the Fechner curve would be impossible for the whole of Powys' stature data, and for 12 out of Macdonell's 26 curves for the English skull! In other words, if the *Abweichungen* of Fawcett and Macdonell and Powys' data are to be used as an argument at all, 38 out of these 62 distributions diverge from the normal curve in a manner which cannot possibly be represented by Fechner's theory!

If we turn from the condition for mesokurtosis to those for differentiation of mode and mean and for skewness we meet other considerations. So far we are not dependent for anything we have said on any theory of frequency other than the Gaussian. On that theory $\beta_2 = 3$. and if the difference $\beta_2 - 3$ be significant the distribution cannot be Gaussian. If we want to distinguish between the mode and the mean, we cannot start from the Gaussian theory, because that theory supposes the two values absolutely the same. On the other hand if we consider asymmetry, we ought to have, within the limits of random sampling, all the odd moments zero, i.e.

$$\mu_3 = \mu_5 = \mu_7 = \ldots = \mu_{2n-1} = 0.$$

Now it is of very little practical value testing the high moments because their probable errors are excessive. The probable error of μ_3 for the normal curve $= \cdot 67449 \sqrt{\frac{6}{N}} \sigma^3$ and of $\mu_5 = \cdot 67449 \sqrt{\frac{945}{N}} \sigma^5$, or in terms of σ as our unit is thirteen times as large. These are the *gross* errors; the percentage probable errors are of course infinite. As a rule it is hardly worth testing these conditions beyond μ_3. We determine whether the third moment is zero within the limits of random sampling. If we wish a relative magnitude we can take $\beta_1 = \mu_3^2/\mu_2^3$, a quantity which occurs over and over again in frequency discussions. The probable error of β_1 is obviously zero for the normal curve, because β_1 is of the square of the order of small quantities. The probable error of $\sqrt{\beta_1} = \cdot 67449 \sqrt{\frac{6}{N}}$, and

$\sqrt{\beta_1}$ for all truly normal distributions ought not to differ by more than two or three times the above expression from zero.

We can form other expressions involving β_1 and β_2 and ask what their value is for the Gaussian curve. We can calculate their probable errors, and determine whether the given distribution satisfies the Gaussian value within the limits of random sampling.

Thus I take the expression:

$$\chi = \frac{1}{2}\frac{\sqrt{\beta_1}(\beta_2+3)}{5\beta_2-6\beta_1-9} = \frac{1}{2}\frac{\sqrt{\beta_1}(6+\beta_2-3)}{6+5(\beta_2-3)-6\beta_1} \quad \text{.....(ii).}$$

Clearly this expression vanishes for the normal curve, and $=\frac{1}{2}\sqrt{\beta_1}$ nearly when $\sqrt{\beta_1}$ and β_2-3 are not very large, i.e. when we have not a very wide deviation from normality. The probable error of this expression, if the distribution be really normal, is $\cdot 67449\sqrt{\frac{3}{2N}}$.

Again, consider the expression:

$$d = \tfrac{1}{2}\sigma\frac{\sqrt{\beta_1}(\beta_2+3)}{5\beta_2-6\beta_1-9} \quad \text{.....(iii).}$$

This is a length which vanishes, if the distribution be truly normal. Its probable error is $\cdot 67449\sqrt{\frac{3}{2N}}\,\sigma$ in the case of the Gaussian curve, and accordingly d should not differ from σ by more than two or three times the above probable error.

Now let us write $\eta = \beta_2 - 3$. Then it is absolutely impossible for any distribution to be looked upon as Gaussian unless χ, d and η are zero within the limits of random sampling. These limits being deduced from their known probable errors.

Now it will be asked why choose such an expression as χ instead of the simpler $\sqrt{\beta_1}$? The answer is quite simple. We want to determine whether the mode coincides with the mean or not, and we cannot do this on the basis of the Gaussian curve where no distinction is made between the two. We must take some curve which is not Gaussian to determine this important quantity from. Now the equation to the Gaussian curve is

$$y = y_0 e^{-\frac{(x-m)^2}{2\sigma_0^2}},$$

where m is the mean value of x and σ_0 the standard deviation, and we have for its differential equation:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x-m}{\sigma_0^2}.$$

Now if we assume that the actually observed character is not x but X, and that X is some function of x, we shall not in plotting the frequencies to X obtain a normal curve, but we ought if Y be the ordinate of this curve to have

$$YdX = ydx \quad \text{or} \quad Y = y\frac{dx}{dX}.$$

Taking logarithmic differentials

$$\frac{1}{Y}\frac{dY}{dX} = \frac{1}{y}\frac{dy}{dx}\frac{dx}{dX} + \frac{d^2x}{dX^2}\Big/\frac{dx}{dX}$$

$$= -\frac{1}{\sigma_0^2}(x-m)\frac{dx}{dX} + \frac{d^2x}{dX^2}\Big/\frac{dx}{dX}.$$

Assume $x - m = f(X)$ and we find:

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma_0^2 F(X)} \qquad \text{.....(iv),}$$

where $$F(X) = Xf'(X)/\{f(X)(f'(X))^2 - \sigma_0^2 f''(X)\}.$$

The form has been so chosen that the origin is the mode, i.e. dY/dX vanishes with X. The proposal to thus generally transform the Gaussian curve is due in a quite different form to Edgeworth*. Kapteyn following Edgeworth and without any acknowledgment takes:

$$f(X) = \beta(X+\kappa)^q$$

where β, κ and q are constants to be determined.

He therefore puts:

$$F(X) = \frac{X(X+\kappa)}{\sigma_0^2(q-1) - mq\beta(X+\kappa)^q - q\beta^2(X+\kappa)^{2q}}.$$

This is a somewhat complex expression. The resulting frequency curve is

$$Y = Y_0(X+\kappa)^{q-1}e^{-\frac{1}{2\sigma_0^2}\{(X+\kappa)^q - m\}^2} \qquad \text{.....(v),}$$

and has been suggested by Kapteyn as a general form of the skew frequency curve. We shall consider it later.

Galton and McAlister as early as 1879 took

$$f(X) = b\log\frac{X}{a} - m,$$

where b and a are constants. Ranke and Greiner, without apparently knowing the history of research in this field, take the same value and attribute to Fechner the well-known Galton-McAlister curve of the geometric mean which results. We find

$$F(X) = \frac{X^2}{\sigma_0^2 + b^2\log\frac{X}{a}},$$

* He has developed it in a long series of papers published in the *R. Statistical Society's Journal*.

whence
$$\frac{1}{Y}\frac{dY}{dX} = -\frac{1}{X}\left(1 + \frac{b^2}{\sigma^2}\log\frac{X}{a}\right),$$

and
$$Y = Y_0 \frac{1}{X} e^{-\frac{b^2}{2\sigma_0^2}\left(\log\frac{X}{a}\right)^2} \quad \text{.....(vi).}$$

Edgeworth himself has made other suggestions as to suitable values for $f(X)$ and accordingly of $F(X)$. Now it is quite clear that assuming the character to be a definite function of another character which really obeys the normal law, there is no more reason for assuming one form of $f(X)$ than another, because we are in absolute ignorance of the nature of this function. Kapteyn's, or Edgeworth's, or Galton's are equally valid, and the only test of their relative suitability lies in the extent to which the resulting curves fit actual data. Clearly to assume $x = f(X)$ is to assume the actual frequency distribution to follow any law whatever. It is only screening the generality of the assumption

$$y = \phi(x),$$

where ϕ is unknown, by an appeal to the supposed universality of the Gaussian curve and by a perfectly arbitrary selection of the subsidiary function f.

But there is another manner of looking at this proposal. Returning to the equation

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma_0^2 F(X)},$$

and writing $\sigma^2 = \sigma_0^2 F(X)$, we see that it becomes identical in form with the normal equation, i.e.

$$\frac{1}{Y}\frac{dY}{dX} = -\frac{X}{\sigma^2}.$$

In other words the distribution of any frequency may be looked upon as given in the neighbourhood of any point by a normal curve of standard deviation $\sigma_0\sqrt{F(x)}$. Hence the conception arises that if the causes which produced variation in the immediate neighbourhood of any value x_0 of the character, were constants for the whole range of variation, we should have a normal curve of standard deviation $\sigma_0\sqrt{F(x_0)}$*. In reality there is a continuous and gradual change of the tendency to variability as we pass from one value of the character to a second†. Analytically

* This method of looking at the matter throws light on another point. If a curve be of limited range, it signifies that $\sigma = 0$ at certain points, or the curve stops because we have reached the limits of local variation. In a curve of unlimited range it is not the capacity for local variation but the absence of individuals to vary, which is the special feature.

† The matter is of such importance relative to some of Ranke's criticisms that I give another proof of equation (iv) here, based on the conception of an infinite number of infinitely small cause groups which Ranke considers can only lead to the normal curve. Let y_{r+1} be the $(r+1)$th term of a binomial, skew or symmetrical, say for simplicity the latter, i.e. $(\frac{1}{2}+\frac{1}{2})^n$. Then

$$\frac{y_{r+1} - y_r}{\frac{1}{2}(y_{r+1} + y_r)} = \frac{n+1-2r}{\frac{1}{2}(n+1)}.$$

Now let c_r be the distance between y_r and y_{r+1} used in plotting these ordinates to obtain a curve, and let it be related to some small value c_0 by the relation $c_r = c_0 \times$ function of $r = c_0 \times \phi(r)$. Let X_r be measured

we may look at it in this way: If δx_0 be the variation in the neighbourhood of x_0, then δx_0 is not independent of x_0, *but correlated with it.* We may have a perfectly continuous population from dwarfs to giants, but it does not follow that the actual tendency to vary of dwarfs and of giants is identical. All proofs that I have seen of the normal curve fail in this respect. They assume that the character x is due to a number of increments, which are due to an indefinitely large number of *independent* cause groups. They assume that δx_0 is not correlated with the already accrued value x_0. All processes like those of Edgeworth, Galton, Kapteyn and Fechner are really devices for getting over this gradual change in the tendency to vary from point to point of the range. It appears to me best to directly acknowledge and face this difficulty by selecting a fitting function for $F(X)$.

If we drop now the distinction between X and x as unnecessary we reach as our frequency equation:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma_0^2 F(x)},$$

or if we use Maclaurin's theorem for $F(x)$:

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2(1 + a_1x + a_2x^2 + a_3x^3 + \ldots)} \qquad \ldots\ldots\ldots\ldots\ldots\text{(vii)}.$$

Now I have shown* how to determine the successive constants σ_0^2, $\sigma_0^2 a_1$, $\sigma_0^2 a_2$, etc. Further all these constants but σ_0^2 are zero, when the distribution is normal, and the series will be found to converge rapidly, when the distribution is

from the largest term of the binomial, then $X_r = c_0\sqrt{(n+1)} \times$ function of $r = c_0\sqrt{(n+1)}\, f(r)$, say, and conversely $r =$ a function of $X_r/\{c_0\sqrt{(n+1)}\}$. Divide both sides of the above equation by c_r, which may be written on the left ΔX, and we obtain:

$$\frac{\Delta Y}{X\Delta X} = \frac{-X_r}{\frac{1}{4}(n+1)\, c_0^2 f(r)\, \phi(r)/\{r/(n+1) - \frac{1}{2}\}}.$$

Put $\sigma_0 = \frac{1}{2}\sqrt{n+1}\, c_0$ and $F(r)$ for the expression $f(r)\,\phi(r)/\{r/(n+1) - \frac{1}{2}\}$ which does not become infinite with $2r = n+1$, because X_r and therefore $f(r)$ vanishes for this value of r, and accordingly $f(r)$ contains $2r - (n+1)$ as a factor. We then have:

$$\frac{\Delta Y}{y\Delta X} = -\frac{X_r}{\sigma_0^2 F(X_r/\sigma_0)}.$$

Now make n infinite and c_0 vanishingly small, then we have if $\sigma_0 = \frac{1}{2}\sqrt{(n+1)}\, c_0$ be still finite

$$\frac{dY}{YdX} = -\frac{X}{\sigma_0^2 F(X/\sigma_0)},$$

a result in agreement with the above investigation. In other words this, and not the Gaussian curve, is the generalised frequency curve we reach if we directly abrogate the third Gaussian principle, that contributory increments of the variate are independent. Of course the first two Gaussian principles simultaneously disappear. This view of the matter occurred to me many years ago, when considering Hagen and Crofton's proofs of the Gaussian law. It was expressed in my memoir of 1894 by the statement that we require curves produced by conditions in which the contributory cause groups are not independent, i.e. in which an increment δx to the variate x depends upon the value of x, or is correlated with it. My method of reaching such curves, however, was a direct appeal to discrete series in which such a condition was fulfilled.

* "Mathematical Contributions to the Theory of Evolution, XIV." p. 6. Dulau and Co., London.

in the least approximately normal. Accordingly if we wish to get a good interpolation curve to determine the distance between the mode and the mean, we may assume

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2(1+a_1x+a_2x^2)} \quad \text{..... (vii) } bis.$$

In this case we discover with the previous notation, that

$$d = \frac{1}{2}\frac{\sqrt{\beta_1}(\beta_2+3)}{5\beta_2-6\beta_1-9}\sigma$$

is the distance between mean and mode, and that $\chi = \frac{\frac{1}{2}\sqrt{\beta_1}(\beta_2+3)}{5\beta_2-6\beta_1-9}$ is the ratio of this distance to the variability, or what I term the skewness, or the asymmetry relative to the variability.

If we leave out a_2 we find the skewness given by $\chi = \frac{1}{2}\sqrt{\beta_1}$ and the distance between mean and mode $d = \frac{1}{2}\sqrt{\beta_1}\sigma$. In practice these give fairly closely the same values as the fuller expressions above, and the fuller expressions are not numerically much modified if we include a_3. Shortly we have got a very fair mathematical process of determining the position of the mode and the degree of asymmetry.

Now the constants of such a curve as (vii) *bis* are absolutely determined by a knowledge of σ, β_1, and β_2; or looked at inversely they suffice to fix σ, β_1, and β_2. In other words the degree of kurtosis (β_2-3), the skewness χ and the distance between mean and mode—all most definite physical constants—are at once fixed by a knowledge of the constants of the curve, or on the other hand, being known, they fix those constants. It is of course allowable to replace any one of the three by the variability of the system. The actual position of the mode and the total magnitude of frequency suffice to fix the position and size of the curve. I have already called $\beta_2-3=\eta$ the degree of kurtosis; I call d the modal divergence. Then unless

η, the degree of kurtosis be zero, subject to probable error, $\cdot 67449\sqrt{24/N}$,

χ, the skewness be zero, subject to probable error, $\cdot 67449\sqrt{1\cdot 5/N}$,

d, the modal divergence be zero, subject to probable error, $\cdot 67449\sigma\sqrt{1\cdot 5/N}$,

no distribution can be legitimately described as normal or Gaussian.

It would be of interest to know how far Ranke and Greiner have applied such tests to any series containing a large number N of individuals. I think if they had done so, they must have come to the same conclusion as the majority of statisticians that the normal curve has only a limited range of application.

Of course if N be small, as in most craniological series, we find our probable errors so large, that it is not possible to say more than that for *short* series the Gaussian curve may roughly describe the result. But for long series in economics, sociology, zoology, botany and anthropometry the Gaussian curve over and over again fails. If in all these cases Ranke and Greiner assert that the material is

heterogeneous they are merely arguing in a circle. The distributions are as continuous and smooth as those which occur in the case of the Gaussian curve, and *they occur for characters in the same group of individuals which present for other characters the normal distribution.*

Thus the length of meropodite of right claw in *Gelasimus pugilator** is quite sensibly normal, but the length of the carpodite of the right claw is almost as certainly platykurtic and skew. If two characters are normal, a third character which is their difference, whether they be correlated or not, should have a normal distribution, yet in the case of *Gelasimus pugilator* for the whole series of measurements the difference distributions are essentially platykurtic†.

The size of the disc in *Ophicoma nigra*‡ has a modal difference in the distribution of 1000 cases of ·271 mm. and the probable error on the basis of a normal curve is ·056. The deviation is thus five times its probable error and the asymmetry undoubtedly significant.

The outer diameter of *Arcella vulgaris*§ in 504 cases gives a modal difference of 3·226 mikrons, and the probable error of this difference is only ·211. The asymmetry is therefore undoubtedly significant.

The distance between the mean and mode in the case of the length of shell of *Nassa obsoleta*‖ from Lloyd Point, Long Island, U.S. was ·68 mm. for 368 individuals, the probable error of this modal difference was ·08. The asymmetry is therefore significant. Other characters of *N. obsoleta* were as definitely asymmetrical, while some from exactly the same individuals were sensibly normal.

The transverse arc in ♂ Naqada skulls has for 115 individuals a modal difference of 2·34 mm. and the probable error of this difference is ·78 mm., or it is probably significant. Yet the breadth of the male Naqada skulls is significantly symmetrical.

The height/length index of 117 English ♀ skulls has a modal difference of ·85 and a probable error of only ·22, the skewness is therefore significant.

The same is true of the distribution of many internal organs in man. For example, if we exclude recognised diseased hearts, we obtain a markedly skew distribution such as is given in the broken line of Fig. 1. This is for 1382 heart-weights. If this be supposed to be due to the great variety of ages, we have only to look at the continuous curve for hearts of 358 young adults, 25 to 35, to see the same asymmetry. This is drawn for four times the scale.

* G. Duncker: *Biometrika*, Vol. II. p. 313.

† There is another point to which I will only refer briefly here. If characters were always distributed according to the Gaussian law the regression curves must be *straight* lines. The generalised Mendelian theory of determinants I have developed makes them, however, hyperbolas, and I have given instances in a recent memoir of a variety of curved regression lines

‡ McIntosh: *Biometrika*, Vol. II. p. 470.

§ R. Pearl and F. J. Dunbar: *Biometrika*, Vol. II. p. 327.

‖ A. C. Dimon: *Biometrika*, Vol. II. p. 29.

Lastly if we consider that the question of health determines the skewness, we have in the dotted curve the weight-distribution of 699 hearts stated to be "healthy." We see that there is still the same essential skewness; the pathologist has merely cut off a small portion of the tail on the left and far too much of the tail on the right, i.e. unusually big hearts were discarded as necessarily "unhealthy." The form of the curve undoubtedly indicates that many of these large hearts are abnormal, but any continuous curve fitted to the remainder, the "healthy hearts," would not only be significantly skew, but would project a long way into the portion of the tail discarded as "unhealthy." The list of asymmetrical distributions might be indefinitely extended, but these must suffice to indicate that asymmetry cannot be lightly put on one side in the manner adopted by Ranke and Greiner.

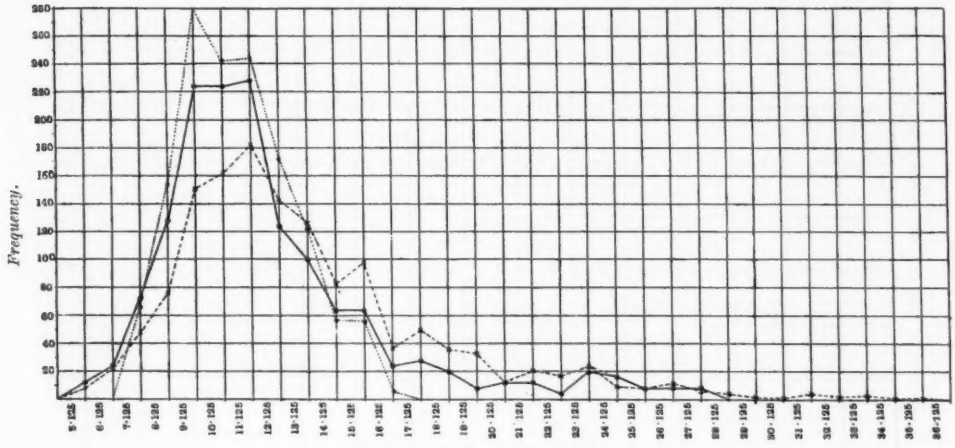


FIG. 1. Frequency Polygons of Weight of Heart in Males (Greenwood*).

I. ⁎------⁎ 25 to 55 years. Without specific disease of heart. Number of cases 1382.
II. ○——○ 25 to 35 years. Without specific disease of heart. Number of cases 358.
III. •——• 25 to 55 years. Definitely sound hearts. Number of cases 669.

The scale is four times as great for II. and twice as great for III. as for I.

If we pass to discrete variates, we find as large a number, if not a larger number of distributions in which skewness is well marked, for example, fertility in the Aphis *Hyalopterus Trirhodus*†, fertility in man‡, fecundity in race-horses§, and fertility and fecundity in mammals generally. I illustrate this with an example of fertility in English mothers in Fig. 2. It will be seen at once that no normal curve could be used to describe this distribution. It is equally

* *Biometrika*, Vol. III. p. 45 *et seq.*
† Warren: *Biometrika*, Vol. I. p. 127.
‡ Pearson: *Phil. Trans.* Vol. 192 A, p. 257, and *The Chances of Death*, Vol. I. p. 63.
§ Pearson: *Biometrika*, Vol. I. p. 292.

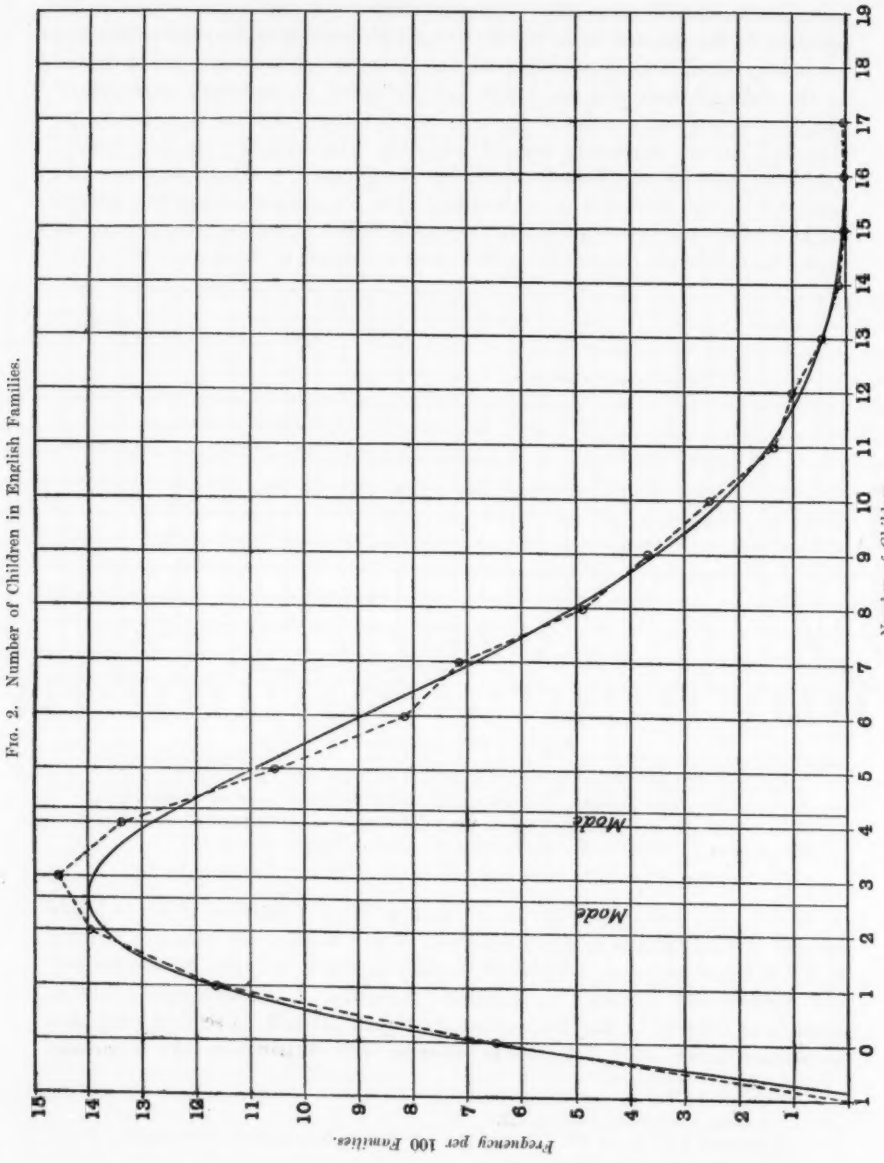


FIG. 2. Number of Children in English Families.

impossible also for a curve of the McAlister-Galton type, for the simple reason that that curve has high contact at both ends of the range. Now I contend that the anthropologist who either neglects such matters as human fertility, or confesses that he has no means of succinctly describing their distributions—and as long as he sticks to the Gaussian curve he certainly will not have—is simply putting on one side a fundamental factor in the science of man. Ranke lays great stress on homogeneity. He does not, however, clearly define what he means by the term. Apparently any series which follows a Gaussian curve is to him homogeneous, any other series is not. I should be glad if he would then consider any craniological series, say, of adult crania. This will involve crania of adults from perhaps 25 or 30 years of age to 50, and these are rather narrow limits considering our paucity of material. Now our data show that the correlation between head measurements and age may be of the order of about $-\cdot1$ to $-\cdot2$. After about 25 to 28 years of age in man, there is a continual shrinkage not only of stature but of skull capacity, brain-weight, circumferences and diameters of the skull. Under the circumstances, what right have we because the Gaussian curve is obtained to call this material "homogeneous"? I will go further; suppose we could, and we can, obtain the measurements on one or two thousand individuals of the same age; are these to be considered as a homogeneous distribution? My reply will be, in man the order of birth is an essential feature in determining the dimensions of the physical characters. My investigations show that physique and health are sensibly correlated with the position of a member in his own family. In mammals others have shown that the physique of an individual is sensibly correlated with the number of members born in that individual's litter. Now with these facts before us what stress can be laid on Ranke's conceptions of homogeneity? The practical anthropologist requires curves which will successfully *graduate* his data. Only on the basis of such graduations can he allow for the influence of disturbing factors like age, order of birth, season or special position of production in the organism. I take this very case: How is it possible to allow for the influence of order of birth, unless you know the size of families or the distribution of births within the community? It can only be achieved provided the distribution can be represented by a few simple constants which allow of definite mathematical handling.

Ranke and Greiner say with considerable asperity that my method of determining the range from given data can be of no service. Yet take this very case of size of families. In the English middle classes for 4390 instances, I find that the *observed* limit is 17, but fitting a skew curve the range is determined as 22 children running practically from just before 0 to over 21 births (Fig. 2). For Denmark by the same process in 34,000 cases the theoretical range is 26 and the observed range 22*. I then proceeded to take statistics for the Argentine Republic, and found for the town of Buenos Ayres, 27,510 births, that the range of the curve was from ·25 to 36·61 births, or 37 possible births. The maximum observed in these 27,510 births was 23. But among the South- and Mid-American

* *The Chances of Death*, Vol. I. *Reproduction Selection.*

populations cases of 24 (Trinidad), 26 (Cuba), 27 (Nicaragua) and 34 (Colombia), the women beginning to bear at 13 and continuing to 50, have been recorded! It will, I think, be realised by the impartial reader that Ranke's statements:

> Für den Anthropologen ist also nur das Gaussche Gesetz von Wichtigkeit. Für seine Probleme beansprucht es aber auch völlige Gültigkeit (S. 327)

and

> So haben anderseits auch die Werte von Variationsumfängen die bislang aufgefunden sind, keinerlei weitere Erkenntniss gebracht (S. 324)

fall wide of the mark. Ranke has either very much circumscribed the field of the anthropologist, or he has not in this, as in other cases noted in his paper, studied the literature of the subject, or finally he has disregarded the results reached.

It is quite true that the range cannot always be determined and being determined does not always give a very good result. The reasons for this are not far to seek. For example, If a discrete quantity has for its minimum 0 units the start of the curve must naturally fall on the negative side of the origin—since its area measures frequency between $-\cdot5$ and $+\cdot5$. Ranke would probably find something mysterious in this "reichen oft ins Negative." Actually it is to be expected, especially if due allowance be given for the probable error of the range. In most biometric statistics, we cannot as in the case of births deal with 20,000 to 30,000 cases and get small errors for our constants. We have only perhaps 500 to 2000 cases and even less than this in craniology. This may denote an error of 14 to 17 p.c. in the calculated range, and it is quite possible that the range may "reichen ins Negative." Take the case worked out by me* of the number of Müllerian glands in the forelegs of ♀ swine. The range theoretically calculated is 18 glands with a probable error of $\pm 2\cdot54$; the start of this range ought not to have exceeded $-\cdot5$. It is actually $-\cdot82$ with a probable error of $\cdot16$. The actual skewness of this distribution is $\cdot31$ with a probable error of $\cdot02$. The distribution is accordingly significantly asymmetrical.

I have cited these cases as sufficient for our present purpose, but there are many other cases in which the discovery of the range has been of biological or special anthropological interest, e.g. the earliest appearance of certain diseases in childhood, the range of cancer attacks, the first occurrence of signs of puberty, etc. It has been applied also effectively to a number of zoological and botanical data. A more striking case, perhaps, of usefulness is the limit to high barometric pressure obtained by dealing with the frequency statistics of barometric height at series of stations†. Throughout the whole of the stations of the British Isles dealt in, there is sensible skewness of distribution, and with one Irish exception, which is sensibly mesokurtic, the whole series of curves are platykurtic, and this deviation from normality cannot be chance, but is a significant character of the frequency distributions. In these cases the limit to high pressure has been found, and appears to be a constant of considerable physical importance for the local climate.

* Pearson and Filon: *Phil. Trans.* Vol. 191 A, p. 289.

† Pearson and Lee: *Phil. Trans.* Vol. 190 A, p. 423 *et seq.*

I again am forced to consider that Ranke has not been aware of what has been published, still less what has been done in this matter. He appears to base his conclusions chiefly on my first paper on skew-variation, and he has not noticed the fact that being the *first* paper much has to be corrected in the light of more recent work in the past ten years. Ranke speaks of the writer's:

> Andwendung von allerlei grösseren und kleineren Änderungen in seiner Methode *ad hoc* in eine für den vorliegenden Zweck nicht zu unwahrscheinliche Form bringen (S. 324).

Now I contend that this gives a grossly unjust description of the paper in question. Had Ranke read recent literature, he would have been aware that the great difficulty with frequency distributions is to obtain the true values of the "moments" from records which merely give data for arbitrary "Spielräume," often far too large and usually selected by the observer without any regard to the needs of the computator. My method is one based on the method of moments, but to deduce the moments from given data is the real difficulty which Ranke never for an instant seems to grasp or at any rate refer to. The standard deviation (which he appears to consider sufficient for anthropologists) will vary, and often very sensibly, with the nature of the grouping of the data. This difficulty was very present in my mind in 1894, and is constantly referred to in my memoir, the "allerlei grössere und kleinere Änderungen in seiner Methode" are no changes in method at all but attempts to obtain some approximation to the true moments of the data. It was not till 1898 that Sheppard showed the correct manner of calculating the moments from the raw data in his important memoir on frequency constants*, for *one* type and one type only of frequency distribution. The curves calculated by Sheppard's method, now in general use, would give better results undoubtedly than are to be found in my memoir of 1894. Further, however, Sheppard's method applies only to curves with high contact with the horizontal axis at both ends. It leaves us still in doubt as to how to find the moments of curves, which cut the axis at the end of the range or are asymptotic at one or both ends to the vertical axis. At such ends of the range, the real solution lies in recording the frequency for very small elements, but this was not provided in any of the statistics which were then before me. It is just these cases of limited range at one or both ends which present difficulty in the determination of the moments. The difficulty will be familiar to all statisticians, if it has escaped Ranke. To some extent it is met in my memoir on the systematic fitting of curves issued in April, 1902†. Yet granting all these difficulties what do we find in my memoir of 1894? An analysis of the cases in which range is dealt with seems justified by the charges made:

Example I. Range determined of Cambridge Barometric Heights. There is nothing physically improbable in the result.

Example VI. Range found for enteric fever runs from $-1{\cdot}35$ years to about 385 years. The probable error of the range is not given, but the whole difficulty

* *Proc. London Math. Society*, Vol. xxix. p. 353 *et seq.* † *Biometrika*, Vol. i. p. 265.

turns upon the *great* changes introduced into the range by different methods of calculating the moments. More recent investigations, in which the sexes are separated, the moments more accurately determined, and larger numbers dealt with, give far better results for zymotic diseases. I presume that one character being age, however, Ranke and Greiner would dismiss these data from consideration under any circumstances.

Example VII. Guesses at 9 tints. Possible range 1 to 9, i.e. curve to run from ·5 to 9·5. Observed guesses run from 1 to 8. Theoretical range of 11 instead of 9. The paucity of the observations gives a probable error of at least 20 to 30 per cent. in the determination of the range, and the result is rather better than might have been anticipated.

Example VIII. Ratio of forehead to body length in *Carcinus moenas*, observed range 30, calculated range 51. This range is probably not very close but it is not in any way that I can see impossible. The material is probably dimorphic.

Example XI. H. de Vries's data for *Ranunculus bulbosus.* Actually observed range 5 to 10 petals. Calculated range 5 to 11 petals.

Example XII. H. de Vries's data for a race of *Trifolium repens.* Actually observed range 0 to 10 high blossoms. Theoretical range in complete agreement.

Example XIV. Pauperism percentages for 632 cases. Observed range 18 for the year 1891 dealt with. Calculated range 31. This range gives 2 units of negative pauperism. Its probable error is, perhaps, 14 per cent.

It will be seen that out of the seven examples in which range is calculated only three *reichen ins Negative*, and that this *reichen* is well within the limits of the errors arising on the one hand from random sampling and on the other from the defective methods of determining the moments, which were alone available in 1894. While quite appreciating the honour done me when other workers use my methods, I must decline to be responsible in any way for their application of my formulae. I have so often found that their failure to fit my curves is due to a misapprehension of my methods or to actual errors in arithmetic, that I have long given up any attempt to set such matters right. The frequent assumption made that statistical methods can be applied without adequate mathematical training is the source of most of the slips in this matter*.

So far then I think we may conclude that Ranke is completely unjustified both in his statement that the Gaussian curve fully describes all the frequency that is of importance to the biologist, and in his attempt to discredit any result of scientific value which flows from endeavouring to measure such differences from the Gaussian law as we find in the distance between mode and mean, the skewness, the kurtosis and range of many actual frequency distributions.

* A good illustration, by no means unique, of this is F. Reinöhl: *Die Variation im Andröceum der Stellaria Media*, 1903. He finds it impossible to fit certain distributions with my curves, owing to ignorance of the full literature and to faulty determination of the moments. He then argues from this want of fit to biological conclusions.

All the leading statisticians, from Poisson to Quetelet, Galton, Edgeworth, and Fechner, with botanists like de Vries, zoologists like Weldon have realised that asymmetry must be in some way described before we can advance in our theory of variation. In innumerable cases the important quantities measured by η, χ and d actually exist; these have each their physical significance and they must be found. It is perfectly open to Ranke and Greiner to criticise my method of determining these quantities, but that they should shut their eyes to their existence appears to me only compatible with a very small acquaintance with the data of variation.

Let us see now how various authorities have met this difficulty of skewness.

B. *The Gaussian Curve**.

Gauss proceeds from the axiom that: *The arithmetical mean of a series of observations gives their most probable value,* i.e. the mean is the value of maximum frequency. This result is not axiomatic. It can only be a result of experience, and if it were true it would make the normal curve as much a result of experiment, i.e. an empirical result, as any other proposed curve of frequency. Gauss's proof demands, however, something more than this first statement. It involves (i) the equal probability of errors in excess of the mean and of errors in defect, (ii) the continuity of magnitude in the errors, and (iii) the independence of all the small contributions to the total error.

Experience shows that Gauss's fundamental axiom as to the mode and mean coinciding is not universally true. It is not true of errors of observations, it is not true of variations in living forms. Gauss reaches a differential equation which leads to the normal curve. His proof seems to me, as it has done to many others, quite invalid, because the equal probability of errors in defect and excess of the mean is not demonstrated, the possible dependence of contributory elements is not discussed, and the question of continuity of errors is not considered.

C. (i) *Laplace and Poisson.*

Laplace and after him Poisson took, I venture to think, much firmer ground. They did not assume (i) and (ii), but they did not realise the importance of (iii). They proceeded by evaluating the terms of the binomial:

$$(p+q)^N.$$

* In writing for Germans I naturally spoke of the Gaussian curve. But I am not clear that precedence is to be given to Gauss. Gauss first gave a proof of the well-known equation $y = y_0 e^{-\frac{1}{2}x^2/\sigma^2}$ in his *Theoria Motus Corporum Coelestium* of 1809. This was three years before the publication of Laplace's *Théorie Analytique des Probabilités* of 1812. But to give absolute priority to Gauss is to disregard Laplace's earlier memoirs, particularly those of 1782, "Sur les approximations des Formules qui sont fonctions des très-grands nombres," and its *Suite du Mémoire* of 1783. On p. 433 of the latter memoir Laplace actually suggests the importance of forming a table of the probability integral $\int e^{-t^2} dt$. The *Théorie des Probabilités* reproduces the substance of this memoir, and on this account some writers have post-dated Laplace's work. Gauss stated that he had used the method of least squares in 1795, but this does not necessarily involve a knowledge of the probability integral, and if it did, it is ten years after Laplace. On the whole my custom of terming the curve the Gauss-Laplacian or *normal* curve saves us from proportioning the merit of discovery between the two great astronomer mathematicians.

Using Stirling's theorem they showed that if the mth be the largest term in the binomial, then the sum p_r of all the terms from $m-r$ to $m+r$ is very nearly given by:

$$p_r = \frac{2}{\sqrt{2\pi}\sigma}\int_0^r e^{-\frac{x^2}{2\sigma^2}}dx + \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{r^2}{2\sigma^2}} \quad \text{.....(viii),}$$

where $\sigma = \sqrt{Npq}$.

Here we have the first appearance of the probability integral as representing a series of *discontinuous* binomial terms. In fact when N is fairly large Laplace and Poisson show that sums of terms of the binomial are closely given by the areas of the probability curve. It is an approximate result based upon Stirling's theorem, and it does not for a moment involve making N infinitely large, or the spacing apart of the binomial terms very small. This representation of a number of finite terms by the probability integral seems to be unfamiliar to Ranke and Greiner, but no practical statistician would calculate the sum of r terms in the binomial $(p+q)^N$ for even moderate values of N. He would simply calculate the standard deviation $\sigma = \sqrt{Npq}$ of the binomial and turn up tables of the probability integral. This fundamental property of the normal curve, i.e. that it closely represents a *discontinuous* series, is passed over in silence by my critics. It is the very purpose for which the probability integral was originally introduced by Laplace. In other words it arises without any consideration of (i) continuity of variation, or (ii) equal probability of negative and positive deviations.

It will be observed that the above approximation to the binomial, i.e. to $(p+q)^N$ is symmetrical, but we can easily allow for some degree of asymmetry. Still writing $\sigma = \sqrt{Npq}$, and for the binomial

$$\beta_1 = (1-4pq)/(Npq), \quad \eta = \beta_2 - 3 = (1-6pq)/(Npq).$$

I have shown*, y_0 being the maximum term in the binomial, that the rth term from the maximum is given by:

$$y_r = y_0 e^{-\frac{r^2}{2\sigma^2}(1-\beta_1+\frac{1}{12}\eta) - \frac{1}{2}\sqrt{\beta_1}\frac{r}{\sigma} + \frac{1}{6}\sqrt{\beta_1}\frac{r^3}{\sigma^3}(1-\frac{3}{2}\beta_1+\frac{3}{2}\eta) - \text{etc.}} \quad \text{.....(ix).}$$

The term in r/σ was, I believe, first added by Poisson, and expresses his attempt to allow for asymmetrical variation. Edgeworth expanding the exponential has adopted for his asymmetrical curve, a form easily deduced from (ix),

$$y_r = y_0 e^{-\frac{r^2}{2\sigma^2}}\left\{1 - \tfrac{1}{2}\sqrt{\beta_1}\left(\frac{r}{\sigma} - \frac{1}{3}\frac{r^3}{\sigma^3}\right)\right\} \quad \text{.....(x).}$$

It will thus be seen that the normal function and the probability integral arise naturally from the expression for a single term or a series of terms of the binomial polygon. This is their historical origin and the historical origin of the conception of asymmetrical variation. Instead of the complex form given above resulting from Stirling's theorem, I approached the subject by looking at the relation of the

* *Phil. Trans.* Vol. 186 A, p. 348, footnote.

normal curve to a symmetrical binomial in a totally different manner. I succeeded in showing that the ordinates and areas of the normal curve gave exceedingly closely the terms and sums of terms of the symmetrical binomial even for relatively small values of n. This had already been done by Laplace. The reader will realise this if he looks at the closeness of the normal curve with

$$\sigma = \sqrt{Npq} = \tfrac{1}{2}\sqrt{N},$$

and the binomial $(\frac{1}{2}+\frac{1}{2})^{10}$ with $N=10$ in the accompanying Figure 3. But my method enabled me to give a simpler expression to the asymmetrical binomial

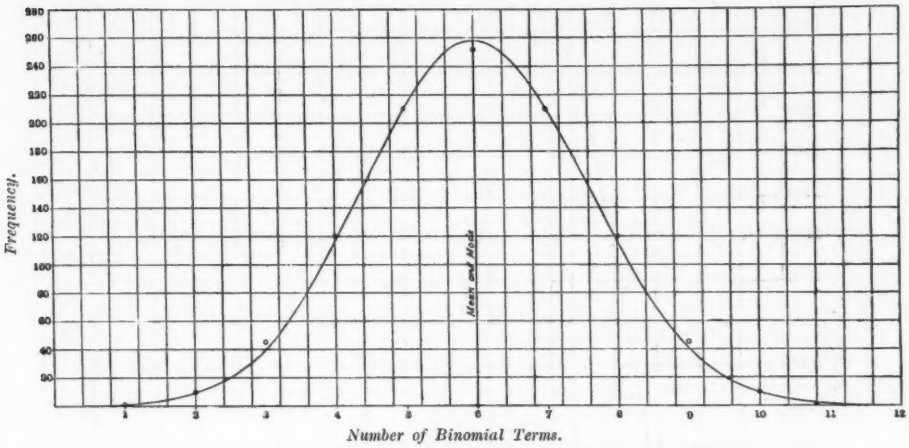


FIG. 3. Comparison of Point Binomial $1024\,(\frac{1}{2}+\frac{1}{2})^{10}$ with the Gaussian Curve.

S.D. $= \sqrt{Npq} = 1{\cdot}5811$. Maximum Ordinate 258·35.

than had been obtained by Poisson or Edgeworth using Stirling's theorem. Figure 4 shows how closely the terms of the asymmetrical binomial $5000\,(\frac{1}{6}+\frac{5}{6})^{12}$ and the sums of terms are reproduced by my curve of Type III., i.e.

$$y = 1536{\cdot}54\left(1+\frac{x}{5}\right)^{14} e^{-3x}.$$

I had no higher ambition—nor could I have had one higher—than Laplace had when he discovered the normal curve. I wanted to find a close mathematical expression for the terms of the asymmetrical binomial for relatively small values of N.

Now Laplace and Poisson had both retained the last of Gauss's limiting conditions, i.e. they had by adopting the binomial supposed each increment of the deviation to be independent of previous increments. It seemed needful to me to get rid of this condition, and I therefore introduced instead of the binomial the hypergeometrical series. Here the successive increments are correlated. In order

to place this new representation on the same footing as the symmetrical binomial to which Laplace approximated with the normal curve, I deduced as I had done for the symmetrical and the asymmetrical binomials, curves which gave the hypergeometrical series and the sum of its terms as closely as Laplace's normal curve gave the symmetrical binomial. This is the complete history of the development of my skew curves. Before I proceed to discuss Ranke and Greiner's criticisms, I must remark that their attack on this point does not concern me only. Every practical statistician uses Laplace's representation of the point binomial by the

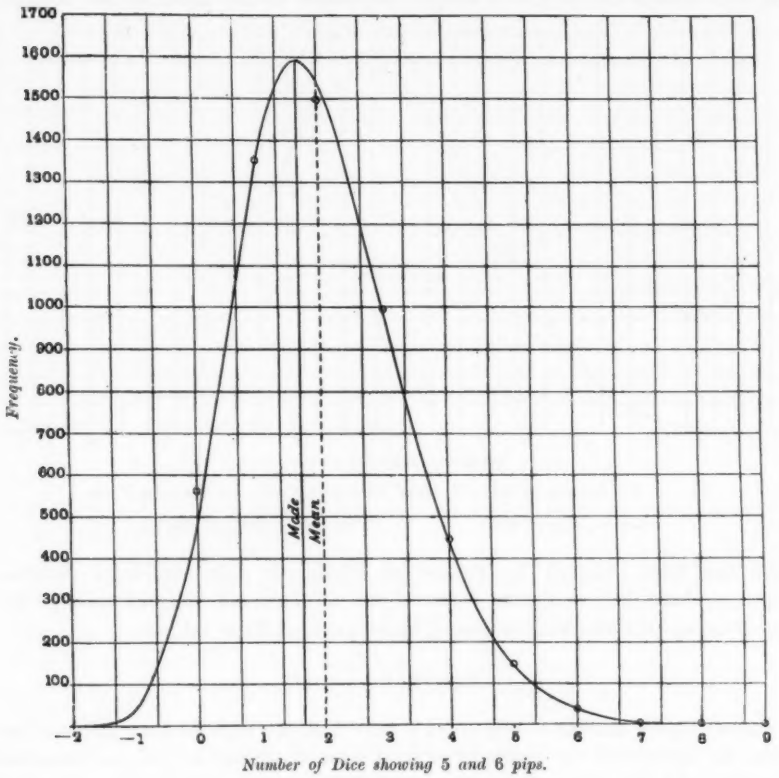


Fig. 4. 5000 Throws with 12 Dice.

○ ○ Points of the Binomial $(\frac{1}{6}+\frac{5}{6})^{12}\times 5000$.

——— Curve $y=1536{\cdot}54\left(1+\frac{x}{5}\right)^{14}e^{-3x}$.

Origin at the mean 2. Mode of curve at $x=-\frac{1}{3}$.

Frequency of each number of 5 and 6 pips	−1	0	1	2	3	4	5	6	7	8	9
Ordinates of Binomial	0	561	1346	1480	987	444	142	33	6	1	0
Areas of Curve	557		1340	1493	982	443	139	37	8	1	0

probability integral, when he is discussing the probable errors of death rates, class indices and a multitude of other problems, and this when the binomial is skew and N relatively small! It is the whole theory of current statistics which Ranke and Greiner are tilting at when they object to the use of what is equivalent to the Euler-Maclaurin theorem, i.e. the mathematical representation of a finite sum of terms by a definite integral*.

There is another point also which may be noted here before we leave the binomial. The quantity β_1 is a measure of the asymmetry. Now consider the ratio β_1/η, or the ratio of this measure of asymmetry to the kurtosis. For the asymmetrical binomial we have $\beta_1/\eta = (1-4pq)/(1-6pq)$. Since p and q if positive must give a product lying between 0 and $\frac{1}{4}$, this ratio cannot take any value between 0 and +1. Hence any curve which gives for β_1/η a value less than unity cannot possibly diverge from the normal curve in the direction of a binomial series. We shall see the application of this later.

C. (ii) *The Galton-McAlister Curve.*

I have already referred to the attempt of Poisson to give the skew binomial by an extra term applied to the Gauss-Laplacian probability integral. Quetelet endeavoured to meet the asymmetry of frequency distributions by placing graphically skew binomials on top of the frequency polygon—a very rough and somewhat deceptive process. The next step in the advance was taken by Francis Galton, who in 1879 suggested that the geometrical mean and not the arithmetical mean is likely to give the most probable result in many vital phenomena†. Galton refers to Gauss's assumption that errors in excess or in defect are equally probable, and says "this assumption cannot be justified in vital phenomena." He cites especially the cases of errors in human judgment, guessing at temperatures, tints, pitch, etc. He appeals to Fechner's law in its simplest form as evidence to the contrary, and placing the matter in the hands of D. McAlister‡, the law of frequency

$$y = y_0 \frac{h}{\sqrt{\pi x}} e^{-h^2\left(\log \frac{x}{a}\right)^2}$$

was deduced, and methods for fitting this curve were discussed. The curve is well-known in England and also on the continent§. It is therefore curious to find Ranke and Greiner attributing this curve to Fechner's work which was not published till 18 years later. It was not till I had made a fairly complete set of experimental determinations of the kind supposed to give this curve, that I finally discarded it. Thus I asked audiences of 100 to 300 persons to match tints in several ways, I asked them to guess heights, to determine mid-lengths, to state which figures in randomly distributed series were most closely circles,

* See Lacroix : *Traité du Calcul différentiel et intégral*, Tom. III. p. 136.

† *R. S. Proc.* Vol. 29, p. 365 *et seq.* "The Geometric Mean in Vital and Social Statistics."

‡ *R. S. Proc.* Vol. 29, p. 367 *et seq.* "The Law of the Geometric Mean."

§ It is cited by Kapteyn, for example.

squares, equilateral triangles, etc. etc. In all these results I found the distribution asymmetrical, but the most probable value was *not* the geometrical mean, nor the distribution the Galton-McAlister curve. One of the striking defects of the curve was its high contact at both ends. The distributions clearly often corresponded to curves in which the contour cut the axis at a finite angle. Another point was that the skewness was *in the opposite direction* to that presupposed by the reasoning from which the curve is deduced. It was precisely this experience which showed me that putting $x = \beta \log(\xi/a)$ in the Gaussian curve is not a sufficient generalisation.

Ranke and Greiner, to say nothing of Fechner himself, are remarkably vague as to the accurate determination of the position and constants of the Galton-McAlister curve. McAlister gives no clear description of how the curve is to be placed if neither the mode nor start of the range is known. I think it desirable therefore, having regard to the inferences I wish to draw, to give the fitting by my method of moments. I write the curve:

$$y = \frac{y_0}{x} e^{-\frac{1}{2c^2}\left(\log \frac{x}{a}\right)^2} \quad \text{.....(xi).}$$

Differentiation shows us at once that the distance x_{mo} of the mode from the origin is given by:

$$x_{mo} = ae^{-c^2} \quad \text{.....(xii).}$$

Integrating the expression $N\mu_n' = \int_0^\infty y x^n dx$ we find if N = total frequency:

$$y_0 = N/(\sqrt{2\pi}c) \quad \text{.....(xiii),}$$

and generally:

$$\mu_n' = a^n e^{\frac{n^2}{2}c^2} \quad \text{.....(xiv).}$$

Thus the distance from the origin to the mean, x_{me}, is given by

$$x_{me} = ae^{\frac{1}{2}c^2} \quad \text{.....(xv).}$$

Now write $e^{c^2} = \lambda$ and we have if μ_n be a moment coefficient about mean:

$$\left.\begin{aligned} \mu_2 &= \mu_2' - \mu_1'^2 = a^2\lambda(\lambda - 1) \\ \mu_3 &= \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = a^3\sqrt{\lambda}(\lambda^4 - 3\lambda^2 + 2\lambda) \\ \mu_4 &= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1^4 = a^4\lambda^2(\lambda^6 - 4\lambda^3 + 6\lambda - 3) \end{aligned}\right\} \quad \text{...(xvi).}$$

Forming the usual constants of frequency we have:

$$\beta_1 = \mu_3^2/\mu_2^3 = (\lambda - 1)(\lambda + 2)^2 \quad \text{.....(xvii),}$$

$$\eta = \beta_2 - 3 = (\lambda - 1)(\lambda^3 + 3\lambda^2 + 6\lambda + 6) \quad \text{.....(xviii),}$$

where $\beta_2 = \mu_4/\mu_2^2$;

$$d = x_{me} - x_{mo} = a(\sqrt{\lambda} - \lambda^{-1}) \quad \text{.....(xix),}$$

$$\chi = \frac{1 - \lambda^{-\frac{3}{2}}}{\sqrt{\lambda - 1}} \quad \text{.....(xx).}$$

We see then that the mean and second and third moments must be found to determine this curve. From the second and third moments we have β_1, whence by equation (xvii) λ is determined. The first equation of (xvi) then gives

$$a = \sigma/\sqrt{\lambda(\lambda-1)}.$$

Then equation (xv) gives the distance of the start of the curve from the known mean. Further since $\lambda = e^{c^2}$, we determine c and finally y_0 is determined by equation (xiii). There is no obscurity or difficulty with the fitting, if we use the method of moments. The cubic equation (xvii) is solvable at once either by Lill's, Mehmke's or Reuschle's mechanisms.

But what are the objections?

(i) The curve touches the axis at the end of the range. Skew curves extremely often cut it at a finite angle.

(ii) The skewness has a definite direction, which to be logically consistent we ought not to neglect, i.e. since λ is always > 1, d remains always of one sign.

(iii) Since $\lambda > 1$, η, the kurtosis, is always positive and the curve can only represent platykurtic distributions. It can never give a curve which deviates from the Gaussian curve in the direction of the Laplace-Poisson skew binomial for $p > \cdot2113 < \cdot7887$, because this is essentially leptokurtic.

(iv) The range of skewness given by χ is very limited. Differentiating χ we find it is a maximum for $\lambda = 1\cdot7200$ and this gives $\chi = \cdot2075$. The Galton-McAlister curve cannot therefore describe any curve whose skewness does not lie between 0 and $\cdot2$. A cursory examination of the observational results reached, shows that the skewness in all kinds of data over and over again exceeds $\cdot2$.

(v) β_1 and η are both functions of λ only*. Hence there is a relation between them or between η and χ. That is to say the kurtosis is determined by the skewness. The kurtosis must vanish with the skewness. But experience shows that many distributions are sensibly symmetrical and yet have far from zero kurtosis, e.g. nasal breadth in English women, etc. etc.

Finally consider the ratio β_1/η. If we approach the normal curve as the limit to a point binomial $(p+q)^N$ we have seen that

$$\beta_1/\eta = (1-4pq)/(1-6pq) \quad \text{.....(xxi),}$$

and this equals nothing if we take the symmetrical binomial. Otherwise it has a finite value depending upon the particular binomial along which we reach the Gaussian curve. The Galton-McAlister curve, if we make $a\sqrt{\lambda}$ infinite, but $a\sqrt{\lambda}\sqrt{\lambda-1}$ finite, approaches the Gaussian curve.

* Actually it is

$$\beta_1^4 - 12\beta_1^3 + 156\beta_1^2 + 64\beta_1 - \eta^3 + 12\eta^2 - 36\eta + 18\beta_1^2\eta - 6\beta_1\eta^2 - 117\beta_1\eta = 0.$$

I have to thank my assistant, Mr J. Blakeman, for much aid in the analysis of this section and the following section of this memoir.

Now for the Galton-McAlister curve

$$\beta_1/\eta = \frac{(\lambda+2)^2}{\lambda^3+3\lambda^2+6\lambda+6},$$

and this approaches the limit 9/16, its maximum value, when λ approaches unity. If we take $(1-4pq)/(1-6pq) = 9/16$, we get imaginary values for p and q. Thus while the normal curve itself gives an indeterminate value for $\beta_1/\eta = 0/0$, and as Laplace has shown describes with fair accuracy any slightly skew binomial with large power, the Galton-McAlister curve cannot describe even approximately any skew binomial, however near to a normal distribution.

On all these grounds we see that the "law of the geometric mean" fails to supply the fundamental need of describing the modal difference, the kurtosis and the skewness of actual frequency distributions. It cannot describe these physical characteristics of the frequency.

C. (iii) *Fechner's Double Gaussian Curve**.

We have noted that the Gaussian curve was first deduced by Laplace to represent a finite number of the terms of a binomial expression, and that Gauss deduced it on hypotheses which amount to the following:

(i) The arithmetic mean is the most probable value.

(ii) Deviations in excess and defect of the mean are equally probable if of the same magnitude.

(iii) The facility of an increment is the same for all values of the character.

Now every one of these assumptions is negatived when the double Gaussian curve is used, and yet the Gaussian curve *which is only deduced by aid of them* is adopted to describe what conflicts with its fundamental axioms. This proceeding is the reverse of logical. However, if the double Gaussian curve be adopted, there is absolutely no reason why we should adopt the rough process by which Fechner determines the mode and obtains the constants of the distribution. The fitting by my method of moments is perfectly straightforward, and as it leads to the points we have to consider it will be indicated here. Let the two half curves be:

$$\left.\begin{aligned} y_1 &= \frac{n_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}}, \quad x>0 \\ y_2 &= \frac{n_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}\frac{x_2^2}{\sigma_2^2}}, \quad x<0 \end{aligned}\right\} \quad \text{.....(xxii).}$$

Then, since the modal value is common, $\sigma_1/n_1 = \sigma_2/n_2$. Further, the total frequency $N = \frac{1}{2}(n_1+n_2)$. Now write $\kappa = \sqrt{2/\pi}$ and $u = \sigma_1 - \sigma_2$, $v = \sigma_1\sigma_2$. Then taking moments round the mode we easily find:

* Here again it is historically incorrect to attribute these curves to Fechner. They had been proposed by De Vries in 1894, and termed "half-Galton curves," and Galton was certainly using them in 1897. See the discussion in Yule's memoir, *R. Statist. Soc. Jour.* Vol. LX. p. 45 *et seq.*

$$\mu_1' = \kappa u, \quad \mu_2' = u^2 + v, \quad \mu_3' = 2\kappa u\,(u^2 + 2v), \quad \mu_4' = 3\,\{u^2\,(u^2 + 3v) + v^2\}.$$

Transferring to the mean we deduce:

$$\mu_2 = u^2 + v - \kappa^2 u^2 \quad \text{.....(xxiii),}$$

$$\mu_3 = \kappa u\,(v - u^2 + 2\kappa^2 u^2) \quad \text{.....(xxiv),}$$

$$\mu_4 = 3\,\{u^2\,(u^2 + 3v) + v^2\} - \kappa^2 u^2\,\{2u^2 + 10v + 3\kappa^2 u^2\} \quad \text{.....(xxv).}$$

μ_2, μ_3, μ_4 will be known quantities as soon as the frequency distribution is known.

Now determine $\beta_1 = \mu_3^2/\mu_2^3$ and write $\lambda' = u/\sqrt{\mu_2}$, we easily find:

$$\left(\frac{6}{\pi} - 2\right)\lambda'^3 + \lambda' - \sqrt{\frac{\pi}{2}}\,\beta_1 = 0 \quad \text{.....(xxvi).}$$

This cubic* gives by its real root the value of $u = \sigma_1 - \sigma_2$. We then easily deduce

$$\left.\begin{aligned} \sigma_1 &= \tfrac{1}{2}\sqrt{\mu_2}\,(\{4 + (4\kappa^2 - 3)\,\lambda'^2\}^{\frac{1}{2}} + \lambda') \\ \sigma_2 &= \tfrac{1}{2}\sqrt{\mu_2}\,(\{4 + (4\kappa^2 - 3)\,\lambda'^2\}^{\frac{1}{2}} - \lambda') \end{aligned}\right\} \quad \text{.....(xxvii).}$$

These determine the different variabilities of the two halves. Then

$$n_1 = \frac{2N\sigma_1}{\sigma_1 + \sigma_2}, \qquad n_2 = \frac{2N\sigma_2}{\sigma_1 + \sigma_2} \quad \text{.....(xxviii)}$$

give the frequencies in each Gaussian curve, while

$$\mu_1' = \kappa\sqrt{\mu_2}\lambda' \quad \text{.....(xxix)}$$

fixes the position of the origin relative to the known mean value of the system. Thus the complete solution depends on a knowledge of the mean, and the second and third moment coefficients. As before the cubic is readily solved by Lill, Reuschle or Mehmke's mechanisms.

The analysis is now a little more complex than in the case of the Galton-McAlister curve. Write $\epsilon = v/u^2 = \sigma_1\sigma_2/(\sigma_1 - \sigma_2)^2$. Then we have:

$$\left.\begin{aligned} \beta_1 &= \kappa^2\,(\epsilon - 1 + 2\kappa^2)^2/(1 - \kappa^2 + \epsilon)^3 \\ \beta_2 &= \{3\,(1 + 3\epsilon + \epsilon^2) - \kappa^2\,(2 + 3\kappa^2 + 10\epsilon)\}/(1 - \kappa^2 + \epsilon)^3 \end{aligned}\right\} \quad \text{.....(xxx).}$$

Thus again we see that β_1 and β_2 are both functions of ϵ only, or the skewness is not independent of the kurtosis†. Whenever the skewness is zero, the kurtosis must also be zero or the curve be normal.

Now consider the expression $1 - \kappa^2 + \epsilon$ which we will write γ, or,

$$\gamma = \cdot 36338 + \sigma_1\sigma_2/(\sigma_1 - \sigma_2)^2.$$

The last term is positive or γ must be $> \cdot 36338$.

* This cubic was, I believe, first given by Edgeworth.

† The actual relation is:

$$29521\eta^2 + 62500\beta_1^2 - 110506\beta_1\eta + 13468\eta^3 - 11345\beta_1 + 15925\eta = 0,$$

which, as in the Galton-McAlister case, has no obvious physical significance.

Now:

$$\eta = \beta_2 - 3 = \frac{\cdot 45352}{\gamma} - \frac{\cdot 05003}{\gamma^2},$$

and this with the above limitation to the value of γ can never become negative. Hence the double Gaussian curve is, like the Galton-McAlister curve, invariably platykurtic. Now consider the value of

$$\beta_1 = \frac{\cdot 63662}{\gamma} - \frac{\cdot 11477}{\gamma^2} + \frac{\cdot 00518}{\gamma^3}.$$

Hence the ratio $\frac{\beta_1}{\eta}$ tends as γ increases to take the value 1·40374. Equating this to $(1-4pq)/(1-6pq)$ we see that the double Gaussian curve approaches the normal curve along the particular platykurtic binomial $p = \cdot 8985$, $q = \cdot 1015$, or it cannot in the neighbourhood of the normal curve represent any skew binomial but this.

Lastly it may be shown that β_1 has its maximum value when $\gamma = \cdot 36338$ or its minimum value. Thus we find that the maximum possible value of β_1 is about ·99. In the same way the maximum skewness is 1·3236. These values are sufficiently high to cover the great bulk of cases, but I have found $\beta_1 = 4 \cdot 071$ for scarlet fever incidence, $= 1 \cdot 9396$ for age of brides who marry men in their 24th year and $= 4 \cdot 1683$ for the distribution of lips in the medusa *P. pentata*. These exceptions suffice to show that the curve is not general enough.

Summing up we conclude that the double Gaussian curve is not satisfactory because theoretically

(i) It starts by denying the very axioms from which alone we can reach the Gaussian curve;

and empirically because

(ii) It can describe no frequency distribution which cuts the axis at a finite angle, and such distributions constantly occur.

(iii) It is essentially platykurtic. Therefore it is not available for leptokurtic curves, nor even for any but very special skew binomials, i.e. those in which p does not lie between ·2113 and ·7887. As we approach close to the normal curve we get nearer and nearer to one definite point binomial, i.e. that in which $p = \cdot 8985$.

(iv) There is always a relation between the skewness and the kurtosis, or these important physical constants are not independent. In particular we cannot have any form of symmetry but the mesokurtic.

(v) The range of β_1 and of the skewness is fairly large, but frequency distributions actually occur markedly outside this range.

(vi) Lastly, and of much import, the kurtosis can never exceed ·8692, or the maximum value of $\beta_2 = 3 \cdot 8692$. This degree of kurtosis is exceeded in a great number of distributions. Thus in the lips of *P. pentata*, in tint guessing, in the breadth of male English skulls, in the nasal breadth of female English skulls, in

no less than eight of Duncker's series in the case of *Gelasimus pugilator* and in various other distributions. In all these cases the platykurtosis is significant, and the double Gaussian curve fails us hopelessly.

C. (iv) *The Edgeworth-Kapteyn Curves.*

Kapteyn*, without recognizing Edgeworth's priority, has proceeded in the manner indicated on p. 178 above. He assumes that some quantity x obeys the normal distribution

$$y = \frac{N}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}x^2/\sigma^2}.$$

He then takes $x = F(X) - M$ and reaches the frequency distribution:

$$Y = \frac{N}{\sqrt{2\pi\sigma}} F'(X)\, e^{-\frac{1}{2}\{F(X)-M\}^2/\sigma^2} \quad \text{(xxxi)}.$$

Thus far (as we have already shown) nothing has been achieved, because this equation may by a proper choice of $F(X)$ represent any curve whatever. As Kapteyn himself says, following Edgeworth, "as $F(x)$ may represent any function, we see that the equation may be made to represent any curve whatever. Therefore it must be the most general form of frequency curve possible" (p. 17). There is, however, one point to be raised here. What is x of which the observed character X is a function? Is it, as in the explanatory illustrations cited by Kapteyn, another characteristic of the organism? If so we ought in some cases to be able to determine it. What is the character which obeys the normal law? For example, sagittal arc in English women is almost exactly normal in its distribution, and nasal breadth is very asymmetrical. Shall we take x = sagittal arc and X = nasal breadth and make

$$x = F(X) - M?$$

Now every biologist knows that such a relation is not in the least true. No two characters in an organism are in any way connected by a mathematical function, such that when one is given the other is determined. The relation is always of the loose kind that we term association or correlation. X does not fix x, and a multitude of x's with varying degree of probability are associated with a given x. This correlation is often of a very low order. Between any two characters of a given organism, no such relation of perfect correlation as that involved in Edgeworth or Kapteyn's relation has ever been discovered. Very imperfect correlation or at any rate all degrees of correlation have been invariably demonstrated to exist. The function x has no real existence as a biological entity. It is only a mechanism for introducing the normal curve, and is not a true character of the organism at all. Supposing, as in English female crania, nasal breadth is asymmetrical, what is the quantity which is symmetrically distributed of which nasal breadth is a function? It has no reality in the organism at all, and Kapteyn proceeds to make it still more impossible in the following manner. If x has

* *Skew Frequency Curves in Biology and Statistics*, Groningen, 1903.

existence at all, its limits lie between $-\infty$ and $+\infty$, i.e. the whole range of the normal curve. But in order to get a range limited at one end, not the whole series of values of X corresponding to x are taken. A value is selected for X, such that X becomes impossible after a certain value of x. In other words, x is a character which although following the normal curve is abruptly terminated as far as X is concerned at a value with a finite frequency! Kapteyn takes*

$$x=(X+\kappa)^q-M,$$

where κ, q and M are to be determined from the data.

For example, in Professor Weldon's data for the measurement of foreheads of *Carcinus moenas*, Kapteyn (p. 39) finds that with our notation

$$\begin{aligned}\sigma &= \cdot002{,}204,\\ M &= \cdot002{,}561,\\ \kappa &= -0{\cdot}5781,\\ q &= 2{\cdot}21.\end{aligned}$$

Thus when $X=-\kappa$, we have $x=-M=-\sigma$ roughly, or the Gaussian frequency curve for x is to be abruptly cut off, and about 15 per cent. of its tail discarded. If it be said that this could be achieved by natural selection of foreheads, the reply is the simple question: Please show what physical character in a crab is given by an abruptly truncated normal curve! The fact is no such character has ever been met with, and it must be recognised that x represents a wholly fictitious variable having no physiological relation to the character X at all, but introduced solely to reduce the frequency by hook or by crook to that fetish distribution the Gaussian curve.

We can now sum up the objections to Kapteyn's method.

Theoretical:

(i) There is no justification whatever for assuming that some character x actually exists which obeys the normal law of distribution, and that the observed character is a function of this. Some characters are found as a rule in any organism which obey the normal law, but no two characters in an organism have ever been found to be the one a mathematical function of a second, they are always imperfectly correlated.

(ii) Kapteyn's hypothesis involves if his normal character were a physiological entity, that distributions of organic characters should occur which would be represented by fragments of Gaussian curves, or such curves abruptly curtailed. We have no experience of such distributions in actual vital statistics. If they did exist they would contradict the first two axioms on which the Gaussian law itself is based, and would thus deprive that law of the sole justification for its application. As it cannot be supposed that all skewly distributed characters X in an organism are functions of one and the same x, for in this case they would be

* This becomes the Galton-McAlister curve for the limit $q=0$.

perfectly correlated with each other, which is contrary to experience, it must follow that if Kapteyn's hypothesis were correct large quantities of characters distributed in truncated Gaussian curves ought to appear when we deal with variation. The total absence of such characters is evidence that the x-characters are shadow variables and of no biological import.

(iii) The previous statements reduce Kapteyn's special choice of

$$F(X)=(X+\kappa)^q$$

to a mere artifice adopted to get an empirical curve of variation by Edgeworth's hypothesis. Many other functions are *à priori* equally valuable, and might be adopted to get curves of limited range, e.g.

$$F(X)=(X+\kappa)^q(X-\kappa')^{q'}.$$

The hypothesis gains nothing in logical consistency by its appeal to the Gaussian curve; that appeal is one adopted for convenience of fitting, and the sole test of Kapteyn's curve is empirical goodness of fit.

Practical:

(iv) *Every frequency curve should be a graduation formula.* Kapteyn's method of fitting is by equating certain *total* frequencies in order to determine his four constants. They thus fail to successfully smooth any special causes tending to exaggerate any particular frequency group. Such screening of special causes of frequency deviation is far less likely to occur when we use the method of moments, which is a true method of graduation*.

(v) *We ought in every law of frequency distribution to be able to judge of the effect of the unit of grouping on the values of the constants.* This has been satisfactorily achieved for, perhaps, the bulk of cases, when the method of moments is used by Sheppard's corrections†. In Kapteyn's process we have no means of ascertaining the extent to which the size of the unit of grouping influences the constants of his distribution.

In his Example II., for instance, he takes his curve to accurately reproduce the total area of the group of houses under £10 annual value. What difference would

* Thus Kapteyn deals with some statistics of the values of house property in England fitted by me (*Phil. Trans.* Vol. 186 A, p. 396). I specially state that £20 was the limit to taxable value, and that accordingly the frequency of houses immediately below this value will be exaggerated. Kapteyn's method fails to indicate such a source of *à priori* recognised irregularity. For example, one of his conditions is that the houses of value less than £10, i.e. more than half the total frequency, shall be *identical* in his result with the observed frequency. He thus cuts away at once any possibility of smoothing this group or allowing for the large probable error in it due to random sampling even. His method leads to a limiting house value of £2. 2*s*., while mine leads to £4. 4*s*. Mine corresponds to a weekly rent of about 2*s*.; his to a weekly rent of 1*s*. The latter rent hardly occurs in England unless the house is given in part payment of wages, or in charity. Kapteyn says that his distribution starts with a zero frequency, and mine with an infinite ordinate. "It seems hardly admissible that the latter solution can be in accordance with nature (*sic*) in this particular." Why not? An infinite ordinate may and does in my case give a finite frequency.

† See reference, p. 187.

be made if the first group had included only houses under £5? We are unable to answer this question.

(vi) *Every frequency curve should be determined by constants of which the probable errors are easily deducible.* The method of moments admits of the probable errors of the moments being easily determined (see *Biometrika*, Vol. II. pp. 273 *et seq.*). My system of skew curves gives all the constants in terms of moments whose probable errors are known.

The moments in Kapteyn's theory depend on the integration of:

$$\int_{-M}^{\infty} (x + M)^{\frac{n}{q}} e^{-\frac{1}{2}x^2/\sigma^2}\, dx,$$

and there is no means of readily evaluating this integral. *In fact the arithmetical mean*, the standard deviation, the skewness and the kurtosis, and the modal divergence are unobtainable from the constants of Kapteyn's theory.* This seems to me sufficient to deprive the method of any practical significance even as an empirical representation.

It has further been shown by Sheppard that the probable errors of constants determined by class frequencies (partial areas) are higher than when these constants are determined by the method of moments. We may give the above statement a separate paragraph as:

(vii) The fundamental physical constants of the frequency distribution are not determinable from Kapteyn's empirical curve.

To illustrate the results of this want of a knowledge of the probable errors, I turn to the three illustrations given by Kapteyn.

Example (i). Observations on the *Threshold of Sensation.* Kapteyn himself shows that his solution is hardly less satisfactory if he uses the Galton-McAlister curve (our equation (xi) p. 194). He does not therefore know whether $q = \cdot 00$ and $q = -\cdot 04$ differ within the probable error of q.

Example (ii). *Valuation of House Property.* Kapteyn fits this with a Galton-McAlister curve for his q comes out ·00. Owing to the difficulty in calculating moments, we cannot do more than approximate to the value of χ the skewness in these data. I make it 1·8. It is certainly well over unity. We have already seen that it is impossible for a Galton-McAlister curve to give a skewness above ·21. The apparent agreement Kapteyn finds for the frequencies is not therefore sufficient evidence that the fundamental constants of the distribution will be really given by reasonable values.

Example (iii). Foreheads of *Carcinus moenas.* Kapteyn fits these first with $q = 2 \cdot 21$. "The agreement seems satisfactory." Then with $q = 0$, or a Galton-McAlister curve, "The representation is hardly less satisfactory." Then with

* "The arithmetic mean of all the X's cannot be *generally* found in a simple and rigorous way," Kapteyn, p. 44.

$q = \infty$, "The representation, though sensibly less satisfactory than that by the [previous] solutions, is still pretty close."

A method by which a fundamental constant of the distribution may take any value between 0 and ∞ and still give a "pretty close" representation, must I think condemn itself. Such a statement demonstrates effectively that the author has not yet determined numerically or even approximately in his own mind the probable errors of the constants he uses.

It will be seen that as far as the three illustrations Kapteyn himself gives go he has not advanced the matter beyond the Galton-McAlister curve. That curve fits reasonably (according to Kapteyn) all his three series. But the skewnesses of the three series are respectively ·72, 1·8 and ·32. I have calculated these roughly, but I think there can be no doubt of their approximate correctness. In every one of these cases the skewness sensibly exceeds the maximum limit of skewness, i.e. ·21, possible for the Galton-McAlister curve which Kapteyn applies to them*.

C. (v) *The General Results which flow without the Third Gaussian Axiom.*

It seems to me accordingly that very grave objections can be raised not only from the theoretical but from the practical standpoint to the methods I have discussed which attempt to allow for asymmetry, i.e.

(i) The Galton-McAlister Geometrical Mean Law

(ii) The Galton-Fechner use of Half Gaussian Curves,

(iii) The Edgeworth-Kapteyn use of transformed Gaussian Curves.

All these experienced statisticians differ *in toto* from the opinion of Ranke and Greiner—that we need not trouble about descriptive curves for asymmetrical distributions—but their methods seem to me unsatisfactory theoretically and insufficient practically, because they still make a fetish of the Gaussian axioms. They do not return to the Laplace-Poisson method of replacing those fundamental axioms by more general conceptions. If a Gaussian curve does not fit, they will consent to deduce their own curves from a truncated Gaussian curve, which some shadow variable of the mathematician is supposed to follow, and of which we have no experience in any organic characters hitherto measured. Indeed if we had such experience, it would at once negative the very axioms on which the Gaussian curve is based.

Now it seems to me that all these attempts, whether embodied in the general method of Edgeworth or in the special hypotheses of Galton-McAlister or Kapteyn, amount to abolishing the third of the Gaussian assumptions, namely that small increments of the variable or the character are independent of the total already reached. That is to say that they amount to saying that increments of the

* I am unable to say how far the general form of Kapteyn allows for the requisite range of skewness and kurtosis, because neither the modal difference, nor the standard deviation, to say nothing of the higher moments, can in general be evaluated.

variate are *correlated* with the value of the variate already reached*. Galton and Fechner made the increment proportional to the variate. But in our ignorance of the actual nature of variation in organisms, we have no reason at all for making such a narrow assumption. We can to please our critics put the matter as I have already indicated in the Gaussian form. We simply assume that if the causes of variation in the immediate neighbourhood of the character x_0 remained the same for the whole range they would give a normal curve, hence we should have a relation of the form:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma^2}.$$

They do not, however, remain the same; the tendency to vary at x is a function of x, in other words $\sigma = \sigma_0\sqrt{F(x)}$, where $F(x)$ is an arbitrary function. We have then

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{\sigma_0^2 F(x)}.$$

This is a result as general as Edgeworth's and more so than Kapteyn's or Fechner's. We now take the simplest possible functional series for $F(x)$, i.e.

$$= a_0 + a_1 x + a_2 x^2 + \ldots.$$

The coefficients a_0, a_1, $a_2 \ldots a_n$ can be found at once in terms of the moments†, and my special curves result if we stop at a_2. Against going to higher powers are the objections I have raised in my memoir on skew correlation‡, namely (i) that the higher powers involve moments of the 5th and higher orders and their probable errors are very large, (ii) that it has not yet been shown that going to a_2 does not suffice to describe all the types of frequency which occur in common practice.

The above is the simplest and most general form into which I would put my theory of asymmetrical frequency for those who feel compelled to approach all frequency from the Gaussian standpoint.

D. *Specific Criticisms of Ranke and Greiner on my Theory.*

I think these may be summed up as follows:

(*a*) That all distributions of variates are continuous, and that accordingly no curves, however closely they may approximate to finite discontinuous series like the binomial and the hypergeometrical series, can be applicable to variation in nature.

* Suppose we draw r cards from a pack, and wish to consider the chance of s being of one suit, we may do so by drawing one card at a time, observing it and returning it, and then drawing again. Here there is not correlation between the successive contributions to r. Or we may draw the r cards, without replacing the individual; here the successive contributions are correlated with the previous contributions, and the third Gaussian principle is upset.

† "Mathematical Contributions to the Theory of Evolution, XIV. On the General Theory of Skew Correlation and Non-Linear Regression," *Drapers' Company Research Memoirs.* Biometric Series, II. (Dulau and Co. 1904) p. 6.

‡ *Ibid.* p. 7.

(*b*) That what I term the "number of cause-groups" must be infinite in number, for without such infinity it is impossible to reach continuity.

(*c*) That looked at from the standpoint of binomial or hypergeometrical series the constants of some one or more of my curves may become unintelligible.

Now not one of these objections has any application to the method which has been used in this paper to deduce the differential equation to my curves. But I still think there are very grave objections to every one of the above statements.

To begin with (*a*). We meet in an immense variety of living forms with discrete variates. For example, the number of teeth on the rostrum of a prawn, the number of lips of a medusa, the number of veins in a leaf, the number of glands in a swine's foot, the number of tentaculocysts in *Ephyra*, the number of individuals in a litter, the number of bands on snails' shells, the number of somites in the body of an earthworm, the number of petals or sepals in a flower, etc. etc. Are we to put all these distributions of variation on one side because Ranke and Greiner hold that all distributions of variates are continuous? We have in these cases probably continuous causes producing discontinuous distributions. Are we not to use the areas of a continuous curve to give the frequency of such discrete variates?

Consider for example the function given by:

$$y = N \times \sum_0^n \left\{ p^{n-r} q^r \frac{\lfloor n}{\lfloor n-r \rfloor r} \frac{1}{\sqrt{2\pi}\sigma_r} e^{-\frac{1}{2}(x-rc)^2/\sigma_r^2} \right\}.$$

This is compounded of $\overline{n+1}$ normal curves, the area of the $(r+1)$th normal curve being $Np^{n-r}q^r \frac{\lfloor n}{\lfloor n-r \rfloor r}$, i.e. the $(r+1)$th term of the binomial $N(p+q)^n$, and this $(r+1)$th normal curve has σ_r for its standard deviation. The origin of the system is at the mode of the normal curve corresponding to $r=0$, and the means of these normal curves are spaced equal distances c apart.

When every $\sigma_r = 0$ we have discrete variation. When σ_r is small, less say than $\frac{1}{8}c$, it would probably be difficult to distinguish the result from discrete variation. Enlarging σ_r we pass on till we get a system which it would be practically impossible to distinguish from continuous variation, even if n were only moderate in magnitude. I lay no stress whatever on the above expression because I am in no sense pledged to any Gaussian curve, but it illustrates well what I want to express: namely, in actual nature the frequency might fundamentally fall on certain values of the character, but that the effect of nurture, environment, and growth may well scatter the values of the variable round the fundamental value, so that continuity of variation is all that can be actually observed. The number of somites in an annulose animal is discrete and probably inherited, but the length of the body may appear as a continuous variate. I do not think for a moment that the distinction made by Ranke between discrete and continuous variation,

and the further statement that variation in man is essentially continuous, is at all valid. Our units of grouping for the numbers available are not very fine, we can hardly successfully classify a few hundred observations into more than 20 groups, and with this unit of grouping it would be practically impossible to distinguish between the apparently continuous distribution and a discrete distribution of a similar number of classes with the variates modified by growth, nurture or any other scattering tendencies. Ranke appears entirely to have overlooked the current biological theory of inheritance summed up in the words *inheritance by determinants*. Such theories, whether they be those of Weismann or Mendel, lead us directly to discrete variation*. The discreteness of the variation will be more or less, in many cases probably entirely, obscured by the environmental influence. In such cases the number of fundamental cause-groups is not infinitely great, and Ranke is overlooking current biological views when he asserts that we *must* take "die Anzahl der Elementarursachen selbst als unendlich gross und die Grösse der Wirkung der einzelnen Ursache als unendlich klein." If the number of determinants which fix a character is finite, that character would correspond to a discrete variation of limited range. If the number of determinants be very large, the distribution would by Laplace's theorem be represented more and more closely by the normal curve.

I toss ten coins into the air and for every head in the result I pay a gramme of gold-dust, the frequency distribution of gold-dust would closely be given by the terms of the binomial $(\frac{1}{2}+\frac{1}{2})^{10}$ as in the points of our Fig. 1. But suppose instead of weighing my gramme of gold-dust accurately, I give a "handful" of sugar. If 6 heads turn up I give six handfuls of sugar, but each of these will not be exactly my standard mean handful. I am unlikely to give either five or seven standard handfuls as my six approximate handfuls, but in some cases even these might be possible; we pass in fact from discrete to continuous variation, and the multimodal character of the discrete variates will disappear with the roughness of the handfuls, or have the peaky appearance of random sampling. The total area up to any midpoint between two discrete groups s and $s+1$ will be given by the continuous integral which represents the first s terms of the binomial. If we have two such total areas, one up to the midpoint between groups s and $s+1$ and the other up to the midpoint between groups $s+1$ and $s+2$, then an interpolated area between these values as given by the continuous integral will be sensibly the same as if, c being the unit of discrete difference, we determined a curve corresponding to the mean binomial frequency in the *Spielraum* c; i.e.

$$\frac{N}{c}p^{n-r}q^r\frac{\lfloor n}{\underline{n-r}\,\lfloor r},$$

by simply fractionising r, i.e. we replaced the factorials by Stirling's theorem or used Γ functions, and supposed r to change continuously from s to $s+1$. This is

* Thus I have shown that a generalised Mendelian theory leads directly to skew binomial distributions of characters in the general population. *Phil. Trans.* Vol. 203 A, pp. 53—86.

merely another way of looking at the change from discrete to continuous variation, due to the influence of a multitude of causes on the discreteness of the variates which fall into a given *Spielraum*. I still find nothing absurd in the statement that the actual effect of the scatter is sensibly equivalent to a fractionising of the indices. It is simply equivalent to the statements, (i) that the ordinates of the Gaussian curve closely give, even for small values of n, the terms of the binomial, (ii) that the ordinate of the Gaussian curve between two terms of the binomial closely gives a fractionised binomial term (owing to Stirling's theorem being true for fractionised factorials or Γ functions), (iii) that we have no knowledge of how the "scatter" within the *Spielraum* may be distributed so as to give a continuous effect*. Now these points are not in the least needful for the deduction of my skew curves, they are merely given here because in our complete ignorance of the nature of the causes, hereditary and environmental, which produce continuous variation, I think we have no warranty for saying that a limited number of cause-groups is impossible, or that no such limited number of fundamental cause-groups could give a continuous variation. In the present state of our knowledge we cannot agree with Ranke in sweeping away as impossible all the discreteness which follows from determinantal theories of inheritance. We cannot afford to be dogmatic as to the continuous or discontinuous character of the ultimate sources of variation and any effective theory must like the Laplace probability integral be equally applicable to the sum of a discontinuous series as well as to the areas of a continuous curve.

(*b*) Any finite series of cause-groups, Ranke tells us, must lead to discontinuity.

I have endeavoured to show above that the discontinuity may be as real and yet as undetectable as the distribution of lengths, say, of the vertebral columns of sharks which yet depends on the number of discrete vertebrae, with a scatter of their individual sizes. But Ranke's argument in itself is a false one, many discontinuous systems lead at once to continuous distributions. In our ignorance of the exact sources of variation, all we can do is to show that a limited number of cause-groups can quite well lead to continuous variation. To take a perfectly arbitrary illustration, suppose that a character can only take values lying between a_1 and a_2 and that this character is to be settled by the determinants derived from $s+1$ ancestors, i.e. suppose all but $s+1$ to be cast out in the successive divisions of the germ-cell. Then it by no means follows that the character will be a *blend* of these $s+1$ determinants, one or other of them may be *dominant*. It does not follow that the dominant one represents either the one with the least or the greatest value of the character. It might be the one with $s-r$ determinants below and r above it.

* For example I have a variety of Binomial machines or "Quicunxes" like that figured in my memoir, *Phil. Trans.* Vol. 186 A, Plate 1. Fig. 2, p. 414. It is quite possible to arrange a quicunx in which there are only a limited number of compartments, but in which the top of the seed in these compartments is not horizontal, but gives a continuous curve, e.g. the greater air draft of the greater frequency might be used to pile up the material in any receptacle on the side of the greater frequency.

In this case the frequency of individuals with character x would be given by the curve

$$y = y_0 (x - a_1)^{s-r} (a_2 - x)^r.$$

This is a perfectly continuous curve, and one of my series of skew curves. Thus it is quite conceivable that a finite number of fundamental cause-groups should lead to an absolutely continuous distribution*.

Now how does Ranke treat this illustration? He first states that all continuity must involve an infinite number of cause-groups, or variation in man being continuous, must be associated with an infinite number of cause-groups. He had this very case before him, and yet he writes:

> Die Analyse der Elementarursachen ergibt uns also unweigerlich die bisher immer angenommene unendliche Anzahl derselben, die unendliche Kleinheit der Wirkung jeder einzelnen Ursache und die Kontinuität der moglichen Wirkungsgrade.
>
> Sie ergibt also wirklich die Verhältnisse, die wir zum Verständniss der kontinuerlichen Variationskurve ganz unumgänglich nötig haben. Denn wie soll eine kontinuerliche Kurve sich aus der Kombination endlicher Bausteine ergeben (S. 321)?

Ranke only gets out of the difficulty by asserting that since the number of causes is finite, but must be infinite for variation, my continuous curve based on a finite number of cause-groups cannot represent variation! A more remarkable specimen of circular reasoning can hardly be conceived. The fact is that Ranke suffers from the old third Gaussian axiom, i.e. the supposition that the increments that go to build up the variate are independent of each other. The fundamental cause-groups are by no means *Bausteine* in the sense that the total variate is the sum of these *Bausteine* placed on top of each other! The causes determine the magnitude of the variate, but not at all necessarily by their sum.

(*c*) Ranke asserts that some of my curves have constants which if we endeavour to interpret them from the standpoint of the binomial give impossible or improbable values for the constants.

The answer to this is that the series were only the scaffolding to deduce the curves. The differential equation to the curves contains the limit to a good many other frequency systems which directly diverge from the fundamental axioms of Gauss. I used the original series as a means of dispensing with the Gaussian axioms in familiar cases, but the result reached involves a good deal more than can be interpreted by the original series. Ranke can only see absurdity in a binomial with a negative p or q. But the nature of the sources of variation is so little known to us that we cannot possibly assert the absurdity of such values. We may not indeed be able to directly interpret them in the case of man, say, but they occur and recur in chance investigations. I will illustrate this in one case only, but such will demonstrate the required possibility and dispose at once of Ranke's argument as to absurdity.

* Making r and a_2 infinite, but $s - r$ finite, we get the curve I have deduced as the limit to a binomial of finite power. In other words, that curve is also shown to correspond to a possible continuity.

Suppose an organ to require the conjunction of exactly n determinants of one kind to fix it, but that the size of the organ depends on how soon this conjunction takes place. Let D be the necessary kind of determinant and π the chance that it is left in the right position after each operation, say a cell-division or cell-fusion. Let D' be any other kind of determinant and κ its chance of appearing, so that $\kappa + \pi = 1$. Then if D appears n times in the first n operations, we have a certain size for the organ, if in the first $\overline{n+1}$ operations another size, and if only in the first $n+r$ operations a third size. But the chances of these respective appearances are the terms of the series

$$\pi^n + n\pi^n\kappa + \frac{n(n+1)}{2}\pi^n\kappa^2 + \ldots + \frac{n(n+1)(n+2)\ldots(n+r-1)}{\lfloor r}\pi^n\kappa^r + \ldots$$

$$= \pi^n(1-\kappa)^{-n} = \left(\frac{1}{\pi} - \frac{\kappa}{\pi}\right)^{-n} = (p-q)^{-n}, \text{ say,}$$

where $p - q = 1$. Here p and q have lost the condition that they are both to be less than unity. I do not for a moment suggest that this is the real interpretation of a binomial with a negative q. I only assert that because we are as yet unable to certainly interpret such expressions, the absence of interpretation does not involve the absurdity which Ranke postulates.

There are many other matters to which I might justifiably take exception in Ranke and Greiner criticism*, but I think I shall have said sufficient to convince the impartial reader of the following points:

(i) The great bulk of modern statisticians are agreed that the Gaussian law is absolutely insufficient to describe observed facts. They may disagree as to the method of supplementing it. I do not think that the opinion of Ranke and Greiner can possibly weigh against those of Poisson, Quetelet, Galton, Edgeworth, Fechner and Kapteyn—all authorities who have had to deal for years with statistical data.

(ii) The original use of the probability integral (the areas of the Laplace-Gaussian curve) as introduced by Laplace was to represent the sum of terms of a discontinuous series. To the mathematical mind there is no absurdity in this replacement of discontinuity by continuity; it is the basis of the Euler-Maclaurin theorem.

(iii) The dogmatic assertion of Ranke that variation in man is due to an infinite number of infinitely small fundamental cause-groups, simply neglects the

* For example, all the discrete variates mentioned on p. 205 have been dealt with by biometric writers, and many others besides, yet Ranke speaks as if such writers had not dealt with discrete variation. He speaks of Ludwig's multimodal curves for flowers as if there had been no controversy as to the actuality of the "Fibonaccizahlen" modes, when due regard is paid to homogeneity of season and environment. He speaks as if Johannsen had demonstrated normal variation in his "Erbsenspopulation," when he has really applied no valid criterion whatever to test for asymmetry, etc. In short he seems to me to have neglected a great deal of the modern literature of the subject, and, if I may venture to say so, to write over-dogmatically on what he has read.

whole determinantal theory of inheritance. A complete theory of asymmetrical frequency must describe in the manner of Laplace's probability integral either continuous or discontinuous variation.

(iv) The apparent or practical continuity of many variation data may be due either to real continuity or to discontinuity effectually masked by: (*a*) the relative paucity of material and roughness of our measurements, which compels us to divide it into groups of the same order of number as the number of determinants, (*b*) the influence of age, nurture and environment superposed upon the pure hereditary results, or (*c*) the fact that many of the characters measured by us are built up of a larger or smaller, but not necessarily an infinite, number of simple organs or characters, which may possibly individually have discontinuous variation*.

(v) The assertion of Ranke that a finite number of fundamental cause-groups *must* lead to discontinuity is disposed of by illustration. It is quite possible to invent a great variety of determinantal systems—the number of the determinants being finite—which lead to continuous variation. In our present ignorance of the sources of variation, especially of the mechanism of inheritance, it would be idle to lay weight on any special interpretation of this kind. It is important, however, to observe that continuity or discontinuity of variation are not, as Ranke asserts them to be, associated with the finite or infinite number of the cause-groups.

(vi) The absurdity which Ranke finds in the values taken by some of the constants of my curves, exists only when a very narrow view is taken of the sources of organic variation. A binomial series with negative power or with negative p or q is capable, as is shown in this paper, of perfectly rational interpretation. But in the present state of our knowledge it would be idle to specify any particular interpretation as the correct one†.

(vii) The problem of variation can be looked at in the following manner without the least loss of generality. Modify Gauss by replacing his third axiom, the independence of contributory increments to the variate, by the postulate that the increments are *correlated* with previous increments‡. Start with any binomial and we reach the generalised probability curve for an infinite number of cause-groups:

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2 f(x/\sigma_0)},$$

where f is an arbitrary function. This theory covers Galton, Edgeworth, Kapteyn and Fechner. Expanding $f(x/\sigma_0)$ in a series of ascending powers of x/σ_0 we have

* Ranke has quite overlooked the work by Galton and myself on the *discontinuity* of the series of individuals even when the population obeys the Gaussian law. See *Biometrika*, Vol. I. pp. 289—299.

† Ranke apparently considers that $(p+q)^n$ with p, q and n positive *is* interpretable. A little philosophical consideration will show that it is merely "familiar," not really intelligible. There is no physiological meaning in p, q, n, and we cannot as yet associate them with any true organic mechanism.

‡ This postulate of course abrogates the first two axioms of the Gaussian theory as well.

my generalised probability curves*. A very few terms of the expansion, however, suffice for describing practical frequency distributions. If we keep only three terms, we see that the same system of curves suffices to describe continuous and discrete variates—an important point. If I lay stress upon this method here, it is because Ranke insists on an infinity of cause-groups and supposes no continuity can arise without them—

> ..."a truth
> Looks freshest in the fashion of the day."

(viii) The important physical constants of a frequency distribution are those which can be determined with the least probable errors. The probable errors of the moment coefficients increase rapidly with the moments. Hence the important physical constants are those which depend on the low moment coefficients, i.e. on the early terms of the expansion of $f(x/\sigma)$. Now these physical constants are (*a*) the mean, (*b*) the modal difference or distance of mean from mode, (*c*) the skewness, and (*d*) the kurtosis. We may replace either (*b*) or (*c*) by the standard deviation. Experience shows that these four physical constants are certainly independent. The constants of my skew curves directly give them and we are able to determine by their probable errors whether they are significant or not.

(ix) With regard to the other theories discussed I have shown:

(*a*) That the Galton-McAlister curve, ascribed by Ranke to Fechner, is not applicable to a great number of cases, for its kurtosis is a function of its skewness and its skewness cannot exceed ·21.

(*b*) That the double Gaussian curves due to Galton and Fechner are illogical, because they reach a Gaussian result by rendering invalid every one of the Gaussian principles. Further, the skewness is always a function of the kurtosis and the kurtosis cannot exceed ·87, a degree which is exceeded in a great variety of data.

(*c*) That the Edgeworth curves as developed by Kapteyn fail from the logical standpoint, for they appeal to a truncated Gaussian distribution which has never been observed in experience. They are not true graduation formulae, and are obtained in such manner that it is not possible to determine any one of the chief physical constants or evaluate their probable errors. Further in the examples given by Kapteyn they all sensibly reduce to the Galton-McAlister curve. But this curve has in every one of the cases dealt with by Kapteyn a skewness significantly less at a maximum than is required by every one of the statistical series involved.

Finally it seems to me that all discussion of asymmetrical frequency must turn in one form or another on the proper form to be given to $F(x)$ in the equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma_0^2 F(x)}.$$

* See "Mathematical Contributions to the Theory of Evolution, XIV." Dulau and Co., London.

If we assume it to be $F(x) = \Sigma(a_r x^r)$ as I have done, we fall back on the normal curve when a_r for $r > 0$ is zero within the limits due to its probable error. Ranke, if he wishes to demonstrate the Gaussian law as general, must show this to be the case. It has been over and over again demonstrated that a_1, a_2, etc. differ significantly from zero for a great variety of series. Another advantage of the form $F(x) = \Sigma(a_r x^r)$ is that it covers as I have shown discrete as well as continuous variation. Considering $\sigma = \sigma_0 \sqrt{F(x)}$ as the standard deviation of the "instantaneous Gaussian curve," we see that the "instantaneous Gaussian curve" varies from one position to a second, like the "instantaneous ellipse" of the astronomers. A reasonable first hypothesis to make is that the local mean square deviation σ^2 is independent of x, we obtain the Gaussian curve. A next assumption is that it is a linear function of x—perhaps it would be better to say that its *mean* local value is a linear function of x, i.e. the mean square of the local variability σ^2 is *correlated* lineally with x. This gives my curve of Type III. The next easiest assumption is to suppose the regression line of σ^2 on x to be parabolic. In this case we obtain the remainder of the curves treated in my II. and XI. memoirs. If we stop at a_q we have what I have termed the skew frequency curves of the qth order*, and we see that this involves a regression curve between the square of the mean local variability and the character of the qth order*. I see, however, at present no *practical* necessity for proceeding beyond skew curves of the 2nd order, although I propose shortly to publish a discussion of skew curves of the 3rd order illustrating some *theoretical* points which arise in their discussion.

To sum up I think Ranke's criticism fails (α) because he has disregarded the universally recognised need of modern statistical science for asymmetrical frequency curves, (β) because he has not appreciated the mathematical transformation by which a number of finite terms are replaced by an integral expression, (γ) because he has not realised that modern theories of heredity lead directly to discontinuous skew distributions, (δ) because continuity does not depend upon infinity of fundamental cause-groups, and lastly (ϵ) because, and this may be due to my fault in the first deduction of my curves, he has quite failed to see either their scope or their real generality.

* "Mathematical Contributions to the Theory of Evolution, XIV. On the Theory of Skew Regression." Dulau and Co.

VARIATION AND CORRELATION IN THE EARTHWORM*.

By RAYMOND PEARL and WILBUR N. FULLER.

1. *Introductory.*

The purpose of this paper is to present the results of a study of the variation and correlation exhibited in certain specific characters of the common garden earthworms.

The earthworm which occurs most abundantly in the region about Ann Arbor and with which we have dealt in the present work, has usually been identified as *Lumbricus herculeus*, Sav. or *L. agricola*, Hoffm. The counting and measuring and a portion of the computing were done by W. N. Fuller in connection with certain class-work in the University. The values of the constants were twice independently computed, and the paper written by R. Pearl.

There were two principal reasons which led to the undertaking of this work. In the first place we wished to determine if possible more exactly than had previously been done the typical condition of the form of earthworm commonly found in this portion of the United States. The earthworm is such a generally used type form, both for laboratory teaching and for investigation in a number of fields, notably regeneration, that an exact determination of the existing type and variation with reference to the several specific characters seemed decidedly desirable. Secondly, the writer hoped to get from this form some light on certain general biometrical problems on which he is collecting data.

The material for the work consisted of sexually adult specimens only. The criterion used to determine whether an earthworm was sexually adult, was the presence of a fully developed clitellum. That this is a justifiable criterion has been shown by Cole†.

* Contributions from the Zoological Laboratory, University of Michigan, No. 89.

† *Zool. Anz.* Bd. xvi. pp. 440 and 453.

The worms were collected on rainy nights, in gardens and lawns near the campus of the University of Michigan. On such nights the worms leave their burrows, and it is comparatively easy, after some experience, to pick up a large number in a reasonably short time.

The worms were brought to the laboratory, stupefied in weak alcohol, straightened to their full length, injected with weak chromic acid or formalin, and hardened and preserved in either alcohol or formalin. As the worms are stupefied in alcohol the muscles become relaxed, and it is easily possible to straighten them to their full length. They are hardened in this straightened position. The specimens used in this investigation were subsequently used by students for laboratory work. The measurements and counts were made during the first half of the fall semester of the college year, 1902–03.

Quantitative determinations of the following characters were made on each worm:

1. Total number of somites in the body.

2. Length of body in centimetres.

3. The number of somites from the anterior end of the worm to the most anterior somite of the clitellum.

4. The number of somites included in the clitellum.

5. The position of the genital openings.

The following points regarding the methods used in making the determinations may be noted. In all counts of somites which began at the anterior end of the worm the prostomium was not counted, but the first somite which was complete on the ventral side of the worm was counted as 1. All the other somites in the body were included in the count. All worms having spiral segments or other segmental anomalies were excluded in this work. Such worms were preserved for use in connection with a general study of teratology in the earthworm. In making the counts it was found very helpful to use a "tally register," such as is used in counting telegraph poles, and by gate keepers at places of amusement for counting people. This instrument has been found very useful in collecting biometrical data on integral variates for a variety of characters (e.g., scale rows and gill-rakers in fish, etc.). Anyone who has ever attempted to make a series of counts of repeated like characters knows how difficult it becomes in a short time to "keep the place," both mentally and on the specimen. All this difficulty is overcome by the use of a tally register, and its use makes it possible to work for a long time without becoming unduly fatigued. The only thing necessary to secure both speed and accuracy is that the operator have some preliminary practice.

With regard to the character "length of body" considerable care had to be exercised in order to get results in any way reliable. Of course, it would be nearly or quite impossible to get any reliable measure of the length of an earthworm

while it was alive, on account of the varying degrees of contraction of the muscles. This source of error is present to a certain degree in worms fixed and hardened in the way described above, but it is very greatly reduced in the fixed as compared with the living worm. Since, however, there was present in all cases a source of error on account of different degrees of contraction, no attempt was made to obtain close measurements of length. What was desired was to get all the worms classified into a comparatively few, rather large length groups, for purposes of correlation. This end was attained by using as our smallest unit of measurement a distance of 2·5 centimetres. A worm was laid on a measuring stick divided into 2·5 cm. spaces, its length observed, and the record made by writing on the record sheet a letter with an algebraic sign, denoting the division of the measuring stick within which the end of the worm fell. Thus a record of $C-$ indicated that that particular worm fell, in length, somewhere between 10 cm. and 12·5 cm. In making the computations all worms falling within the class $C-$ were considered to have been 11·25 cm. long, or in other words it was assumed that within a group the lengths were arranged symmetrically about the centre of that group. The assumption is, of course, the same as is usually made in dealing with the frequency distribution of graduated variates.

In the counts for the position and number of somites in the clitellum the following arbitrary rules were followed. The number of somites from the anterior end of the worm back to the *first somite on which any clitellar tissue was present*, was recorded as "Number of somites to clitellum." *All somites on which any clitellar tissue was present* were recorded as "Number of somites in clitellum." Some such arbitrary rules were necessary in dealing with the cases in which the clitellum included only a part of one or both of its terminal somites. A record was kept, according to a plan which will be described later, of the exact extent and form of the clitellum in each particular worm.

The only variations which were found in the collection of worms with reference to the position of the genital openings were regarded as teratological and the specimens exhibiting such variations were excluded from our tabulations. Abnormality in position of the genital pores in the very few cases in which it appeared was found associated with other abnormalities. The position of these openings is perfectly constant in normal worms. There is no "fluctuating" or continuous variation. On this account these characters will not be discussed further in this paper.

It should be stated that the collection was made entirely at random. No selection on the basis of any characteristic of the worm, with the exception of the presence or absence of the clitellum as noted above, was exercised in forming the collection. The question of the homogeneity of the material will be considered farther on in the paper.

We may turn now to the results.

2. *Total Number of Somites and Length of Body.*

In Table I. is shown the frequency distribution for the total number of somites in the body.

TABLE I.

Total Somites.

Number of Somites	Frequency	Number of Somites	Frequency
79	1	*130*	1
80—88	—	*131*	4
89	2	*132*	7
90—92	—	*133*	4
93	1	*134*	3
94	—	*135*	5
95	1	*136*	6
96—98	—	*137*	9
99	2	*138*	10
100	1	*139*	6
101	—	*140*	5
102	2	*141*	17
103	—	*142*	27
104	3	*143*	21
105	1	*144*	24
106	1	*145*	37
107	2	*146*	32
108—112	—	*147*	31
113	1	*148*	29
114	3	*149*	26
115	3	*150*	51
116	—	*151*	15
117	—	*152*	15
118	2	*153*	12
119	—	*154*	9
120	5	*155*	9
121	—	*156*	10
122	1	*157*	4
123	2	*158*	—
124	2	*159*	2
125	5	*160, 161*	—
126	2	*162*	2
127	1	*163*	—
128	5	*164*	1
129	4		
		Total	487

This distribution is shown graphically in Fig. 1. There is hardly any doubt that the apparently multimodal condition of this histogram is due simply to the fact that we have a limited amount of material and a very wide range of variation. Our sample is not sufficiently large to cover smoothly the whole of the range.

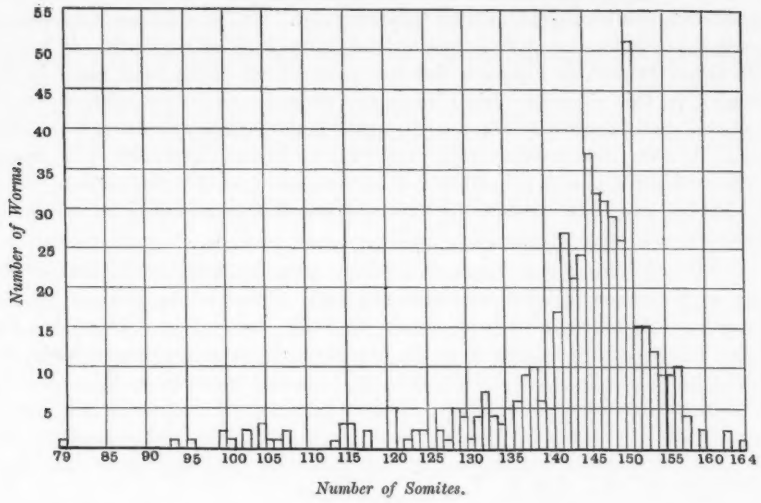
The values for the mean, standard deviation (with Sheppard's correction), and coefficient of variation deduced from this frequency distribution are as follows:

Mean = 142·715 ± ·362 somites.

Standard Deviation = 11·850 ± ·256 somites.

Coefficient of Variation = 8·303 ± ·181 %.

FIG. 1. Variation in number of Somites in body of Earthworm.



Turning to the character "length of body" we have the frequency distribution given in Table II.

TABLE II.

Length of Worms. Class unit = 2·5 cms.

Length in centimetres	Frequency
10 —12·5	7
12·5—15	50
15 —17·5	75
17·5—20	153
20 —22·5	137
22·5—25	55
25 —27·5	9
27·5—30	1
Total	487

The constants deduced from this distribution have the following values:

Mean = 19·171 ± ·094 cms.

Standard Deviation = 3·077 ± ·067 cms.

Coefficient of Variation = 16·049 ± ·356 %.

From these values the following points are to be noted:

(*a*) The range of variation in respect to number of somites in the body is very great. The explanation of this fact is simple. The sample of worms under consideration is not homogeneous with respect to age. Up to a certain age at least, *Lumbricus* adds somites at the posterior end of the body with growth. It is clear from either Table I. or Figure 1, that the great extent of the total range in the variation in this character arises from the presence of a comparatively small number of individuals with few somites which unduly extend the lower end of the range. It seems reasonably certain from what we know of the method of growth in the earthworm that if this sample of worms could have been allowed to go on growing the range of variation in total number of somites would have steadily decreased with lapse of time.

The fact that there is no apparent tendency towards extreme extension of the range at the upper end indicates that the bulk of the worms included in the sample have either completed the process of adding somites or at least that the process is going on very slowly if at all. So then the sample serves to define the existing typical condition of the worms in the region with respect to this character with a reasonable degree of accuracy. It is to be regretted that an entirely homogeneous sample with respect to age cannot be obtained, but in the nature of the case this is practically impossible. It would involve following each individual worm from the time that its development began.

(*b*) The earthworm is apparently more variable in respect to length of body than in respect to number of somites. This would indicate that the observed variation in length is not due primarily to variation in *number* of somites, but rather to variations in the *lengths* of the individual somites in different worms. Of course, it immediately occurs to one that possibly some of the greater variation in length is due to varying degrees of contraction in individual worms. As has been stated above, however, this source of error has been largely eliminated by taking large length classes. The range of variation in length due to degree of contraction, when the worms are fixed in the way described, is very probably considerably less than one length class unit of 2·5 cm. The conclusion, that the earthworm is more variable in length than in number of somites, may seem at first sight paradoxical, but it is really no more so than to conclude, what is very obviously the fact, that man is more variable in respect to sitting height than in respect to number of vertebrae. It simply indicates that in the earthworm we have two kinds of growth occurring together: one the addition of somites, the other increase in size of the individual somites. That these two kinds of growth are acting together will be clear when the correlation surface between total number of somites and length of body is examined.

(*c*) The average length of the individual somite in this sample of worms is evidently

$$\frac{191{\cdot}71}{142{\cdot}715} = 1{\cdot}34 \text{ mm.}$$

This value so obtained accords very well with observation on the single somites. Its significance from a taxonomic standpoint will be considered later.

We may turn next to the question of the correlation between total number of somites and length of body. The correlation surface is exhibited in Table III.

Computing the value of the coefficient of correlation from this table by the usual equation

$$r = \frac{S(xy)}{N\sigma_1\sigma_2},$$

we get

$$r = {\cdot}260 \pm {\cdot}028.$$

This is a rather low coefficient and indicates that, so far as can be judged from this sample, there is no very close relationship between length and number of somites in the earthworm.

It is seen at once by mere inspection that the relationship between the two characters is not the same in all parts of the correlation table. Thus the worms in the two lowest length classes (i.e. from 10 to 15 cm. in length) are not, in general, individuals having few somites, but instead these arrays centre well towards the right end of the table. The five worms having the smallest number of somites all fall in or above the third length class.

This apparently paradoxical result of low correlation between length and number of somites is evidently to be explained as a consequence of the fact that the length of a worm depends not only on the number of somites in the body, but also on the size of the individual somites. It is the same sort of result as would be expected if the sitting height and number of vertebrae in man were correlated.

This factor of increase in length of the individual somite can now be analysed somewhat more precisely, through the medium of the regression equations. For the regression of length on number of somites we have the coefficient of regression:

$$b_1 = {\cdot}260 \times \frac{3{\cdot}077}{11{\cdot}850} = {\cdot}0675,$$

and for the regression of number of somites on length the regression coefficient

$$b_1 = {\cdot}260 \times \frac{11{\cdot}850}{3{\cdot}077} = 1{\cdot}001.$$

The characteristic equations, referred to the means as origin, are

(i) $L = 9{\cdot}538 + {\cdot}0675\ S$, and

(ii) $S = 123{\cdot}525 + 1{\cdot}001\ L$,

TABLE III.

Correlation between Length of Body and Number of Somites.

Number of Somites.

Length in centimetres	79	89	93	95	99	100	102	104	105	106	107	113	114	115	118	120	122	123	124	125	126	127	128	129	130	131	132	133
10 —12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—
12·5—15	—	—	—	—	1	—	1	1	1	—	1	—	2	2	—	—	—	—	—	2	—	—	—	1	—	—	2	—
15 —17·5	1	—	1	—	—	—	1	1	—	1	1	—	—	—	—	1	—	1	1	1	1	1	—	—	—	—	3	1
17·5—20	—	2	—	1	1	1	—	—	—	—	—	1	1	1	—	2	—	1	1	2	1	—	2	—	1	3	1	1
20 —22·5	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2	—	1	—	—	—	—	—	2	2	—	1	1	2
22·5—25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—
25 —27·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
27·5—30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	2	1	1	2	1	2	3	1	1	2	1	3	3	2	5	1	2	2	5	2	1	5	4	1	4	7	4

Continued

Length in centimetres	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	159	162	164	Totals
10 —12·5	—	—	—	—	1	—	—	—	1	—	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	7
12·5—15	—	1	1	1	—	—	1	1	1	2	1	3	4	4	4	2	3	1	3	1	—	—	1	—	—	—	1	50
15 —17·5	—	—	1	—	2	1	—	1	7	1	3	10	6	5	4	5	8	1	1	—	2	—	—	—	—	1	—	75
17·5—20	1	4	3	3	6	3	2	6	7	6	8	14	8	8	7	6	18	3	5	2	1	3	3	2	—	1	—	153
20 —22·5	2	—	1	4	1	2	2	6	8	8	10	7	10	7	8	11	11	6	3	6	6	1	2	2	1	—	—	137
22·5—25	—	—	—	1	—	—	—	3	3	4	2	2	2	4	5	2	10	3	2	1	—	4	4	—	1	—	—	55
25 —27·5	—	—	—	—	—	—	—	—	—	—	—	—	1	3	1	—	—	1	1	2	—	—	—	—	—	—	—	9
27·5—30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	3	5	6	9	10	6	5	17	27	21	24	37	32	31	29	26	51	15	15	12	9	9	10	4	2	2	1	487

where L denotes the length of the body in centimetres, and S the total number of somites. The probable error made in estimating length from number of somites is 2·004 cms., and in estimating somite number from length the probable error is 7·718 somites.

From (i) it is seen that an increase of one somite is associated, on the average, with an increase in length of the worm of ·6 mm. But, as has been shown above, the average length of an individual somite is more than twice this amount (1·34 mm.). Now if the individual somites did not increase in size after they are added the value given in equation (i) and the average value obtained by dividing the total length by the number of somites should agree. The values actually obtained lead us to conclude that the individual somite after being laid down about doubles in length. This result is of interest as showing the quantitative relation of the two factors considered in the growth of this simple segmented animal.

The regressions are sensibly linear.

3. *Variation and Correlation in the Clitellum.*

In the discussion of this subject the term "position of the clitellum" will be used to indicate the number of somites from the anterior end of the body back to the first somite which bears any clitellar tissue. It will be recalled that "number of somites in the clitellum" is taken to include all somites on which any clitellar tissue is present.

The frequency distributions for these two characters are given in Tables IV. and V. respectively.

TABLE IV.

Position of Clitellum.

Number of Somites	Frequency
29	1
30	117
31	376
32	1
Total	495

TABLE V.

Somites in Clitellum.

Number of Somites	Frequency
6	394
7	84
8	17
Total	495

The very small range of variation in both cases is noteworthy. In the case of the position of the clitellum practically all the individuals are included within a range of two classes. The number of somites in the clitellum with a total range of variation of only three somites has approximately four-fifths of the individuals in a single class.

The variation constants for position of clitellum deduced from Table IV. have the following values:

Mean = 30·762 ± ·013 somites.
Standard Deviation = ·436 ± ·009 somites.
Coefficient of Variation = 1·416 ± ·030 °/₀.

For the number of somites in the clitellum the values are:

Mean = 6·238 ± ·015 somites.
Standard Deviation = ·500 ± ·011 somites.
Coefficient of Variation = 8·019 ± ·173 °/₀.

It is at once evident that the variation in respect to number of somites in the clitellum is significantly greater, as indicated both by the standard deviation and the coefficient of variation, than the variation in the position of the clitellum. The difference between the coefficients of variation in the two cases is very large (6·603 °/₀). The difference between the two standard deviations is absolutely small (·064), but relatively it is large enough to be significant, being more than three times its probable error (which is ± ·014). The probable reason for the difference in variability between these two characters will be discussed later.

Turning now to the correlation between position and number of somites in the clitellum, we have the following table:

TABLE VI.

Correlation between Number of Somites in Clitellum and Position of that Organ.

Position of Clitellum.	Number of Somites in Clitellum.			
	6	*7*	*8*	Totals
29	—	1	—	1
30	39	62	16	117
31	354	21	1	376
32	1	—	—	1
Totals	394	84	17	495

The value of the coefficient of correlation deduced from this table is

$$r = -\cdot 629 \pm \cdot 018.$$

This rather high negative value for the coefficient is of considerable interest from the morphogenetic standpoint. It shows that as the number of somites anterior to the clitellum increases, the number included in the clitellum itself tends to decrease, and *vice versâ*. Or in other words, it means that the posterior end of the clitellum is a definitely fixed morphological point, subject to very little variation.

That the posterior end of the clitellum shows the least variation of any point in connection with this organ is further shown by direct measurement. The frequency distribution concerned is as follows:

Number of Somites to posterior end of Clitellum	Frequency
36	40
37	416
38	38
39	1
Total	495

The mean deduced from this is 37 somites and the standard deviation ·407, giving a coefficient of variation of 1·100 %, which is significantly lower than that for the position of the anterior end of the clitellum.

This relatively fixed position of the posterior end of the clitellum is very probably associated with the fixed position of the openings of the male ducts and the other parts of the reproductive system in this form. The clitellum is to be regarded as an accessory organ, at least, of the reproductive system. It has been shown by teratological work that the various portions of the reproductive system proper vary, so far as position is concerned, as a unit*. Whenever one part develops in another somite than that in which it normally occurs all the other parts appear in abnormal positions such that the normal relations between them are preserved. Beddard† has shown that in the highly variable form *Perionyx* the same sort of relationship holds between the position of the clitellum and the other organs of the reproductive system proper.

We may now turn to the consideration of the exact form which the clitellum takes in different cases. It was found during the collection of the data, that the clitellum did not, in all cases, completely include all the somites which it touched. In other words, it was rather frequently observed that clitellar tissue would only partially cover a somite at the anterior or the posterior end of the organ, or, in rare cases at both ends. It is of interest to know how frequently this condition occurs, and accordingly we have prepared a table to bring out this point. The following method was used in recording clitellum form: in case the clitellar tissue ended at both extremities of the organ, precisely at intersegmental grooves, two parallel, unbroken vertical lines were made, with a figure indicating the number of somites in the clitellum between them. Thus |6| indicates that the clitellum

* Cf. Bateson, *Materials for the Study of Variation*, pp. 159—167, and a paper by the present writer in the *Anat. Anz.* Bd. XVIII. pp. 123—127 describing a case in which this close interconnection of the parts in respect to position is very clearly shown.

† *Proc. Zool. Soc.* 1886, pp. 298—314.

contains six somites and ends anteriorly and posteriorly in an intersegmental groove. Whenever the clitellar tissue overlapped so as to cover only a part of a somite at one end or the other, either the left or right vertical line was dotted, according as the overlapping occurred at the anterior or posterior end of the clitellum. Thus ⋮6| indicates that the clitellar tissue is present on six somites, and that it only covers a portion of the most anterior one of these six somites. |6⋮ signifies that it is the most posterior somite which is only partially covered, and ⋮6⋮ indicates that overlapping occurs at both ends.

In Table VII. are given the results of a determination of clitellum form in 495 worms, the form classes being arranged in the descending order of the frequency of their occurrence.

TABLE VII.

Clitellum Form.

Form Class	|6|	|7|	|8|	⋮7|	⋮6|	|7⋮	⋮8⋮	|8⋮	⋮7⋮	|6⋮	Total
Frequency	389	72	14	9	4	2	2	1	1	1	495

In 95·9 % of all the worms the clitellum ends exactly in an intersegmental groove at both ends. In the remaining 4·1 % it will be seen that overlapping occurs much more frequently at the anterior than at the posterior end of the clitellum. Thus in only 1·4 % of all the worms is there any overlapping at the posterior end, including the three cases, or ·6 % of the whole number, in which overlapping occurs at both ends of the clitellum.

So far as may be judged from the present sample of worms, then, it appears that (1) clitellar tissue does not extend over more than eight whole somites; (2) in a majority of cases it extends over exactly six whole somites; (3) in the great majority of cases clitellar tissue ends exactly on intersegmental grooves, both anteriorly and posteriorly, and (4) when overlapping does occur, it is more apt to be at the anterior than at the posterior end of the clitellum.

Now this spreading out or "overlapping" of the clitellum beyond the limits set by intersegmental grooves would apparently indicate a growth in longitudinal extent of the clitellum after it is first laid down. It seemed very desirable to determine exactly whether any such growth actually does occur, because it is usually supposed that the clitellum is an organ of definite extent which does not change so far as size is concerned. The matter can be definitely settled by determining whether large worms have extensive clitella, or, in other words, whether extent of clitellum and length of worm are sensibly correlated.

In order to test this matter we have made use of Pearson's recently published *Method of Contingency**. This extension of the theory of correlation makes it

* "Math. Cont. to the Theory of Evolution, XIII." *Drapers' Company Research Memoirs*, Biometric Series I. pp. 35, Two plates.

possible to deal with a variety of problems which could not before be attacked by biometric methods on account of inherent peculiarities of the data.

At the outstart the following contingency table (Table VIII.) between length of worm and extent of clitellum was formed:

TABLE VIII.

Length. Class unit 2·5 cm.

Form class—Clitellum.	C−	C+	D−	D+	E−	E+	F−	F+	Totals
|6|	5	35	43	129	120	38	8	1	379
⋮6|	—	—	1	—	2	—	—	—	3
|6⋮	—	—	—	—	1	—	—	—	1
|7|	2	14	23	19	8	6	—	—	72
⋮7|	—	—	—	3	1	3	1	—	8
|7⋮	—	—	—	—	—	2	—	—	2
⋮7⋮	—	—	—	—	—	1	—	—	1
|8|	—	—	6	4	2	2	—	—	14
|8⋮	—	—	—	—	—	1	—	—	1
⋮8⋮	—	—	—	—	1	—	—	—	1
Totals	7	49	73	155	135	53	9	1	482

This gives a table of 80 sub-contingencies, a comparatively large number for this method. Following the notation of Pearson in the memoir referred to, we find for the square contingency from this table

$$\chi^2 = 101{\cdot}5385,$$

whence the mean square contingency

$$\phi^2 = {\cdot}2107.$$

The mean square contingency coefficient

$$C_1 = \sqrt{\frac{\phi^2}{1+\phi^2}},$$

hence equals ·4172.

From the same grouping

$$\psi = {\cdot}1167,$$

whence, interpolating by the diagram in Pearson's Table I., the mean contingency coefficient

$$C_2 = {\cdot}34.$$

It is at once evident that these values do not approach very closely the relation $C_1 = C_2$, which should be the case if the proper grouping is used in the

table and the correlation is normal. It seemed probable that the discrepancy was due to too fine grouping in the table. In this particular case with the grouping adopted a considerable number of compartments have no observed occurrences falling in them.

With the hope of bettering the results a somewhat coarser grouping was tried, the table then taking the form shown in Table IX.

TABLE IX.

Length. Class unit 2·5 cm.

Form class—Clitellum.	*C*−	*C*+	*D*−	*D*+	*E*−	*E*+	*F*−	*F*+	Totals
|6| and ⋮6|	5	35	44	129	122	38	8	1	382
|6⋮ and |7|	2	14	23	19	9	6	—	—	73
⋮7| and |7⋮	—	—	—	3	1	5	1	—	10
⋮7⋮ and |8|	—	—	6	4	2	3	—	—	15
|8⋮ and ⋮8⋮	—	—	—	—	1	1	—	—	2
Totals	7	49	73	155	135	53	9	1	482

This gives only 40 sub-contingencies and a correspondingly larger number of the compartments are filled with observations. With this grouping we find the following values:

$$\chi^2 = 70{\cdot}615,$$
$$\phi^2 = \ {\cdot}1465,$$
$$C_1 = \ {\cdot}3575.$$

For the mean contingency from the same grouping we find:

$$\psi = {\cdot}1093,$$

whence $C_2 = {\cdot}33.$

This evidently comes fairly close to satisfying the relation $C_1 = C_2$, and further manipulation of the grouping seems hardly worth while. We may conclude, then, that there is a very sensible degree of correlation between the size of the worm and the extent of the clitellum, the value of r being about ·357 with a probable error of somewhat less than ± ·054.

This result implies that growth occurs in the clitellum after it is formed. Putting all the results together we are able to form a fairly complete and connected picture of the morphogenetic phenomena connected with the position and extent of this organ. These phenomena we conceive to have probably the following relations: (*a*) the clitellum develops at such a point on the body that its posterior end lies a definite and, except in teratological cases, a fixed number of somites behind the openings of the genital ducts. (*b*) The clitellum when formed has a

variable number of somites but as a consequence of (*a*) any increase in the number of somites in this organ must necessarily entail a decrease in the number of somites between the anterior end of the worm and the anterior end of the clitellum. Hence a negative correlation between these two characters arises. (*c*) There is a tendency for the clitellum after being formed to increase its extent by growth. This growth tends to extend the clitellum both anteriorly and posteriorly. In a sample of worms non-homogeneous with respect to age any extension in the posterior direction tends to increase the variation in the number of somites in the clitellum without correspondingly affecting the amount of variation in the number of somites *lying in front of* this organ. (*d*) The increase in extent of the clitellum by growth is more marked at the anterior than at the posterior end of the organ.

If this analysis be correct the factors concerned in the form taken by the clitellum after it is first laid down are fairly simple and understandable. The problem as to what causes the clitellum to originally appear in the precise region where it does is not so simple however. It is a particular case falling under the general problem of the "localization of morphogenetic phenomena." While natural selection is capable of accounting for the facts when the worm is entirely normal, this explanation fails to account for such cases as those decribed by Bateson for *Perionyx* and referred to above, where the position of the clitellum bears the same relation to the other organs of the reproductive system, whatever its absolute position.

4. *The Taxonomic Position of the Form Studied.*

It was supposed that the sample of worms studied in connection with this work was sufficiently large to enable a very precise determination of the systematic position of the form to be made. The constants obtained for the various characters from this collection were compared with the values given by Beddard* for the same characters in the various species of *Lumbricus*. Somewhat to our surprise it was found that our specimens differed widely in some points from the condition found in any described species of the genus. The worms come nearest to *L. herculeus*, Sav., and should evidently be classed with, or at least very close to this species. The points of distinction and agreement are shown in the following table:

Character	Ann Arbor form	*L. herculeus* Sav. (as given by Beddard)
Number of Somites ...	Mean 142·715	180
Length	„ 191·7 mm.	360 mm.
Somites to Clitellum ...	„ 30·76	31
Somites in Clitellum ...	„ 6·24	6

(Other diagnostic characters in agreement in the two cases.)

* *A Monograph of the Order of Oligochaeta.* Oxford, 1895, pp. 721—724.

In position and extent of the clitellum there is essential agreement between the Ann Arbor form and the typical *L. herculeus*. In size and number of somites, however, there is a striking discrepancy. It would appear that the earthworms typical for this region approach towards a dwarf condition of the type of the species, being on the average only a little more than half as long and having about 40 fewer somites. The typical *L. herculeus*, according to the above figures, has for the single somite an average length of 2 mm., as against 1·34 mm. for our sample. Applying equations (i) and (ii) (p. 219) we find the mean length of an array of worms each having 180 somites, 216·9 mm. instead of 360 mm., as given for *L. herculeus*, and for the mean number of somites for an array having a length type of 360 mm., 159·6 as against 180.

Is this Ann Arbor form to be regarded as a true dwarf variety, or is the small size to be explained in some other way? Several possible explanations occur to one but none of them seems adequate. In the first place it might be maintained that the specimens in our sample were young, and had not completed their growth. This is, undoubtedly, true for some of the specimens in the sample. The earthworm continues to add somites for a long period, possibly throughout its life. If such addition does occur throughout the life of the worm it must, after a time, be at a very slow rate. This is evidenced by the fact that in the present sample there is no tendency towards extreme extension of the range at the upper end. The form of the frequency polygon for these worms, as has been pointed out above, indicates that the majority have either stopped growing or are growing at a very slow rate.

Another possibility is that in the collection of these worms an unconscious and unavoidable selection was made on the basis of size. This might arise in the following way. Suppose it were a habit of very large (old) worms not to leave the burrows on rainy nights. Then it would necessarily result that any collection made in the way the present one was made, would contain no or few very large worms. Now it is possible that this is the case, but it seems to me hardly probable, and for the following reason: in the large number of earthworms which have been collected for class-work in this laboratory during the last five years, including several thousand individuals at least, specimens have not been found of twice the size of the average of the present collection. Furthermore worms collected by digging are of no larger size than those collected in the way described in this paper*. Finally no student of earthworm habits has recorded, so far as I am able to find, any tendency towards a habit of the sort mentioned.

Again it might be thought that the relatively small size of the worms in this sample as compared with the type of the species was due to the method of killing and fixing. That this cannot be the case is shown, first by comparison with living

* These last two facts also argue strongly against the first point raised; viz., that the worms in the present sample are all young specimens. Surely in five years collecting a fair number at least of old worms would be found.

worms from this same locality, and secondly because such an explanation would not in any way account for the smaller *number* of somites.

Altogether, it must be concluded that the typical form of *Lumbricus* of this locality is very considerably smaller and contains fewer somites than the type of the species to which it is most closely related; viz. *L. herculeus**. There is no evidence available as to what is the cause of this dwarfing, but it is not unlikely that it is due to direct environmental influence.

5. *Summary.*

A study of variation and correlation in several characters of the earthworm *Lumbricus herculeus*, Sav., leads to the following conclusions:

1. The earthworm is more variable in length of body than in number of somites.

2. There is a rather low degree of correlation ($r = \cdot 260$) between these two characters.

3. The results just stated (1 and 2) arise from the fact that the organism increases in length as a consequence of the combined action of two processes, (*a*) the addition of new somites, and (*b*) the growth of somites already added.

4. There is greater variability in the number of somites in the clitellum than in the number of somites lying in front of that organ. The factors concerned in this greater variability are analysed.

5. The number of somites included by the clitellum is negatively correlated to a relatively high degree with the number of somites making up the portion of the body anterior to that organ.

6. This implies that the posterior end of the clitellum is a relatively fixed point. This conclusion is confirmed by direct measurement.

7. There is a strongly pronounced tendency for the clitellum to end both anteriorly and posteriorly exactly at intersegmental grooves. Overlapping occurs more frequently at the anterior than at the posterior end of the organ.

8. The clitellum increases in extent as the worm grows in length.

9. The form of *Lumbricus* found at Ann Arbor is shorter and contains fewer somites than the typical *L. herculeus*, Sav.

In conclusion I wish to acknowledge my indebtedness to the officials of the Carnegie Institution for a grant to aid in the carrying on of certain biometrical work of which this paper forms a part.

* There remains, of course, still another possibility, namely, that the values for the type of the species are based on a few unusually large specimens, but if this is the case it is difficult to understand why some one has not called attention to the matter. Especially does this seem remarkable when we remember that there is no organism more universally used as a type in zoological laboratories.

MISCELLANEA.

I. Per la risoluzione delle curve dimorfiche.

NOTA DEL DR FERNANDO DE HELGUERO, Roma.

Il Prof. Karl Pearson ha trattato il problema della decompozizione di una curva dimorfica risultante da componenti normali in una memoria inserita nelle *Phil. Trans.* Vol. 185 A, pp. 71–110.

Egli riduce tutta la difficolta del problema alla risoluzione del sistema:

$$\begin{cases} \mu_3^2 - 4\mu_3 p_3 - 2p_3^2 - \lambda_4 p_2 + 6p_2^3 = 0 \ldots\ldots\ldots\

Siano μ_2, μ_3, μ_4, μ_5 i momenti della curva complessa rispetto all' asse delle ordinate, a l' area.

Si calcolino: $$\nu_4=\frac{a\mu_4-3\mu_2^2}{\mu_3^{\frac{4}{3}}a^{\frac{2}{3}}}, \quad \nu_5=\frac{a\mu_5-10\mu_2\mu_3}{\mu_3^{\frac{5}{3}}a^{\frac{1}{3}}}.$$

Si trovino p_2', p_3' per mezzo del sistema (A), o p_2' per mezzo della nonica (B) e p_2' per mezzo della

$$p_3'=\frac{-8p_2'^3+3p_2'(2\nu_4+p_2'\nu_5)+2}{2p_2'^3+3\nu_4p_2'+4}. \quad \text{..... (C)}$$

Noti p_2', p_3' si calcoli $p_1'=\frac{p_3}{p_2'}$, poi γ_1 e γ_2 che sono le radici della

$$\gamma^2-p_1'\gamma+p_2'=0.$$

I parametri delle curve normali componenti sono allora date dalle formule:

$$b_1=\gamma_1\sqrt[3]{\frac{\mu_3}{a}}, \quad b_2=\gamma_2\sqrt[3]{\frac{\mu_3}{a}},$$

$$c_1=\frac{a\gamma_2}{\gamma_2-\gamma_1}, \quad c_2=\frac{a\gamma_1}{\gamma_1-\gamma_2},$$

$$\sigma_1^2=\frac{\mu_2}{a}+b_1^2\left\{\frac{1}{3\gamma_1}\left(2p_1-\frac{1}{p_2}\right)-1\right\}, \quad \sigma_2^2=\frac{\mu_2}{a}+b_2^2\left\{\frac{1}{3\gamma_2}\left(2p_1-\frac{1}{p_2}\right)-1\right\}.$$

L' equazione (B) può ammettere più radici reali, così il sistema (A). Per il nostro problema interessano solo le soluzioni con p_2' negativo. A ciascuna soluzione corrisponde una coppia di curve normali componenti.

Riguardo al numero di queste soluzioni si osservi che esso è sempre pari: se $\nu_4>-7$ si hanno o nessuna o due soluzioni; se $\nu_4<-7$ esistono, al più, quattro soluzioni, due sono certo reali se $\nu_4<-5{\cdot}94587$.

La dimostrazione di questi risultati, come pure dei metodi per la risoluzione del sistema (A), si troveranno in una speciale memoria che verrà pubblicata altrove.

[In this *Note* μ_n stands for the nth moment about the centroid of the frequency system—the notation of my memoir of 1893—and not $a\mu_n$ the more usual notation now. K.P.]

II. Albinism in Sicily. A further Correction.

In *Biometrika*, Vol. III. Pt. IV. I pointed out that Dr Arcoleo states explicitly that he never met with any case of an albino child being born to an albino parent, and consequently that Professor Weldon's use of this authority to prove the production of two pigmented children (+3 albinos) by albino parents was erroneous.

Professor Pearson replied that the meaning of the explicit statement I quoted from Arcoleo was not clear to him, and that "Dr Arcoleo states explicitly that the mother of the four albinos in Family No. 6 was 'una albina di belle forme,' &c."

Happily we need not examine the various discrepancies and self-contradictions which Professor Pearson's version would presuppose in Arcoleo's table and text. It is merely a

mis-translation. The beautiful "*albina*" was not the mother of Family No. 6, but the *friend* of the mother. The text reads:—

> ....." Il Cav. N. N. senti grande simpatia per una albina di belle forme e desiderò avere una figlia che le rassomigliasse. Con tale scopo indusse la moglie a farlese amica, e già ambedue vagheggiarono la medesima idea. Il desiderio fu soddisfatto colla nascita di una bella bambina; ma moriva a due anni, &c."

The words "indusse.....amica" mean that he induced *la moglie*, his wife to make friends with the "*albina*"; literally, *far*, to make, *le*, her [the *albina*], *amica*, a friend, *si*, to herself. Arcoleo evidently attributes the albino births to "sympathy," or other mysterious influence, in which case he might well call the occurrence a "fatto curiosissimo." Nevertheless his memoir is perfectly consistent, save for a numerical slip in one place, and when he wrote that he knew no instance of hereditary albinism he meant no more or less than he said, doubtful as this has seemed to Professor Pearson.

W. BATESON.

CAMBRIDGE, 27 *Feb.* 1905.

[I am sorry that I overhastily read Arcoleo's paragraph and thus stated that Cav. N. N. made an *amica* of the "fair albino." It is clear that he caused his wife to make a friend of her with the object that his wife and not the albino lady should provide the albinotic daughter. Arcoleo's consistency is thus established, and I can only hope that his powers of scientific observation are not detrimentally influenced by the capacity thus exhibited for archaic belief. K.P.]